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[PREPARED IN THE ORDNANCE COLLEGE.]

TEXT BOOK

OF

GUNNERY.

PART II.

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ERRATA.

Page 15, line 4.—Δ^{2r-1} *should read* $\Delta^{2r-1}t_{s-(r-1)l}$.

Page 15, line 9 from bottom.—(3) *should read* (20).

Page 24, line 5.—0·7799 *should read* 0·07809.

Page 26, line 13 from bottom.—t_n *should read* t_2.

Page 28, line 4.—(14) *should read* (17).

Page 36, line 7. $\frac{180g}{\pi}\frac{g}{v}dt$ *should read* $-\frac{180}{\pi}.\frac{g}{v}.dt$.

Page 47, line 9.—*Dele* "$-\mathrm{B}_a$."

Page 48, line 16.—*For* $+\mathrm{B}_a$ *read* $-\mathrm{B}_a$.

Page 66, in diagram.—y_1 *is* length BC and *not* BE.

Page 72, line 2 from bottom.—*Read* "U = " *instead of* "V =."

Page 76, line 9.—*Dele* "pseudo-."

Page 88, line 15 from bottom.—"Entering" *should read* "referring to."

Page 94, line 21.—*Read* $0{\cdot}7987\epsilon$ *instead of* $0{\cdot}9024\epsilon$.

TEXT BOOK OF GUNNERY.

CHAPTER I.

THE RESISTANCE OF THE AIR.

In Chapter II., Part I., of the Text Book of Gunnery, the practical importance of a Ballistic Table has been illustrated by various examples, in which, by the use of the table, the solution was effected of a Ballistic Problem; it is necessary now to examine the theory upon which a Ballistic Table is based, and the experimental data upon which the calculation is founded.

The first requirement is the experimental determination of the *Resistance of the Air* to a projectile moving with a velocity within the limit of that found useful in artillery fire.

The most important series of experiments carried out in this country are those of the Rev. F. Bashforth, B.D., the first Professor of Mathematics to the Advanced Class of Artillery Officers.

These experiments were conducted in 1865–1870 and in 1878–1879, and the results are tabulated in the *Reports on the Experiments made with the Bashforth Chronograph*, &c., 1865–1870; and *Final Report on Experiments with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Elongated Projectiles*, 1878–1880.

The projectiles employed in these experiments were of various weights and sizes, and were fired from guns of 3, 5, 6, 7, and 9 inches calibre; the external shape was nearly uniform for all, consisting of a cylindrical body with a flat or slightly rounded base, and provided with an ogival-pointed head, struck with a radius of $1\frac{1}{2}$ diameters.

In 1902–1906 the Ordnance Committee (now the Ordnance Board), with Sir G. Greenhill as technical adviser, carried out experiments to determine the resistance of the air to the motion of elongated projectiles, struck with a radius of 2 diameters. The Ballistic Tables at the end of this book are founded upon the data obtained from these experiments, and these tables are used in the present text. Previous editions (containing the Bashforth ballistic tables) were written by Sir G. Greenhill, to whom the scientific development of gunnery owes so much.

As a first result of the experiments it was found that the resistance was proportional, at the same velocity, to the cross-sectional area or to the square of the diameter.

The resistance R can thus be split up into two factors, one of which is d^2, where d denotes the diameter of the shot in inches; and the other is the resistance of the air at the same velocity to a similar 1-inch projectile; this is denoted by p, so that

$$R = d^2p.$$

These values of p refer to a certain standard density of the air, of 534·22 grains per cubic foot, which is the density of dry air at sea-level, in the latitude of Greenwich, at a temperature of 62° F. and a barometric height of 30 inches.

Assuming that a projectile travels during its flight so that its axis coincides with the tangent of the trajectory, then the forces acting on the projectile are the force of gravity, vertically downwards, and considered as constant; and the variable resistance of the air which acts continually in a direction opposite to that of the motion of the projectile. Experiment

shows that this resistance is proportional to the cross-section area or square of the diameter of the projectile; hence, putting $p = f(v)$, where $f(v)$ is a function of the velocity of the shot,

$$R = d^2 p = d^2 f(v).$$

It is assumed further that the resistance is proportional to the density of the air; so that if the density changes to δ grains per cubic foot, we must put $R = \tau_0 d^2 p$, where

$$\tau_0 = \frac{\delta}{\Delta},$$

and

$$\Delta = 534 \cdot 22.$$

Therefore

$$\tau_0 = \frac{\delta}{534 \cdot 22},$$

and Table VIIA, calculated from the formula

$$\tau_0 = \frac{b}{30} \cdot \frac{460 + 62}{460 + F} = \frac{b}{30} \cdot \frac{1}{1 + \frac{F - 62}{522}} \quad . \quad . \quad . \quad (1),$$

derived from the laws of Boyle and Charles, gives the value of τ for different Fahrenheit temperatures F and barometric heights b inches; this applies to dry air, so that a further correction is required from the hygrometrical tables given by the readings of the wet- and dry-bulb thermometers, as damp air is perceptibly lighter than dry air at the same temperature and pressure; the air is supposed to be two-thirds saturated, so that a pressure two-thirds of the pressure in inches of mercury of the aqueous vapour at the temperature F is added. The effect of the vapour correction is to reduce the standard temperature from 62° to 60° F.

This factor τ_0 is called the *coefficient of tenuity at the ground level*, but if a projectile attain any considerable height above ground, then a correction in τ_0 must be made to allow for the decrease in density (and temperature) in the air above the ground; a good average, as shown in Chapter III, for the mean density of air of the whole trajectory is obtained by considering the average height h of the trajectory to be at two-thirds of the maximum height, hence the working formula for the coefficient of tenuity for a trajectory is

$$\tau_h = \frac{\tau_0}{f}, \quad \text{so that} \quad f = \frac{\tau_0}{\tau_h},$$

where f is *the altitude factor*, which corrects for a mean height above ground of h feet, f is 1 at the ground level or near it, and is greater than 1 for higher levels. Hence

$$R = \frac{\tau_0}{f} d^2 p = \tau_h d^2 p \quad . \quad . \quad . \quad (2).$$

In all carefully conducted experiments the value of τ_0 should be calculated and allowed for from day to day.

For shooting under water, the coefficient of tenuity becomes 800, because the density of water is about 800 times that of ordinary air.

The extract from Glaisher's *Hygrometrical Tables*, Table VII at the end of the book, gives δ, the density in grains/ft.^{3}, for the reading of the barometer in inches, and of the wet- and dry-bulb thermometer in degrees Fahrenheit of the meteorological record on the day of an experiment, and hence the value of

$$\tau_0 = \frac{\delta}{\Delta}.$$

In using the hygrometric table, look out the number corresponding to the readings of the thermometer, wet and dry bulb; this gives the grains per cubic foot at a barometric height of 29 inches; and then the table of proportional parts shows the addition to be made

for the extra height of the barometer above 29 inches. The standard barometric height in the table is taken very low, at 29 inches, so as to avoid negative proportional parts.

As examples, find the value of τ_0 when the meteorological record is

Barometer	... ...	29·95
Wet bulb	... ...	37° F.
Dry bulb	... ...	39° F.

$$\left\{\begin{array}{l} 538\cdot 9 \text{ for } 29 \text{ inches,} \\ +\ 16\cdot 7 \text{ for } 0\cdot 9 \text{ difference,} \\ +\ \ 0\cdot 9 \text{ for } 0\cdot 05 \text{ difference,} \end{array}\right.$$

Value of $\delta = 556\cdot 5$.

Therefore

$$\tau_0 = \frac{\delta}{\Delta} = \frac{556\cdot 5}{534\cdot 22} = 1\cdot 042.$$

For

Barometer	... ...	30·25
Wet bulb	... ...	42° F.
Dry bulb	... ...	45° F.

$$\left\{\begin{array}{r} 532\cdot 3 \\ 18\cdot 3 \\ 3\cdot 7 \\ 0\cdot 9 \end{array}\right.$$

$$\delta = 555\cdot 2$$

$$\tau_0 = \frac{555\cdot 2}{534\cdot 22} = 1\cdot 039.$$

To find the value of the altitude factor f (see Note at end of Chapter).

1*st Case.*—If the decrease in density of the air at a height h feet is taken as due only to the decrease in the pressure of the air, the temperature being supposed uniform, then

$$f = e^{\frac{h}{k}} \text{ (see Note at end of Chapter)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where k is the height of the homogeneous atmosphere, that is, the height of an atmosphere of uniform density which will give the barometric pressure, and it is equal to 27,800 feet on the average, but varying with the temperature from 26,000 to 30,000 feet.

For a moderate value of h (3) may be written

$$f = 1 + \frac{h}{k} \quad \text{and} \quad \frac{1}{f} = 1 - \frac{h}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

where $k = 27{,}800$.

2*nd Case.*—Experiments show that the temperature, as well as the pressure, decreases with the increase of altitude. Considering *dry air only*, suppose that the temperature at any height is that which the surface air would have if it expanded adiabatically (*i.e.*, without loss or gain of heat) from the surface, then the value of f is given by

$$f = \left(1 - \frac{\gamma - 1}{\gamma}\frac{h}{k}\right)^{-\frac{1}{\gamma - 1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

where $\gamma = 1{\cdot}4$ is the ratio of the specific heat of dry air at constant pressure to its specific heat at constant volume (see Note at end of Chapter), and with $\gamma = 1{\cdot}4$,

$$f = \left(1 - \frac{h}{97{,}300}\right)^{-2\cdot 5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Table VIIA gives the value of τ_0, and the value of f found for values up to $h = 10{,}000$ feet, these are plotted in fig. 1, when the ground temperature is 60° F.

For moderate values of h (5) may be written

$$f = 1 + \frac{h}{\gamma k} = 1 + 0\cdot000026h,$$

where h is in feet.

3rd Case.—In 1st case no account is taken of change of temperature when ascending in the atmosphere; in the 2nd case no account is taken of the humidity of the atmosphere and the correct value for γ is probably less than $1\cdot4$; a smaller value of γ in the 2nd case would bring closer together the values of f obtained by the 1st and 2nd cases.

A well-known and practical rule* is that there is :—

1 inch in fall of barometer per 1000 feet altitude,
1° F. fall of temperature „ 300 „ „

Working upon this and employing Table VIIA, the value of $f = \frac{\tau_0}{\tau_h}$ is readily obtained and then plotted for various values of h feet.

On fig. 1 the curve of f for various values of h is seen as obtained from the three cases considered, that got from the 3rd case is seen to be a mean between the other two, it has therefore claims for being considered suitable, and, if employed, there is no need of a separate table for f, but it is got direct from Table VIIA as soon as the barometer and thermometer readings at the ground surface are known.

As an example, suppose a shot attain a maximum height of 900 feet and that the mean height of the trajectory is taken as 600 feet. To correct for this, when the readings at the earth's surface are

$$\left.\begin{matrix} 30'' \\ 56^\circ \text{ wet bulb} \\ 60^\circ \text{ dry bulb} \end{matrix}\right\},$$

neglecting the wet-bulb reading, $\tau_0 = 1$ from Table VIIA, and from the same table τ_h for

$$\left.\begin{matrix} 30 - \cdot6 = 29''\cdot4 \\ 60^\circ - 2^\circ = 58^\circ \end{matrix}\right\} \text{is equal to } 0\cdot9836.$$

So that

$$f = \frac{\tau_0}{\tau_h} = \frac{1}{0\cdot9836} = 1\cdot016.$$

The following tabulated values of f are now set out for each rise of 1000 feet.

---

* Captain R. K. Hezlet, R.A., finds that, from an examination of the British Association Report on the upper atmosphere (Winnipeg, 1909), a better value for the mean temperature gradient up to 5000 feet altitude would appear to be 2·5° F. per 1000 feet. Taking this rate of decrease of temperature and 1 inch fall of barometer per 1000 feet, it will be found that for barometer 30″, thermometer 60° F. on ground, up to 5000 feet,

$$\frac{1}{f} = \tau_h = 1 - 0\cdot000027\,h.$$

This gives slightly higher values than in the table below.

Altitude Correction f in Ballistic Coefficient.

$$C = f\frac{w}{\kappa\sigma\tau_0 d^2}; \quad f = \frac{\tau_0}{\tau_h}.$$

Value of f at 60° F. calculated from τ_0 and from Table VIIA, allowing:—

1 inch fall of pressure per 1000 feet of altitude,
1° F. fall of temperature per 300 feet of altitude.

h feet.	f.	Differences. Δ
0	1·000	
		0·028
1,000	1·028	
		0·029
2,000	1·057	
		0·032
3,000	1·089	
		0·034
4,000	1·123	
		0·037
5,000	1·160	
		0·040
6,000	1·200	
		0·044
7,000	1·244	
		0·047
8,000	1·291	
		0·053
9,000	1·344	
		0·056
10,000	1·400	

Working with Glaisher's table to find f for a height of 600 feet, and supposing the wet-bulb fall of temperature to be the same as that of the dry-bulb temperature, then for surface readings of

$$\left.\begin{array}{l}30''\\56^\circ\\60^\circ\end{array}\right\}\delta_0 = 515\cdot9 + 17\cdot8 = 533\cdot7,$$

and these readings at 600 feet elevation become

$$\left.\begin{array}{l}29''\cdot4\\54^\circ\\58^\circ\end{array}\right\}\text{for which } \delta = 518\cdot1 + 7\cdot1 = 525\cdot2;$$

so that

$$f = \frac{533\cdot7}{525\cdot2} = 1\cdot016.$$

The resistance of the air is reduced considerably in a modern projectile by giving it a greater length and a sharper point; and a factor κ, called the *coefficient of shape*, is brought in to allow for this change.

For an ogival head of 2 calibres $\kappa = 1$ in the present new ballistic tables (at end of book) and for other shaped heads

$$\kappa = \frac{2}{m}\sqrt{\frac{4m-1}{7}}, \text{ approximately,}$$

where m is the number of calibres in the radius of the ogival head.

For a 3-calibre ogive

$$\kappa = \tfrac{2}{3}\sqrt{\tfrac{11}{7}} = 0{\cdot}835.$$

Bashforth's tables were constructed for a $1\frac{1}{2}$-calibre ogive, whereas the present tables are for a 2-calibre ogive.

For a so-called flat-headed *proof* projectile $\kappa = 2$ on the average; this has rounded edges, and were it really flat, κ would be greater than 2.

For spherical shot, such as a shrapnel bullet, κ is not constant, and a separate ballistic table (Table IX) is constructed; but $\kappa = 1{\cdot}7$, on the average.

Lastly, to allow for the superior centring of the projectile obtained with breech-loading guns, Bashforth introduced a factor σ, called the *coefficient of steadiness.*

This steadiness may vary during the flight of the projectile, as the shot is often unsteady for some distance after leaving the muzzle, and finally steadies down afterwards, sometimes becoming unsteady again in high-angle howitzer fire.

With the present new tables and with modern breech-loading guns σ may be taken as unity ($\sigma = 1$).

Collecting all the coefficients, κ, σ, τ, we now put the resistance of the air in pounds,

$$R = nd^2p \quad \ldots\ldots \quad (7),$$

where

$$n = \kappa\sigma\tau$$

is called the *coefficient of reduction.*

Denoting the weight of the shot by w lbs., and the retardation of the shot by r f/s per second,

$$\frac{R}{w} = \frac{r}{g},$$

$$r = R\frac{g}{w} = nd^2p\frac{g}{w} = \frac{nd^2}{w}pg \quad \ldots\ldots \quad (8);$$

put

$$\frac{w}{nd^2} = C,$$

then C is called the *ballistic coefficient* of the shot, and from (8)

$$r = \frac{pg}{C},$$

that is

$$Cr = pg \quad \ldots\ldots \quad (9)$$

Until the time of Benjamin Robins, and of his invention of the Ballistic Pendulum (1740), the vaguest ideas prevailed as to the velocity of shot and the resistance of the air.

It was never realised that such an attenuated elastic medium could offer so enormous a resistance, in spite of Newton's caution (Ex Medii subtilitate resistantia projectilium celerrime motorum non multum diminuitur. *Philosophiæ Naturalis Principia Mathematica*, lib. ii, prop. xxxiii, cor. 5), so that artillerists were in the habit of neglecting this resistance, and of employing Galileo's parabolic theory for unresisted motion; and thereby the velocity of the shot was considerably under-estimated.

Thus, for instance, the velocity V required with an elevation of 9° to attain a range of 3500 yards is, according to this parabolic theory (Chapter II, § 4, Part I).

$$V = \sqrt{(gX \operatorname{cosec} 2\alpha)},$$

where $X = 10{,}500$, the range in feet, and $2\alpha = 18°$; hence

$$V = 1047 \text{ f/s}.$$

But it is found that the modern magazine rifle, with an initial velocity of 2000 f/s, can hardly attain a range of 3500 yards, whatever elevation is given; and the resistance of the air to the bullet at the outset is now estimated at about $1\frac{1}{4}$ lbs., or 40 times the weight of the bullet.

So also Robins found, in an experiment (*New Principles of Gunnery*, 1742, Chap. II, Prop. II) by firing at his ballistic pendulum at ranges of 25, 75, and 125 feet, that the mean velocities of impact were 1670, 1550, and 1425 f/s.

The musket employed was a 12-bore, so that the bullets weighed 12 to the pound; and the charge of powder was half the weight of the bullet.

Denoting by R the average resistance in pounds over the range of 100 feet during which the velocity fell from $V = 1670$ to $v = 1425$,

$$R = \frac{w(V^2 - v^2)}{2g \times 100} = 10 \text{ lbs., about,}$$

or 120 times the weight of the bullet; this may be taken as the resistance of the air to a spherical bullet of this description, $\frac{3}{4}$ of an inch in diameter, moving with the velocity of 1550 f/s, at the mean range of 75 feet.

Mr. A. Mallock, F.R.S., has repeated the Robins experiment with a ballistic pendulum similar to that described in Part I, using a 0·303-inch calibre, and attaining a maximum velocity of 4500 f/s with a light aluminium bullet weighing 23·3 grains or 300 to the lb. (*Proceedings, Royal Society*, Nov. 1904).

The loss of velocity Δv at velocity v in a range of 5 yards, or $\Delta s = 15$ feet, was about 700 f/s.

So that with $v = 4500$, $\Delta v = 700$, the retardation

$$r = \frac{\Delta v}{\Delta t} = v\frac{\Delta v}{\Delta s} = 4500\,\frac{700}{15} = 210{,}000.$$

Also

$$\frac{r}{g} = \frac{R}{w} = 6500,$$

hence, at the velocity of 4500 f/s the retardation is found to be 210,000 f/s², and the resistance about 6500 times the weight of the bullet.

The conclusions of Robins naturally met with great opposition from the teachers of the ancient theory; thus, for instance, Professor Müller, in his *Treatise of Artillery, Supplement*, 1768, p. 110, proves that "the velocity from a 42-pr. can never amount to 914·7 f/s, and consequently much less in a smaller calibre."

But the experimental results, obtained by the modern method of shooting through electric screens, confirm Robins' results; and, according to Bashforth, these results of Robins, obtained from experiments with musket balls, are more accurate than those obtained 50 years later in Hutton's experiments with cannon balls and a larger ballistic pendulum.

The practical details of the construction and use of modern electro-ballistic apparatus are given in Chapter IV, § 2, Part I.

An electro-ballistic experiment consists essentially in recording the instant of time,

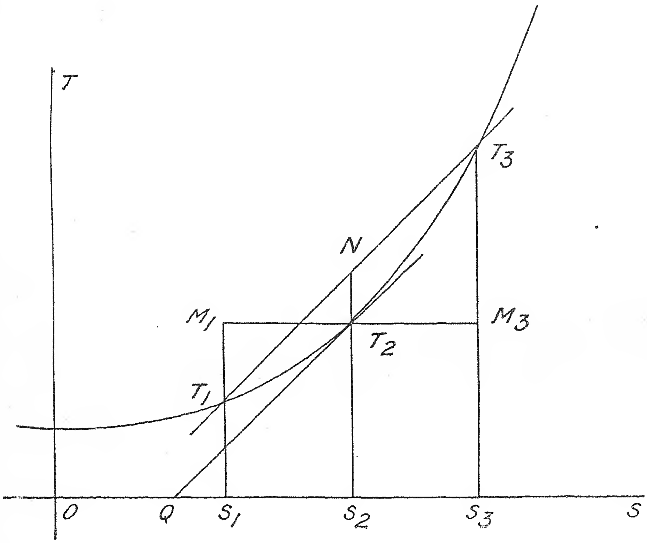
$$t_1, t_2, t_3 \ldots \text{seconds},$$

at which an electric screen at distance

$$s_1, s_2, s_3 \ldots \text{feet},$$

measured from a fixed point, is cut by the passage of a shot flying nearly horizontally.

Fig. 2.



Taking s and t as co-ordinates, a fair curve is drawn through the points

$$(s_1, t_1), (s_2, t_2), (s_3, t_3) \ldots \quad \text{(fig. 2)}.$$

And now the problem is to determine the most appropriate analytical expression for this curve, in the form

$$t = f(s);$$

and thence to derive

$$\frac{dt}{ds} \quad \text{and} \quad \frac{d^2t}{ds^2}.$$

The distance s must be taken as the independent variable in screen records, and not the time t, so that v denoting the velocity, and r the retardation,

$$\frac{dt}{ds} = \frac{1}{v},$$

$$\frac{d^2t}{ds^2} = \frac{d}{ds}\left(\frac{dt}{ds}\right) = -\frac{1}{v^2}\frac{dv}{ds} = -\frac{1}{v^3}\frac{vdv}{ds} = -\frac{1}{v^3}\frac{dv}{dt} = \frac{r}{v^3},$$

so that

$$r = -\frac{dv}{dt} = -v\frac{dv}{ds} = \frac{d^2t}{ds^2}v^3 \quad \ldots\ldots\ldots\ldots \quad (10),$$

and if w lbs. be the weight of the shot, and R the resistance of the air in pounds,

$$nd^2p = R = w\frac{r}{g} = w\frac{d^2t}{ds^2}\cdot\frac{v^3}{g} \quad \ldots\ldots\ldots\ldots \quad (11).$$

Writing $\frac{w}{nd^2} = C$, the ballistic coefficient,

$$pg = Cr = C\frac{d^2t}{ds^2}v^3 = C\frac{d^2t}{ds^2}10^9\left(\frac{v}{1000}\right)^3 = K\left(\frac{v}{1000}\right)^3 \quad \ldots\ldots \quad (12),$$

where

$$K = C\frac{d^2t}{ds^2} \times 10^9 \quad \ldots\ldots\ldots\ldots\ldots \quad (13).$$

This value of K is as defined by Bashforth, and the definition still holds good. The advantage of the notation is that in the reduction of screen records the retardation r and the resistance R is divided into two factors, one of which is the cube of the velocity, the other factor is

$$\frac{d^2t}{ds^2}, \quad \text{and} \quad w\frac{d^2t}{ds^2}\cdot\frac{1}{g} \quad \ldots\ldots\ldots\ldots \quad (14),$$

which are given immediately by the difference of the screen records, but as they are very small decimals, beginning with seven or eight zeroes, Bashforth found it more convenient to reckon velocity in thousands of f/s, and to multiply by 10^9; the formulas for r, R, are thus

$$pg = Cr = K\left(\frac{v}{1000}\right)^3 \quad \ldots\ldots\ldots\ldots \quad (15),$$

$$R = w\frac{r}{g} = w\frac{K}{Cg}\left(\frac{v}{1000}\right)^3 \quad \ldots\ldots\ldots\ldots \quad (16),$$

and

$$K = C\frac{d^2t}{ds^2} \times 10^9 \quad \ldots\ldots\ldots\ldots\ldots \quad (17),$$

where v is in f/s, r in f/s^2.

The value of $\frac{d^2t}{ds^2}$ and hence that of K must be determined from the reduction of the screen records. The method employed by Bashforth is that of *Finite Differences*, and his method is the one now in use for the determination of v and K from the screen records.

Method of Finite Differences.

In the notation of this subject, t_s or $f(s)$ denotes the value of t from a fixed point, say one of the screens, to any distance s, to a given screen, for instance; and then t_{s+l} or $f(s+l)$ will denote the value of t to any extra distance $s+l$, say to the next screen, l feet beyond; and generally, as required for the problem in hand, t_{s+nl} or $f(s+nl)$ will denote the time to the nth screen beyond the given screen, and t_{s-nl} or $f(s-nl)$ will denote the time to the nth screen in front of the given screen, the screens being spaced equally l feet apart.

Again, in the subject of Finite Differences, the symbol Δ is employed as a prefix (not as a factor) to denote the operation of differencing; and thus

$$t_{s+l} - t_s \text{ is denoted by } \Delta t_s;$$

or

$$f(s+l) - f(s) \text{ is denoted by } \Delta f(s);$$

while

$$\Delta t_{s+l} - \Delta t_s \text{ is denoted by } \Delta^2 t_s;$$

or

$$\Delta f(s+l) - \Delta f(s) \text{ is denoted by } \Delta^2 f(s);$$

$$\Delta^2 t_{s+l} - \Delta^2 t_s \text{ is denoted by } \Delta^3 t_s;$$

and so on.

Then, since

$$\Delta t_s = t_{s+l} - t_s,$$

therefore

$$\begin{aligned} \Delta^2 t_s &= \Delta t_{s+l} - \Delta t_s \\ &= t_{s+2l} - t_{s+l} - t_{s+l} + t_s \\ &= t_{s+2l} - 2t_{s+l} + t_s, \end{aligned}$$

and similarly,

$$\begin{aligned} \Delta^3 t_s &= \Delta t_{s+2l} - 2\Delta t_{s+l} + \Delta t_s \\ &= t_{s+3l} - 3t_{s+2l} + 3t_{s+l} - t_s; \end{aligned}$$

and generally, by induction,

$$\Delta^n t_s = t_{s+nl} - n t_{s+(n-1)l} + \frac{n(n-1)}{2} t_{s+(n-2)l} - \dots \qquad (18),$$

analogous to the Binomial Theorem.

Again—

$$\begin{aligned} t_{s+l} &= t_s + \Delta t_s, \\ t_{s+2l} &= t_{s+l} + \Delta t_{s+l} \\ &= t_s + 2\Delta t_s + \Delta^2 t_s, \\ t_{s+3l} &= t_s + 3\Delta t_s + 3\Delta^2 t_s + \Delta^3 t_s, \end{aligned}$$

and generally, by induction,

$$t_{s+nl} = t_s + n\Delta t_s + \frac{n(n-1)}{2!}\Delta^2 t_s + \frac{n(n-1)(n-2)}{3!}\Delta^3 t_s + \dots \qquad (19),$$

again analogous to the Binomial Theorem.

But if t_{s+nl} or $f(s+nl)$ is expanded by Taylor's Theorem in ascending powers of nl, then

$$t_{s+nl} = f(s) + nl\frac{df(s)}{ds} + \frac{n^2l^2}{2!}\frac{d^2f(s)}{ds^2} + \frac{n^3l^3}{3!}\frac{d^3f(s)}{ds^3} + \frac{n^4l^4}{4!}\frac{d^4f(s)}{ds^4} + \dots \qquad (20).$$

The general, $(r+1)$th, term in the series (19) can be written

$$\begin{aligned} &\frac{n(n-1)\dots(n-r+1)}{r!}\Delta^r t_s \\ &= -(-1)^r\, n(1-n)\left(1-\frac{n}{2}\right)\dots\left(1-\frac{n}{r-1}\right)\frac{\Delta^r t_s}{r} \\ &= -(-1)^r\left\{n - n^2\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\dots+\frac{1}{r-1}\right)+\dots\right\}\frac{\Delta^r t_s}{r}. \end{aligned}$$

Collecting the coefficients of n, n^2 and n^3 in (19),

$$\begin{aligned} t_{s+nl} = t_s &+ n\left\{\Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \tfrac{1}{4}\Delta^4 t_s + \dots(-1)^r\frac{\Delta^r t_s}{r}+\dots\right\} \\ &+ n^2\left\{\frac{\Delta^2 t_s}{2} - \frac{\Delta^3 t_s}{3}(1+\tfrac{1}{2}) + \frac{\Delta^4 t_s}{4}(1+\tfrac{1}{2}+\tfrac{1}{3}) - \frac{\Delta^5 t_s}{5}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}) + \dots\dots\right. \\ &\left. + (-1)^{r-1}\frac{\Delta^r t_s}{r}\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\dots+\frac{1}{r-1}\right)+\dots\right\} \\ &+ n^3\left\{\frac{\Delta^3 t_s}{6} - \frac{\Delta^4 t_s}{4}+\dots\right\} + n^4\left\{-\frac{\Delta^4 t_s}{24}+\dots\right\} + \dots \qquad (21); \end{aligned}$$

so that, equating the coefficients of n and n^2 in these two different expressions for t_{s+nl}, given in (20) and (21),

$$l\frac{dt_s}{ds} = \Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \dots - (-1)^r\frac{\Delta^r t_s}{r} + \dots \qquad (22),$$

$$l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \tfrac{11}{12}\Delta^4 t_s - \dots + 2(-1)^r\frac{\Delta^r t_s}{r}\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\dots+\frac{1}{r-1}\right)+\dots \qquad (23).$$

With an unlimited number of screens, l feet apart, and their time records, the successive differences of the records can be found according to the following scheme.

It will be noticed that the series of numbers

$$t_s,\ \Delta t_s,\ \Delta^2 t_s,\ \Delta^3 t_s,\ \Delta^4 t_s,\ \dots,$$

run in a diagonal line slanting downwards, so that the preceding formulas (22) and (23) are suitable for employment at the initial screens of a series.

$t.$	$\Delta t.$	$\Delta^2 t.$	$\Delta^3 t.$	$\Delta^4 t.$	$\Delta^5 t.$	$\Delta^6 t.$	$\Delta^7 t.$	$\Delta^8 t.$
t_{s-7l}								
	Δt_{s-7l}							
t_{s-6l}		$\Delta^2 t_{s-7l}$						
	Δt_{s-6l}		$\Delta^3 t_{s-7l}$					
t_{s-5l}		$\Delta^2 t_{s-6l}$		$\Delta^4 t_{s-7l}$				
	Δt_{s-5l}		$\Delta^3 t_{s-6l}$		$\Delta^5 t_{s-7l}$			
t_{s-4l}		$\Delta^2 t_{s-5l}$		$\Delta^4 t_{s-6l}$		$\Delta^6 t_{s-7l}$		
	Δt_{s-4l}		$\Delta^3 t_{s-5l}$		$\Delta^5 t_{s-6l}$		$\Delta^7 t_{s-7l}$	
t_{s-3l}		$\Delta^2 t_{s-4l}$		$\Delta^4 t_{s-5l}$		$\Delta^6 t_{s-6l}$		$\Delta^8 t_{s-7l}$
	Δt_{s-3l}		$\Delta^3 t_{s-4l}$		$\Delta^5 t_{s-5l}$		$\Delta^7 t_{s-6l}$	
t_{s-2l}		$\Delta^2 t_{s-3l}$		$\Delta^4 t_{s-4l}$		$\Delta^6 t_{s-5l}$		$\Delta^8 t_{s-6l}$
	Δt_{s-2l}		$\Delta^3 t_{s-3l}$		$\Delta^5 t_{s-4l}$		$\Delta^7 t_{s-5l}$	
t_{s-l}		$\Delta^2 t_{s-2l}$		$\Delta^4 t_{s-3l}$		$\Delta^6 t_{s-4l}$		$\Delta^8 t_{s-5l}$
	Δt_{s-l}		$\Delta^3 t_{s-2l}$		$\Delta^5 t_{s-3l}$		$\Delta^7 t_{s-4l}$	
t_s		$\Delta^2 t_{s-l}$		$\Delta^4 t_{s-2l}$		$\Delta^6 t_{s-3l}$		$\Delta^8 t_{s-4l}$
	Δt_s		$\Delta^3 t_{s-l}$		$\Delta^5 t_{s-2l}$		$\Delta^7 t_{s-3l}$	
t_{s+l}		$\Delta^2 t_s$		$\Delta^4 t_{s-l}$		$\Delta^6 t_{s-2l}$		$\Delta^8 t_{s-3l}$
	Δt_{s+l}		$\Delta^3 t_s$		$\Delta^5 t_{s-l}$		$\Delta^7 t_{s-2l}$	
t_{s+2l}		$\Delta^2 t_{s+l}$		$\Delta^4 t_s$		$\Delta^6 t_{s-l}$		$\Delta^8 t_{s-2l}$
	Δt_{s+2l}		$\Delta^3 t_{s+l}$		$\Delta^5 t_s$		$\Delta^7 t_{s-l}$	
t_{s+3l}		$\Delta^2 t_{s+2l}$		$\Delta^4 t_{s+l}$		$\Delta^6 t_s$		$\Delta^8 t_{s-l}$
	Δt_{s+3l}		$\Delta^3 t_{s+2l}$		$\Delta^5 t_{s+l}$		$\Delta^7 t_s$	
t_{s+4l}		$\Delta^2 t_{s+3l}$		$\Delta^4 t_{s+2l}$		$\Delta^6 t_{s+l}$		
	Δt_{s+4l}		$\Delta^3 t_{s+3l}$		$\Delta^5 t_{s+2l}$			
t_{s+5l}		$\Delta^2 t_{s+4l}$		$\Delta^4 t_{s+3l}$				
	Δt_{s+5l}		$\Delta^3 t_{s+4l}$					
t_{s+6l}		$\Delta^2 t_{s+5l}$						
	Δt_{s+6l}							
t_{s+7l}								

At the final screens the numbers end off in a diagonal line sloping upwards, containing the typical terms

$$t_s,\ \Delta t_{s-l},\ \Delta^2 t_{s-2l},\ \Delta^3 t_{s-3l},\ \Delta^4 t_{s-4l},\ \ldots$$

But

$$t_{s-l} = t_s - \Delta t_{s-l},$$

$$\begin{aligned} t_{s-2l} &= t_{s-l} - \Delta t_{s-2l} \\ &= t_s - \Delta t_{s-l} - \Delta(t_{s-l} - \Delta t_{s-2l}) \\ &= t_s - 2\Delta t_{s-l} + \Delta^2 t_{s-2l}, \end{aligned}$$

and so on; so that generally

$$\begin{aligned} t_{s-nl} &= t_s - n\Delta t_{s-l} + \frac{n(n-1)}{2\,!}\Delta^2 t_{s-2l} - \ldots \\ &= t_s - n(\Delta t_{s-l} + \tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{3}\Delta^3 t_{s-3l} + \ldots) \\ &\qquad + n^2(\tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{2}\Delta^3 t_{s-3l} + \tfrac{11}{24}\Delta^4 t_{s-4l} + \ldots) \quad . \quad . \quad (24); \end{aligned}$$

and therefore, as before,

$$l\,\frac{dt_s}{ds} = \Delta t_{s-l} + \tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{3}\Delta^3 t_{s-3l} + \ldots \quad . \quad . \quad . \quad (25),$$

$$l^2\,\frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-2l} + \Delta^3 t_{s-3l} + \tfrac{11}{12}\Delta^4 t_{s-4l} + \ldots \quad . \quad . \quad . \quad (26),$$

the formulas appropriate at the final screens of a series.

But at the middle screens the numbers which run horizontally are typified by

$$t_s,\ \frac{\Delta t_{s-l}}{\Delta t_s},\ \Delta^2 t_{s-l},\ \frac{\Delta^3 t_{s-2l}}{\Delta^3 t_{s-l}},\ \Delta^4 t_{s-2l},\ \ldots$$

The formulas required are now

$$l\frac{dt_s}{ds} = \tfrac{1}{2}(\Delta t_{s-l} + \Delta t_s) - \frac{1}{3\,!}\tfrac{1}{2}(\Delta^3 t_{s-2l} + \Delta^3 t_{s-l})$$

$$+ \frac{1^2 \,.\, 2^2}{5\,!}\,\tfrac{1}{2}(\Delta^5 t_{s-3l} + \Delta^5 t_{s-2l}) \ldots$$

$$-(-1)^r \frac{1^2 \,.\, 2^2 \,.\, 3^2 \ldots (r-1)^2}{(2r-1)\,!}\,\tfrac{1}{2}(\Delta^{2r-1} t_{s-rl} + \Delta^{2r-1}) \quad \ldots \ldots \quad (27)$$

$$+ \ldots$$

$$l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-l} - \frac{1}{3\,!}\frac{\Delta^4 t_{s-2l}}{2} + \frac{1^2 \,.\, 2^2}{5\,!}\frac{\Delta^6 t_{s-3l}}{3} - \ldots$$

$$-(-1)^r \frac{1^2 \,.\, 2^2 \,.\, 3^2 \ldots (r-1)^2}{(2r-1)\,!}\frac{\Delta^{2r} t_{s-rl}}{r} + \ldots \quad \ldots \ldots \quad (28),$$

the first (27) involving odd differences, and the second (28) even differences only (De Morgan, *Differential and Integral Calculus*, p. 544).

This is proved, if equation (19) is replaced by an equivalent formula,

$$t_{s+nl} = t_s + n\Delta t_s + \frac{n(n-1)}{2\,!}\Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3\,!}\Delta^3 t_{s-l} + \ldots$$

$$+ \frac{(n+r-1)\ldots(n-r)}{(2r)\,!}\Delta^{2r} t_{s-rl} + \frac{(n+r)\ldots(n-r)}{(2r+1)\,!}\Delta^{2r+1} t_{s-rl} + \ldots \quad . \quad (29).$$

Putting

$$\Delta t_s = \Delta t_{s-l} + \Delta^2 t_{s-l},$$

and, generally,

$$\Delta^{2n+1} t_{s-nl} = \Delta^{2n+1} t_{s-(n+1)l} + \Delta^{2n+2} t_{s-(n+1)l},$$

formula (29) is equivalent to

$$t_{s+nl} = t_s + n\Delta t_{s-l} + \frac{n(n+1)}{2\,!}\Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3\,!}\Delta^3 t_{s-2l} + \ldots$$

$$+ \frac{(n-r+1)\ldots(n+r)}{(2r)\,!}\Delta^{2r} t_{s-rl} + \frac{(n-r)\ldots(n+r)}{(2r+1)\,!}\Delta^{2r+1} t_{s-(r+1)l} + \ldots \quad (30).$$

Taking the half sum of (29) and (30),

$$t_{s+nl} = t_s + n\tfrac{1}{2}(\Delta t_{s-l} + \Delta t_s) + \frac{n^2}{2\,!}\Delta^2 t_{s-l} + \frac{(n+1)\,n\,(n-1)}{3\,!}\,\tfrac{1}{2}(\Delta^3 t_{s-2l} + \Delta^3 t_{s-l}) + \ldots$$

$$+ \frac{n\,(n-r+1)\,(n-r+2)\ldots(n+r-1)}{(2r)\,!}\Delta^{2r} t_{s-rl}$$

$$+ \frac{(n-r)\ldots(n+r)}{(2r+1)\,!}\,\tfrac{1}{2}(\Delta^{2r+1} t_{s-(r+1)l} + \Delta^{2r+1} t_{s-rl}) + \ldots \quad (31),$$

and equating the coefficients of n and n^2 in this equation and in (3) will lead to the two required formulas (27) and (28), already stated.

Having thus determined

$$l\frac{dt_s}{ds} \text{ and } l^2\frac{d^2 t_s}{ds^2}$$

by the successive differences of the screen records, the velocity v is the reciprocal of $\frac{dt}{ds}$ while the retardation r is given by

$$r = -\frac{dv}{dt} = -v\frac{dv}{ds} = \frac{d^2 t}{ds^2}v^3, \text{ as on page 13.}$$

Numerical illustrations taken from the *Reports on Experiments made with the Bashforth Chronograph, to determine the Resistance of the Air to the Motion of Projectiles*, 1865–1870 and

1878–1880, or from *A Revised Account of the Experiments made with the Bashforth Chronograph* (Cambridge, 1890), will make the preceding theory more clear.

The following examples are given by Sir G. Greenhill, F.R.S., to whom is due the explanation of the method of Finite Differences given in this chapter and in previous editions of the present work, also a great many examples in the 1902 edition of the Text Book of Gunnery.

Round 463, Report VIII, Table I, has been selected as a specimen for showing the nature of reduction employed, arranged in the scheme annexed, the chronograph records being taken to five decimals from the *Bashforth Chronograph*, p. 41, 1890; the date must be added for the meteorological record in the calculation of the tenuity factor T, by means of the extract from the Hygrometrical Table.

To obtain velocity to one decimal, five-figure logarithms are required; but the Slide Rule will give the accuracy permissible in the calculation of K, r, R, ... which depend on the two-figure observed values of $\Delta^2 t$.

Round 1 is taken also for its historical interest, October 7th, 1867, in which a solid shot, weighing 12 lbs., was fired from a 3-inch gun, with a charge of 2 lb. of powder; the instants of time at which the 10 screens, 150 feet apart, were cut by the shot, are recorded to 4 decimals in the following table, where the time differences are also given.

Round 1, Report III, Table I, p. 27, Table X, p. 54.

October 7th, 1867.	Barometer.	Thermometer. Dry.	Thermometer. Wet.
10 a.m.	29·66	53	53
3 p.m.	29·62	52	48

and Bashforth takes $\tau = 1\cdot002$, $w = 12$, $d = 2\cdot92$,

$$C = \frac{w}{\tau d^2}, \quad \log C = 0\cdot1476, \quad C = 1\cdot405.$$

Screen	t	Δt	$\Delta^2 t$
1	0·0000		
		0·1247	
2	1247		0·0019
		1266	
3	2513		21
		1287	
4	3800		22
		1309	
5	5109		21
		1330	
6	6439		20
		1350	
7	7789		23
		1373	
8	0·9162		24
		1397	
9	1·0559		0·0023
		0·1420	
10	1·1979		

It will be noticed that the second difference is very nearly constant, and on the average equal to 0·0021, or 0·0022, and that the higher differences are illusory; this is because the chronograph does not record smaller intervals of time than the ten-thousandth of a second, recorded in the fourth place of decimals; and this figure is therefore subject to a correction, which may reach to nearly $\pm$ 5 in the fifth place.

The calculation of v, depending chiefly on Δt, will be given within the unit place by the four decimals; but in calculating K the irregularity in Δ^2 must be smoothed down by a smoothing figure in the fifth decimal place, to obtain a result in accordance with Bashforth.

No fixed method can be laid down for this smoothing operation, but a convenient way is to plot $\Delta^2 t$ to a large scale, say one inch to one ten-thousandth of a second, join up the points by a broken line, and then draw a straight line by eye to run as evenly as possible between the points; the corrected values of Δ^2 being measured to this line, then Δ^3 will be constant; and Δt is derived by the successive additions of Δ^2 and Δ^3.

This operation may show an over-correction of t at some screen; but this can be cancelled by sharing it between the screen intervals by a suitable correction in Δt, left untouched before, thus making a constant correction throughout of Δt.

On this method, with Round 1, we shall adopt the corrections of Δ^2 as follows:—

Screen.	t.	Δt.	Δ^2.	Correction adopted in Δ^2.	Δ^3.
1	0·00000				
		0·12470			
2	12470		0·00211	+0·00021	
		12681			
3	25151		212	+ 2	
		12893			
4	38044		213	− 7	
		13106			
5	51150		214	+ 4	
		13320			0·00001
6	64470		215	+ 15	
		13535			
7	78005		216	− 14	
		13751			
8	0·91756		217	− 23	
		13968			
9	1·05724		0·00218	−0·00012	
		0·14186			
10	1·19910				

This makes $t_{10} = 1{\cdot}19910$, or 0·00120 above the measured value, and this difference can be shared between 9 intervals by a correction − 0·00013 in Δt; and now we find the corrected values of t for Round 1 given in the *Chronograph*, p. 33, as shown here.

ROUND 1.

Number of screen.	t.	Δt.	$\Delta^2 t$.	$\Delta^3 t$.
1	0·00000			
		0·12457		
2	12457		0·00211	
		12668		
3	25125		00212	
		12880		
4	38005		00213	
		13093		
5	51098		00214	0·00001
		13307		
6	64405		00215	
		13522		
7	77927		00216	
		13738		
8	0·91665		00217	
		13955		
9	1·05620		0·00218	
		0·14173		
10	1·19793			

The calculation of v and K can proceed now as in Round 463.

Round 463 is now set out completely; $\overline{\mathrm{K}}$ and $\overline{p}$ represent the mean value of K and of p obtained from the results of a large number of rounds.

Since

$$pg = \mathrm{C}r = \mathrm{K}\left(\frac{v}{1000}\right)^3,$$

$$\frac{p}{\overline{p}} = \frac{r}{\overline{r}} = \frac{\mathrm{K}}{\overline{\mathrm{K}}}$$

for a definite velocity $= v$, and

$$\frac{p}{r} = \frac{\mathrm{C}}{g};$$

therefore

$$\frac{p}{\overline{p}} = \frac{r}{\overline{r}} = \frac{\mathrm{K}}{\overline{\mathrm{K}}} = \frac{\overline{\mathrm{C}}}{\mathrm{C}} = \frac{\sigma}{\overline{\sigma}},$$

since

$$\mathrm{C} = \frac{w}{\kappa\sigma\tau d^2},$$

and here σ is the uncertain factor, hence since $\sigma > \overline{\sigma}$ in Round 463, it shows that the coefficient of steadiness σ is above the normal.

Round 463, March 7th, 1879. Report VIII, Table I.

Thermometer $\left\{\begin{array}{l}\text{Wet, } 37^\circ \text{ F.}\\ \text{Dry, } 39^\circ \text{ F.}\end{array}\right\}$ Barometer, $29\cdot95\left\{\begin{array}{r}538\cdot9\\16\cdot7\\0\cdot9\end{array}\right\}$, $\tau = \dfrac{556\cdot50}{534\cdot22} = 1\cdot042$, $w = 70$, $d = 6$, $\kappa = 1$, $C = \dfrac{w}{\kappa\tau d^2} = 1\cdot866.$

$\delta = 556\cdot5.$

Screen.	t	Δt	$\Delta^2 t$	$\Delta^3 t$	$l\dfrac{dt}{ds}$	v	$l^2\dfrac{d^2t}{ds^2}$	$K = C\dfrac{d^2t}{ds^2}\cdot10^9$	$r = \dfrac{d^2t}{ds^2}\times v^3$	$\dfrac{R}{w} = \dfrac{r}{g}$	$R = w\dfrac{r}{g}$	$p = \dfrac{R}{\tau d^2}$	$\bar{p}$	$\sigma = \dfrac{p}{\bar{p}} = \dfrac{K}{\bar{K}}$
1	0·00000				0·07679	1953·4	0·00090	74·6	298·2	9·26	648·5	17·28	16·19	1·068
		0·07724												
2	·07724		0·00091		7770	1930·6	90	74·6	287·8	8·94	625·5	16·66	15·81	1·064
		7815		0·00001										
3	·15539		92		7860	1908·4	94	77·9	290·4	9·03	632·3	16·85	15·47	1·088
		7907		2										
4	·23446		94		7954	1885·8	94	77·9	280·3	8·71	610·0	16·24	15·17	1·070
		8001		0										
5	·31447		94		8048	1863·8	94	77·9	270·5	8·40	588·0	15·67	14·88	1·053
		8095		0										
6	·39542		94		8142	1842·3	94	77·9	261·3	8·11	568·0	15·13	14·59	1·037
		8189		0										
7	·47731		94		8236	1821·3	94	77·9	252·4	7·85	549·5	14·63	14·29	1·024
		8283		1										
8	·56014		95		8330	1800·6	95	78·8	246·4	7·66	536·0	14·27	14·01	1·018
		8378		1										
9	·64392		96		8426	1780·2	96	79·7	240·7	7·47	522·9	13·94	13·74	1·014
		8474		1										
10	·72866		97		8522	1760·1	97	80·4	235·8	7·44	520·5	13·86	13·47	1·029
		8571		0·00001										
11	·81437		0·00098		8620	1740·2	98	81·3	229·4	7·14	499·5	13·60	13·19	1·029
		0·08669												
12	0·90106				0·08718	1720·5	0·00099	82·1	224·0	6·96	487·3	12·96	12·91	1·002

In the Bashforth system of measurement two parallel lines are traced on a cylindrical drum, which is spun round a vertical axis with a peripheral speed of about 10 to 12 inches a second. On one line an astronomical clock marks the seconds, and on the other line the shot registers its passage through each screen as it is broken, each record being made by an electric signal actuating a stylus.

The clock mark gives the time scale of the movement of the cylinder, and the problem is to determine the intermediate instant of time corresponding to the mark caused by the fracture of a screen by the passage of the shot.

For Round 479 the distance measured on the paper was as follows, on a linear scale of about 3 units to 1 inch, the record being measured carefully under a microscope:—

ROUND 479.

Seconds.	Seconds—linear measurement.	Screens.	Screens—linear measurement.
1	16·576		
2	49·130		
		1	73·856
		2	76·000
		3	78·180
		4	80·390
3	81·410		
		5	82·640
		6	84·910
		7	not observed
		8	89·210
		9	91·994
		10	94·376
		11	96·814
		12	99·314
4	113·454		
5	145·350		
6	176·994		

The screen records extend from 73 to 100, lying between the 2-second and 4-second record, and it is sufficient to form a complete record in this interval of time to every tenth of a second.

First, the half-second record is found by interpolation, just as well as if the clock had given a record every half-second.

The record gives—

t	s	Δs	$\Delta^2 s$	$\Delta^3 s$
1	16·576			
		32·554		
2	49·130		−0·274	
		32·280		0·038
3	81·410		−0·236	
		32·044		0·088
4	113·454		−0·148	
		31·896		
5	145·350		−0·252	
		31·644		
6	176·994			

in which the negative value of Δ^2 indicates the retardation of the drum.

Putting $n = \frac{1}{2}$ in formula (19)

$$s_{t+n} = s_t + n\Delta s + \frac{n(n-1)}{2!}\Delta^2 + \frac{n(n-1)(n-2)}{3!}\Delta^3 + \dots,$$

$$s_{2+\frac{1}{2}} = s_2 + \tfrac{1}{2}\Delta_2 - \tfrac{1}{8}\Delta_2^2 + \tfrac{1}{16}\Delta_2^3,$$

$$s_{3+\frac{1}{2}} = s_3 + \tfrac{1}{2}\Delta_3 - \tfrac{1}{8}\Delta_3^2 + \tfrac{1}{16}\Delta_3^3.$$

	$s_{2\cdot5}$	$s_{3\cdot5}$
s	49·1300	81·4100
$\frac{1}{2}\Delta$	16·1400	16·0220
$-\frac{1}{8}\Delta^2$	0·0295	0·0185
$\frac{1}{16}\Delta^3$	0·0055	0·0055
	65·3050	97·4560

The time-table now reads :—

t	s	Δ_s	Δ_s^2	Δ_s^3
2·0	49·130			
		+16·175		
2·5	65·305		−0·070	
		+16·105		0·011
3·0	81·410		−0·059	
		+16·046		0·011
3·5	97·456		−0·048	
		+15·998		
4·0	113·454			

A further application of the interpolation formula (19), with $n = \pm\frac{1}{5}, \pm\frac{2}{5} \dots$, will lead to the clock record for every tenth of a second, between 2·5 and 3·6, between which the screen records are found.

Thus for $s_{2\cdot6}$ the expression is

$$s_{2\cdot5+\cdot\frac{1}{5}} = s_{2\cdot5} + \tfrac{1}{5}\Delta + \frac{\frac{1}{5}(\frac{1}{5}-1)}{2!}\Delta^2 + \frac{\frac{1}{5}(\frac{1}{5}-1)(\frac{1}{5}-2)}{3!}\Delta^3,$$

$$s_{2\cdot6} = s_{2\cdot5} + 0\cdot2\Delta - 0\cdot08\Delta^2 + 0\cdot0048\Delta^3;$$

whilst

$$s_{2\cdot7} = s_{2\cdot5} + \tfrac{2}{5}\Delta + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2!}\Delta^2 + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}-2)}{3!}\Delta^3$$

$$= s_{2\cdot5} + 0\cdot4\Delta - 0\cdot12\Delta^2 + 0\cdot0064\Delta^3,$$

and so on, so that

	$s_{2\cdot6}$	$s_{2\cdot7}$	$s_{2\cdot8}$	s
$s_{2\cdot5}$	65·30500	65·30500		
$n\Delta$	3·22100	6·44200		
$\frac{n(n-1)}{2!}\Delta^2$	+0·00472	0·00708		
$\frac{n(n-1)(n-2)}{3!}\Delta^3$	+0·00058	0·00070		

To find the corresponding record for $t = 3{\cdot}5, 3{\cdot}6, 3{\cdot}7 \ldots 3{\cdot}9$, make use of formula (24). For example, to find $s_{3\cdot6}$:—

$$3{\cdot}6 = 4{\cdot}0 - 0{\cdot}5\left(\tfrac{4}{5}\right), \quad \text{where} \quad 0{\cdot}5 = l, \ \tfrac{4}{5} = n,$$

then

$$s_{4-nl} = s_{3\cdot6} = s_4 - n\Delta s_3 + \frac{n(n-1)}{2!}\Delta^2 s_2 - \frac{n(n-1)(n-2)}{3!}\Delta^3 s_1,$$

$$\begin{aligned} s_{3\cdot6} &= 113{\cdot}454 - \tfrac{4}{5} \text{ of } 15{\cdot}998 + \tfrac{4}{50} \text{ of } 0{\cdot}048 - \tfrac{4}{125} \text{ of } 0{\cdot}011 \\ &= 113{\cdot}454 - 12{\cdot}7984 + 0{\cdot}00384 - 0{\cdot}0004 \\ &= 100{\cdot}659, \end{aligned}$$

and then the table reads :—

Time.	Record.	Δ^1	Δ^2	Time.	Record.	Δ^1	Δ^2
2·5	65·305			3·1	84·624		−0·003
		+3·226				+3·211	
2·6	68·531		−0·002	3·2	87·835		−0·002
		+3·224				+3·209	
2·7	71·755		−0·003	3·3	91·044		−0·002
		+3·221				+3·207	
2·8	74·976		−0·003	3·4	94·251		−0·002
		+3·218				+3·205	
2·9	78·194		−0·002	3·5	97·456		−0·002
		+3·216				+3·203	
3·0	81·410		−0·002	3·6	100·659		
		+3·214					

Here Δ^2 is so small that its influence becomes insensible, and the interpolation formula reduces to the rule of proportional parts, equivalent to neglecting the retardation of the drum in so short an interval of time as one-tenth of a second.

The smoothed record of Round 479 is—

Screen.	Smoothed Record.	Δ^1	Δ^2
1	73·856		
		2·144	
2	76·000		
		2·178	
3	78·178		
		2·212	
4	80·390		
		2·246	
5	82·636		
		2·280	
6	84·916		+ 0·034
		2·314	
7	87·230		
		2·348	
8	89·578		
		2·382	
9	91·960		
		2·416	
10	94·376		
		2·450	
11	98·826		
		2·484	
12	99·310		

The screen record in length of travel of the drum must now be converted into decimals of a second, using proportional parts for the interval of one-tenth of a second.

Thus the record 73·856 of the first screen lies in the interval 2·7 to 2·8 seconds, and closer to 74·976, the 2·8-second record, than to 71·755, the 2·7-second record; using proportional parts,

$$\begin{array}{r} 73\cdot856 \\ 74\cdot976 \\ \hline \end{array} \qquad \begin{array}{r} 2\cdot800 \\ -0\cdot03477 \\ \hline \end{array}$$

$$\frac{-\ 1\cdot120}{3\cdot221} = 0\cdot3477 \qquad t_1 = 2\cdot76523$$

and so for t_2, t_3, ...

The resulting table of time is given by Captain J. H. Hardcastle, late R.A.:—

Screen.	t'' Smoothed records transformed to seconds by scale.	These are the figures used in 1879 reduction $t''-t_1$	Figures published in 1879. t''	Figures published in 1890.	Δ^1	Δ^2
1	2·76523	0·00000	0·0000	0·00000		
					0·06659	
2	83182	0·06659	0·0666	0·06659		0·00109
					0·06768	
3	89950	0·13427	0·1343	0·13427		0·00110
					0·06878	
4	96828	0·20305	0·2031	0·20305		0·00109
					0·06987	7
5	3·03815	0·27292	0·2729	0·27292	4	0·00109
					0·07096	13
6	10909	0·34386	0·3439	0·34388	7	0·00109
					0·07205	
7	18116	0·41593	0·4159	0·41593	6	0·00109
					0·07314	8
8	25432	0·48909	0·4891	0·48907		0·00110
					0·07424	
9	32856	0·56333	0·5633	0·56331		0·00110
					0·07534	
10	40390	0·63867	0·6387	0·63865		0·00110
					0·07644	
11	48034	0·71511	0·7151	0·71509		0·00110
					0·07754	
12	55788	0·79265	0·7927	0·79263		

These second differences of the *five*-figure records being just smoothed as shown account for the difference between the 1879 and the 1890 reports.

Second differences slightly irregular = 0·00107 or 113, 10 or 12. This is due to the transformation of space into time.

The figures published in 1890 and 1879 are slightly discrepant, depending upon different estimates of the fifth decimal in the screen times or on one hundred-thousandth of a second.

See a paper published by Captain J. H. Hardcastle, late R.A., concerning Round 479 in the *Proceedings of the R.A. Institution*, vol. XXX, 1903.

Taking the figures published in 1890 for Round 479, with $d = 6$, $w = 50$, $\tau = 1\cdot014$, $k\sigma = 1$, and screen 150 feet apart,

$$C = \frac{w}{\kappa\sigma\tau d^2} = 1\cdot37,$$

$$l = 150', \quad \Delta^3 t_s = 0.$$

For a *middle screen*, say the 6th, from (27) and (28)

$$l\frac{dt}{ds} = \tfrac{1}{2}(\Delta t_{s-1} + \Delta t_s) = \tfrac{1}{2}(\Delta t_5 + \Delta t_6),$$

i.e.,

$$\frac{l}{v} = \tfrac{1}{2}(0 \cdot 07096 + 0 \cdot 07205) = 0 \cdot 0715$$

and

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-l} = \Delta^2 t_5 = 0 \cdot 00109.$$

For a final screen, say the 12th, from (25)

$$l \frac{dt_s}{ds} = \Delta t_{s-l} + \tfrac{1}{2}\Delta^2 t_{s-2l},$$

i.e.,

$$\frac{l}{v} = 0 \cdot 07754 + \frac{0 \cdot 00110}{2} = 0 \cdot 7799\,;$$

from (26)

$$l^2 \frac{d^2 t_s}{ds} = \Delta^2 t_{s-2l} = 0 \cdot 00110,$$

and for an initial screen, say the 1st, from (22)

$$l \frac{dt_s}{ds} = \Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \ldots,$$

$$\frac{l}{v} = 0 \cdot 06659 - \frac{0 \cdot 00109}{2} = 0 \cdot 06605\,;$$

from (23),

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \ldots = 0 \cdot 00109\,;$$

the results from Round 479 for each screen are tabulated thus :—

Number of screen . . .	1	2–5	6	7–11	12
$\frac{l}{v}$	0·06605		0·0715		0·7799
$\log \frac{l}{v}$	$\bar{2}$·8199				
$\log l$ (= log 150)	2·1761				
$\log v$	3·3562				
v f/s	2271		2098		1923
$l^2 \frac{d^2 t}{ds^2}$	0·00109		0·00109		0·00110
$\log \frac{d^2 t}{ds^2} 10^9$	1·6852				
log C	0·1367				
$\log K = \log\left(C \frac{d^2 t}{ds^2} 10^9\right)$	1·8219				
K	66·36		66·36		67
$p = \frac{K}{g}\left(\frac{v}{1000}\right)^3$	24·14		19·02		14·65
$pg = Cr = K\left(\frac{v}{1000}\right)^3$	776·9		612·7		471·7
$R = n d^2 p$	881·2		695		535

The method of finite differences is very useful for detecting any irregularity in the records or any error in transcribing them; also it often enables a general formula to be deduced for connecting variables in an experiment.

Thus, to find the formula connecting the figures 4, 11, 22, 37, 56, 79, 106 :—

n	t	Δt	$\Delta^2 t$	$\Delta^3 t$
0	4 $(=t_0)$			
		7 $(=\Delta t_0)$		
1	11 $(=t_1)$		4 $(=\Delta^2 t_0)$	
		11 $(=\Delta t_1)$		
2	22 $(=t_2)$		4 $(=\Delta^2 t_1)$	
		15		
3	37		4	0
		19		
4	56		4	
		23		
5	79		4	
		27		
6	106			

From (19),

$$t_{s+nl} = t_s + n\,\Delta t_s + \frac{n(n-1)}{2\,!}\,\Delta^2 t_s + \ldots\,;$$

here $s = 0$, and

$$t_{nl} = t_n = t = t_0 + n\,\Delta t_0 + \frac{n(n-1)}{2\,!}\,\Delta^2 t_0 + \ldots$$

$$= 4 + n\,.\,7 + \frac{n(n-1)}{2\,!}\,4 + 0,$$

giving

$$t = 4 + 5n + 2n^2.$$

This result can be obtained direct from (21), thus

$$t_n = t_0 + n\left(\Delta t_0 - \tfrac{1}{2}\,\Delta^2 t_0 + \tfrac{1}{3}\,\Delta^3 t_s\right) + n^2\left(\frac{\Delta^2}{2} - \frac{\Delta^3}{2} + \ldots\right) + n^3\left(\frac{\Delta^3}{6} - \frac{\Delta^4}{4}\right)$$

$$= 4 + n\left(7 - \tfrac{1}{2}4 + 0\right) + n^2\left(\tfrac{4}{2} - 0\right),$$

i.e.,

$$t = 4 + 5n + 2n^2.$$

The second differences were constant, and the equation connecting n and t is a parabola.

To find a formula giving the pressure p in tons/in.^{2} in a closed vessel for any density of loading δ from the results of the following experiment :—

δ	p
0·05	3·40
0·10	6·80
0·15	10·47
0·20	14·68
0·25	19·70

Writing n for 20δ and employing formula (21), replacing t by p,

$$p_n = p_0 + n\left(\Delta t_0 - \tfrac{1}{2}\Delta^2 t_0 + \tfrac{1}{3}\Delta^3 t_0 + \ldots\right) + n^2\left(\frac{\Delta^2 t_0}{2} - \frac{\Delta^3 t_0}{2} + \Delta^4 t_0\,\tfrac{11}{24}\right)$$

$$+ n^3\left(\frac{\Delta^3 t_0}{6} - \frac{\Delta^4 t_0}{4} - \ldots\right) + n^4\left(-\frac{\Delta^4 t_0}{24} + \ldots\right).$$

n	δ	p	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
0	0	0 $(=p_0)$				
			3·40 $(=\Delta p_0)$			
1	0·05	3·40 $(=p_1)$		0 $(=\Delta^2 p_0)$		
			3·40 $(=\Delta p_1)$		0·27 $(=\Delta^3 p_0)$	
2	0·10	6·80		0·27 $(=\Delta^2 p_1)$		
			3·67		0·27	0
3	0·15	10·47		0·54		
			4·21		0·27	
4	0·20	14·68		0·81		
			5·02			
5	0·25	19·70				

$$p_n \text{ or } p = 0 + n(3\cdot 40 + 0\cdot 09) + n^2(-0\cdot 135) + n^3\left(\tfrac{0\cdot 27}{6}\right) + 0$$
$$= 0\cdot 045 n^3 - 0\cdot 135 n^2 + 3\cdot 49 n.$$

Replacing 20δ for n gives

$$p = 360\delta^3 - 54\delta^2 + 69\cdot 8\delta,$$

which gives the formula for finding the pressure in a closed vessel for any density of loading δ.

If the screens, instead of being equidistant, were placed at distances

$$s_1,\ s_2,\ s_3,\ \ldots\ ,\ s_n,$$

from a fixed origin, and if

$$t_1,\ t_2,\ t_3,\ \ldots\ ,\ t_n$$

denoted the corresponding time records, then, according to Lagrange's Interpolation Formula, the simplest algebraical expression for t may be written

$$\begin{aligned} t = {} & \frac{{}^*\ (s-s_2)(s-s_3)\ldots(s-s_n)}{{}^*(s_1-s_2)(s_1-s_3)\ldots(s_1-s_n)}t_1 \\ & + \frac{(s-s_1)^*(s-s_3)\ldots(s-s_n)}{(s_2-s_1)^*(s_2-s_3)\ldots(s_2-s_n)}t_2 \\ & + \ldots \\ & + \frac{(s-s_1)(s-s_2)\ldots{}^*\ldots(s-s_n)}{(s_r-s_1)(s_r-s_2)\ldots{}^*\ldots(s_r-s_n)}t_n \\ & + \ldots \\ & + \frac{(s-s_1)(s-s_2)\ldots(s-s_{n-1})^*}{(s_n-s_1)(s_n-s_2)\ldots(s_n-s_{n-1})^*}t_n, \end{aligned}$$

a formula which agrees in giving

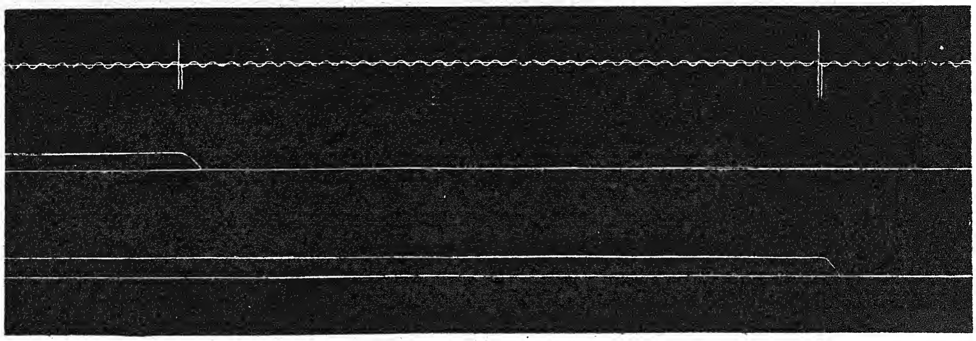
$$t = t_1, \text{ when } s = s_1;$$
$$t = t_2, \text{ when } s = s_2;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$t = t_r, \text{ when } s = s_r;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$t = t_n, \text{ when } s = s_n;$$

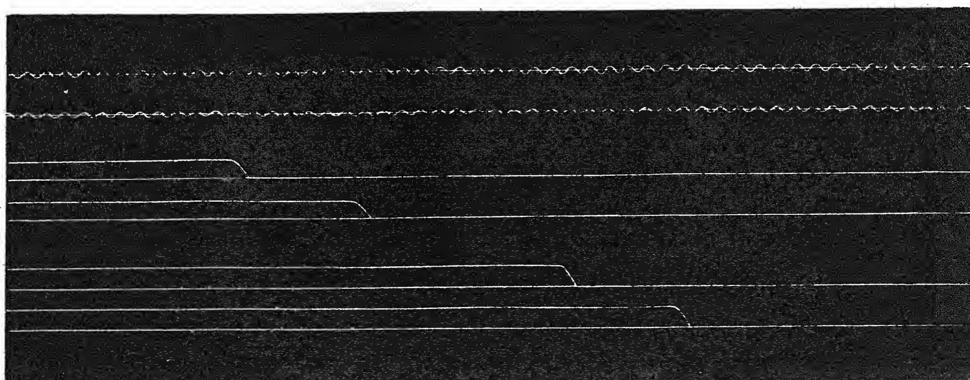
the asterisk * showing the position of the omitted vanishing factors.

Now to find $\frac{dt}{ds}$ and $\frac{d^2t}{ds^2}$ at the rth screen, and thence v and K, we make $s_n = 0$ by replacing any s_n by $s_m - s_n$, and then pick out the coefficient of s and $\frac{1}{2}s^2$ in t.

Plate I.

JERVIS SMITH CHRONOGRAPH TRACE.





Weller & Graham, Ltd Litho, London.

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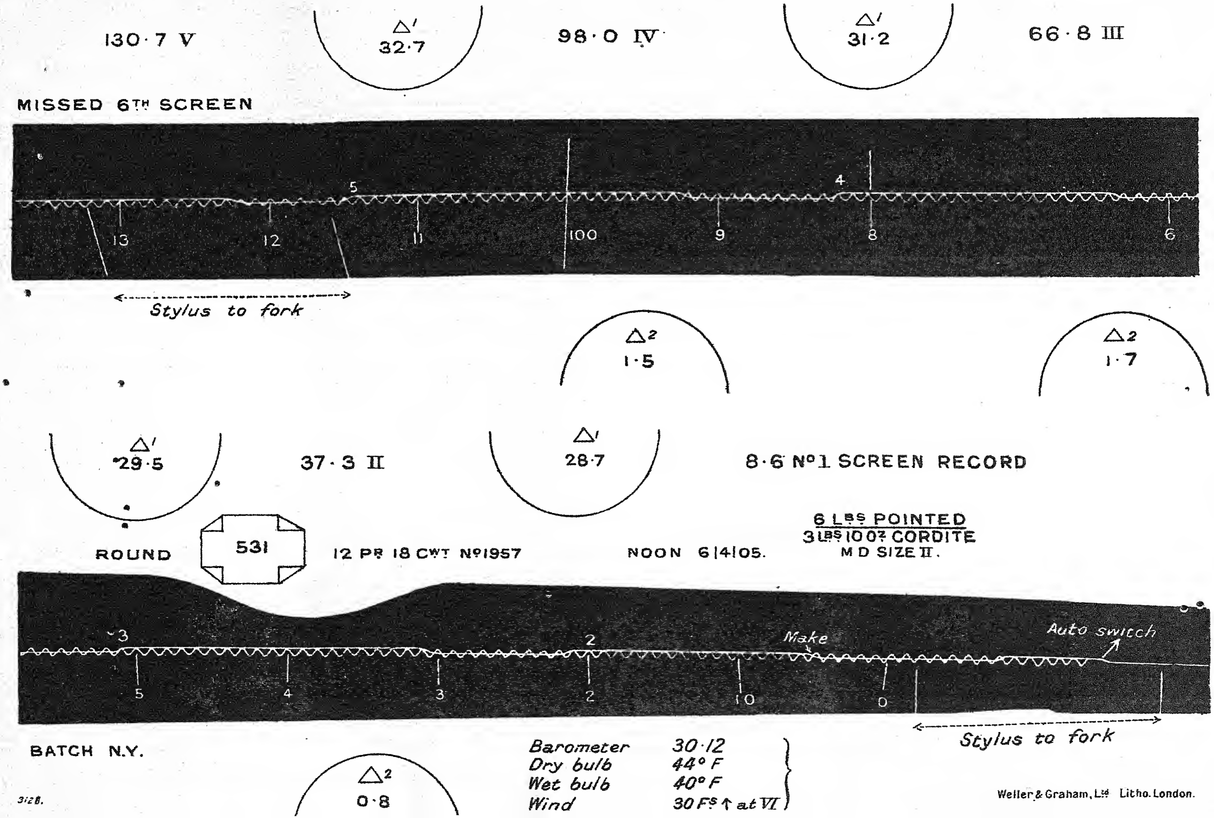
Plate II.



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The heavy work formerly required in the conversion of the time scale is obviated now by the use of a timing-fork trace, shown in the diagrams of a record with the chronograph, which gives a continuous time record (see Plates I and II); a chronograph of this kind was employed for the 1902–1906 experiments for finding K and the resistance of the air, the time of a complete vibration of the tuning fork being $\frac{1}{348\cdot 3}$ second (at 55° F.).

The upper figure gives the chronograph record of two screens, made each by its own stylus, and the measurement on the sinuous time record has been transferred from the first indication of the movement of a stylus. If more screens are employed than two, an additional stylus is required for each screen.

The lower figure shows the record of two rounds on the same plate of smoked glass; *and so a number may be taken on the same plate, and measured up at the end of the day.*

The new Ballistic Tables, founded upon the experiments carried out at Shoeburyness, are based upon a standard density of air of 534·2 grains per cubic foot, a standard ogival head of two calibres, so that κ, the coefficient of shape, is equal to unity for such ogival head.

The new Ballistic Table is calculated by integration, to be explained in the next chapter, by assuming a monomial law for K $\left(= \mathrm{C}\frac{d^2t}{ds^2}10^9\right)$ of the form

$$\mathrm{K} = \lambda v^m,$$

in which the index m can be determined by plotting the experimental value of K on a logarithmic chart; and the value of m and K adopted in the Ballistic Table re-calculated is given in the following table, for each of eight regions of velocity.

A calculation can be made in one step from one end to the other of a region.

Region.	v	m	K	K
	5000		17·6044	
I		−1·55		$[2\cdot 3290234]\left(\frac{v}{1000}\right)^{-1\cdot 55}$
	4000		24·8789	
II		−1·33		$[2\cdot 1965702]\left(\frac{v}{1000}\right)^{-1\cdot 33}$
	2600		44·1221	
III		−1·5		$[2\cdot 2671157]\left(\frac{v}{1000}\right)^{-1\cdot 5}$
	2000		65·3989	
IV		−1·2		$[2\cdot 1768067]\left(\frac{v}{1000}\right)^{-1\cdot 2}$
	1460		95·4076	
V		0		$[1\cdot 9795830]\left(\frac{v}{1000}\right)^{0}$
	1190		95·4076	
VI		3·45		$[1\cdot 7189459]\left(\frac{v}{1000}\right)^{3\cdot 45}$
	1040		59·9392	
VII		0		$[1\cdot 7777107]\left(\frac{v}{1000}\right)^{0}$
	840		59·9392	
VIII		−1·4		$[1\cdot 6717017]\left(\frac{v}{1000}\right)^{-1\cdot 4}$
	400		169·3627	

In the above the bracket [ ] signifies the antilogarithm of the number in the bracket

thus in Region VIII $[\bar{1}\cdot 6717017] \equiv 46\cdot 957$, this being the antilogarithm of $\bar{1}\cdot 6717017$, and so for the other regions of velocity.

In Regions VII and V of the velocity, where 840 < v < 1040, and 1190 < v < 1460, a constant value of K exists, so that from (14)

$$K = C \frac{d^2t}{ds^2} 10^9 = \text{constant},$$

or

$$\frac{d^2t}{ds^2} = \text{constant} = 2b, \text{ say.}$$

Integrate with respect to s,

$$\frac{dt}{ds} = \frac{1}{v} = 2bs + \text{const.},$$

when $s = 0$, $\frac{dt}{ds}$ is equal to $\frac{1}{V}$ if V denote the initial muzzle velocity, and v the remaining velocity at the end of a range s feet.

Therefore

$$\frac{dt}{ds} = 2bs + \frac{1}{V},$$

from which

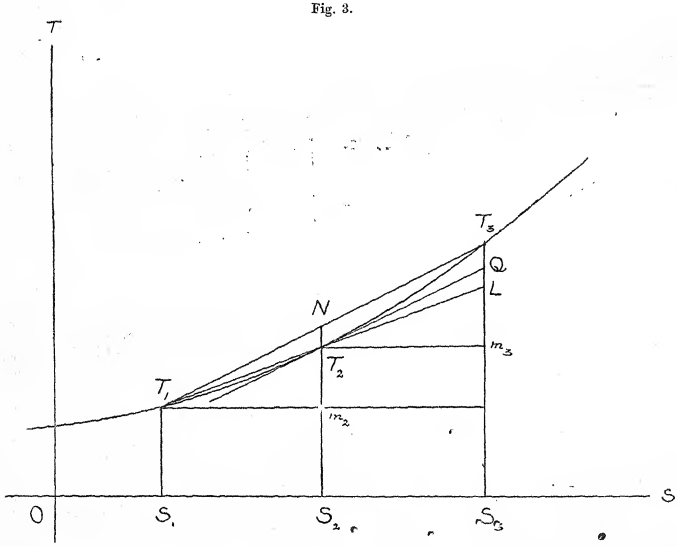
$$t = bs^2 + \frac{s}{V} + t_0,$$

where t_0 is a constant.

The curve $T_1T_2T_3$ on the figure (3), or the curve $f(s, t) = 0$, is a parabola.

Let $OS_1 = s$ feet and $S_1S_2 = S_2S_3 = l$ feet, then S_2 being the mid point between S_1 and S_3

Fig. 3.



and $T_1T_2T_3$ a parabola, the tangent T_2Q at T_2 is parallel to the chord T_1T_3, so that with screen distance $S_1S_3 = 2l$,

$$\left.\begin{aligned} S_1T_1 &= t_s \\ S_2T_2 &= t_{s+l} \\ S_3T_3 &= t_{s+2l} \end{aligned}\right\}\text{also}\quad \begin{aligned} T_2m_2 &= t_{s+l} - t_s = \Delta t_s \\ \\ T_3m_3 &= t_{s+2l} - t_{s+l} = \Delta t_{s+l} \end{aligned}$$

$$\Delta^2 t_s = \Delta t_{s+l} - \Delta t_s = T_3m_3 - T_2m_2.$$

Join T_1T_2 and produce it to L, then from similar triangles $T_1T_2m_2$, T_2Lm_3, $Lm_3 = T_2m_2$.

Also $T_3L = 2NT_2$ and $T_3Q = NT_2$.

Therefore

$$\Delta^2 t_s = (T_3L + Lm_3) - T_2m_2 = T_3L = 2NT_2 = 2T_3Q,$$

or

$$\frac{\Delta^2 t_s}{2} = NT_2.$$

And therefore

$$T_3Q = QL = NT_2 = \frac{\Delta^2 t_s}{2}.$$

The initial velocity when $s = 0$ is V, and denoting by (v_1, t_1), (v_2, t_2), (v_3, t_3), the velocity and time at s_1, s_2, s_3 respectively,

$$\frac{1}{v_1} = 2bs + \frac{1}{V}, \qquad t_1 = bs^2 + \frac{s}{V} + t_0,$$

$$\frac{1}{v_2} = 2b(s+l) + \frac{1}{V}, \qquad t_2 = b(s+l)^2 + \frac{s+l}{V} + t_0,$$

$$\frac{1}{v_3} = 2b(s+2l) + \frac{1}{V}, \qquad t_3 = b(s+2l)^2 + \frac{s+2l}{V} + t_0.$$

The average velocity U over the distance $s_1s_3 (= 2l)$ is given by

$$\frac{1}{U} = \frac{t_3 - t_1}{2l} = \frac{4bl(l+s) + \frac{2l}{V}}{2l} = 2b(s+l) + \frac{1}{V}$$

$$= \tfrac{1}{2}\left(\frac{1}{v_1} + \frac{1}{v_3}\right) = \frac{1}{v_2} \text{ from above,}$$

hence

$$U = v_2,$$

so that the average velocity U is the harmonic mean of the initial velocity and final velocity over any distance $2l$, and U is the actual velocity at the half distance.

This is the rule employed in determining velocity with a Boulangé chronograph at proof, where $2l$ is the distance in feet between the screens, and t_3, t_1 are the initial and final chronograph record of time, so that

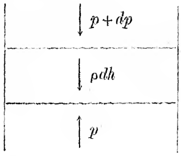
$$\frac{2l}{t_3 - t_1} = U$$

gives the average velocity of the shot between the screens, and this is taken to be the actual velocity at the point midway between the screens, but it depends upon the assumption that the resistance of the air varies as the cube of the velocity.

Proof of Mathematical Formulas on page 5 for Finding f.

To find the tenuity factor $f = \frac{\tau_0}{\tau_h}$ for an altitude h feet above sea level.

Take the foot as the unit of length, the pound avoirdupois as the unit of weight; the density of a substance is the weight of the unit of volume, in this case unit volume is 1 cubic foot.



Let p_0, ρ_0 denote the pressure and density in lbs/ft^2 and lbs/ft^3 of the air at sea level; p, ρ the pressure and density at a height h feet.

Then, for a small increase of height dh feet, let the pressure at $h+dh$ feet be $p+dp$.

The pressure at height h is greater than at height $h+dh$ by the weight of the column of air, dh feet high (and a square foot in section), therefore

$$dp = -\rho dh \quad \ldots\ldots \quad \text{I.}$$

Also, by definition,

$$f = \frac{\tau_0}{\tau_h} = \frac{\rho_0}{\rho} \quad \ldots\ldots \quad \text{II.}$$

1*st Case*: *If the temperature is constant.*

$$pv = \text{const.}, \quad \text{or} \quad p = \lambda\rho$$

and

$$p_0 = \lambda\rho_0,$$

therefore

$$p = \frac{p_0}{\rho_0}\rho.$$

Suppose the density of the atmosphere uniform throughout and equal to ρ_0, then $p_0 = \rho_0 k$, where k is the height of the homogeneous atmosphere, that is, the height of an atmosphere of uniform density which will give the barometric pressure, p_0.

Therefore

$$\frac{p_0}{\rho_0} = k = 27{,}800 \text{ about},$$

and

$$p = k\rho \quad \ldots\ldots \quad \text{III.};$$

from III.,

$$dp = kd\rho \quad \ldots\ldots \quad \text{IV.};$$

from I. and IV.,

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^h \frac{dh}{k};$$

hence

$$\log\frac{\rho}{\rho_0} = -\frac{h}{k},$$

therefore

$$\frac{\rho}{\rho_0} = e^{-\frac{h}{k}};$$

from II.,

$$f = \frac{\tau_0}{\tau_h} = \frac{\rho_0}{\rho} = e^{\frac{h}{k}} \quad \text{and} \quad \tau_h = \frac{\tau_0}{f} = \tau_0 \,.\, e^{-\frac{h}{k}} \quad \ldots\ldots \quad \text{V.}$$

2nd Case: Suppose the rate of diminution of temperature constant per foot upwards of the atmosphere.

In this case, assuming that the adiabatic law follows, namely :—

$$pv^{\gamma} = \text{const.} = \lambda,$$

or

$$p = \lambda\rho ,$$

and

$$p_0 = \lambda\rho_0{}^{\gamma}.$$

Therefore

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma} \quad \ldots\ldots\ldots\ldots \quad \text{VI.}$$

Differentiate VI.,

$$dp = \frac{p_0}{\rho_0{}^{\gamma}} \cdot \gamma\rho^{\gamma-1}\,d\rho \,. \quad \ldots\ldots\ldots\ldots \quad \text{VII.}$$

From I. and VII.,

$$\gamma \cdot \frac{p_0}{\rho_0{}^{\gamma}}\,\rho^{\gamma-1}\,d\rho = -\rho\,dh,$$

or

$$\int_{\rho_0}^{\rho} \rho^{\gamma-2}\,d\rho = -\int_0^h \frac{\rho_0{}^{\gamma}}{\gamma p_0}\,dh,$$

$$\frac{\rho^{\gamma-1}}{\gamma-1} - \frac{\rho_0{}^{\gamma-1}}{\gamma-1} = -\frac{\rho_0{}^{\gamma-1}}{\gamma} \cdot \frac{h}{k} \quad \text{since} \quad \frac{p_0}{\rho_0} = k,$$

$$\left(\frac{\rho}{\rho_0}\right)^{\gamma-1} = 1 - \frac{\gamma-1}{\gamma} \cdot \frac{h}{k},$$

$$f = \frac{\rho_0}{\rho} = \left\{1 - \frac{\gamma-1}{\gamma} \cdot \frac{h}{k}\right\}^{-\frac{1}{\gamma-1}} \quad \ldots\ldots\ldots\ldots \quad \text{VIII.}$$

CHAPTER II.

CONSTRUCTION OF A BALLISTIC TABLE.

THE experimental determination of the *resistance of the air* having been obtained by the experiments carried out in 1902–1906, the calculation of a Ballistic Table can be carried out.

In the previous chapter the value of K is given for each region of velocity, and in the same chapter it is shown that

$$pg = Cr = K\left(\frac{v}{1000}\right)^3$$

(see (15), (17), of Chapter I), where

$$K = C\frac{d^2t}{ds^2}10^9$$

and K is of the form (see p. 27)

$$[A_0]\left(\frac{v}{1000}\right)^m,$$

where $[A_0]$ denotes the antilogarithm of A_0, hence Cr is of the form

$$[A_0]\left(\frac{v}{1000}\right)^{m+3} = \frac{[A_0]}{(1000)^n}v^n, \text{ where } n = m + 3$$

$$= A]v^n. \quad \ldots \quad (1)$$

Thus, in Region VIII of velocity, for $v < 840$

$$K = [1\cdot6717017]\left(\frac{v}{1000}\right)^{-1\cdot4};$$

hence for this region

$$Cr = \frac{[1\cdot6717017]}{(1000)^{1\cdot6}}v^{-1\cdot4+3} = [\bar{4}\cdot8717017]v^{1\cdot6}.$$

In the Region III of velocity $2000 < v < 2600$,

$$K = [2\cdot2671157]\left(\frac{v}{1000}\right)^{-1\cdot5};$$

Therefore for this region

$$Cr = \frac{[2\cdot2671157]}{(1000)^{1\cdot5}}v^{-1\cdot5+3}$$

$$= [\bar{3}\cdot7671157]v^{1\cdot5}.$$

The following table gives the value of Cr for each region of velocity:—

Region.	v	$pg = Cr = K\left(\dfrac{v}{1000}\right)^3$.
I	5000–4000	$[\bar{3}\cdot9790234]\,v^{1\cdot45} = [A_1]\,v^{n_1}$
II	4000–2600	$[\bar{3}\cdot1865702]\,v^{1\cdot67} = [A_2]\,v^{n_2}$
III	2600–2000	$[\bar{3}\cdot7671157]\,v^{1\cdot5} = [A_3]\,v^{n_3}$
IV	2000–1460	$[\bar{4}\cdot7768067]\,v^{1\cdot8} = [A_4]\,v^{n_4}$
V	1460–1190	$[\bar{8}\cdot9795830]\,v^{3} = [A_5]\,v^{n_5}$
VI	1190–1040	$[\overline{18}\cdot3689459]\,v^{6\cdot45} = [A_6]\,v^{n_6}$
VII	1040–840	$[\bar{8}\cdot7777107]\,v^{3} = [A_7]\,v^{n_7}$
VIII	840–0	$[\bar{4}\cdot8717017]\,v^{1\cdot6} = [A_8]\,v^{n_8}$

The above values of Cr are for a standard projectile of 2 calibres ogive, moving under standard conditions in air of standard density, and these values of Cr are required for computing Ballistic Tables given at the end of the book.

To find t, the time it takes in seconds for the velocity of a projectile, d inches in diameter and weighing w pounds, to fall from any initial velocity, V f/s, to any final velocity, v f/s.

If dt seconds is the time during which the resistance R of the air causes the velocity to fall dv f/s, so that the velocity drops from $v+\frac{1}{2}dv$ to $v-\frac{1}{2}dv$ in passing through the mean velocity v, then, in accordance with the laws of motion,

$$R\,dt = \text{loss of momentum of the shot, in second-pounds}$$

$$= w\frac{v+\frac{1}{2}dv}{g} - w\frac{v-\frac{1}{2}dv}{g} = w\frac{dv}{g},$$

and from (7), Chapter I,

$$R = n\,d^2p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2);$$

hence

$$dt = \frac{w}{nd^2}\frac{dv}{pg} = C\frac{dv}{pg},$$

$$\frac{dt}{C} = \frac{dv}{pg} = \frac{dv}{Cr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where C is the *ballistic* coefficient of the shot.

Since r is a retardation and v decreases as t increases, then with dt, dv very small,

$$\int_0^t \frac{dt}{C} = \int_V^v -\frac{dv}{Cr} = +\int_v^V \frac{dv}{[A]\,v^n} = \sum_v^V \frac{dv}{Cr},$$

giving

$$\frac{t}{C} = T(V) - T(v) = \sum_v^V\left(\frac{dv}{Cr}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

which is employed for calculating Table II, and if V and v are both within the same region of velocity, the integration can be carried out at one step; if V and v are in different regions of velocity, two (or more) integrations will be required (see p. 37).

Next, if the shot advances a distance ds feet in the time dt seconds, during which the velocity drops from $v+\frac{1}{2}dv$ to $v-\frac{1}{2}dv$,

$$\mathrm{R} \,.\, ds = \text{loss of kinetic energy in foot-pounds}$$

$$= w\frac{(v+\frac{1}{2}dv)^2}{2g} - w\frac{(v-\frac{1}{2}dv)^2}{2g} = w\frac{v\,dv}{g};$$

so that

$$ds = \frac{w}{nd^2}\frac{v\,dv}{pg} = \mathrm{C}\frac{v\,dv}{pg},$$

$$\frac{ds}{\mathrm{C}} = \frac{v\,dv}{pg} = v\frac{dv}{\mathrm{C}r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

and the distance s feet through which the shot advances whilst its velocity drops from the initial V f/s to the final v f/s is given by

$$\int_0^s \frac{ds}{\mathrm{C}} = \int_{\mathrm{V}}^{v} - v\frac{dv}{\mathrm{C}r} = \int_v^{\mathrm{V}} + \frac{v\,dv}{[\mathrm{A}]\,v^n} = \int_v^{\mathrm{V}} \frac{dv}{[\mathrm{A}]\,v^{n-1}},$$

giving

$$\frac{s}{\mathrm{C}} = \mathrm{S}(\mathrm{V}) - \mathrm{S}(v) = \sum_v^{\mathrm{V}}\left(\frac{v\,dv}{\mathrm{C}r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

which is employed for calculating Table III; as before, if V and v are in the same region of velocity, one integration only is necessary, but more than one if these are not in the same region (see p. 41).

A third table (Table IV), is useful for determining the change in direction of motion of a projectile while the velocity drops from any initial value V to any final value v.

To explain the theory of this table, let the tangent at the point of the trajectory, where the velocity is v, make an angle i radians with the horizon.

Then, if di denotes the infinitesimal *decrement* of i in the infinitesimal increment of time dt, resolving normally in the trajectory (see *Notes on Dynamics*, 2nd edition, 1908, Sir G. Greenhill, published by Wyman and Sons),

$$v\frac{di}{dt} = g\cos i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

This may be proved in the following manner: Suppose that in passing through the point P on the trajectory, where the inclination is i radians, the velocity drops from

$$v + \tfrac{1}{2}\Delta v \;\text{ to }\; v - \tfrac{1}{2}\Delta v \text{ f/s},$$

as the shot passes from Q to R, where the inclinations are

$$i + \tfrac{1}{2}\Delta i \text{ and } i - \tfrac{1}{2}\Delta i \text{ radians.}$$

Measure off the length TU and TV from T, the point of intersection of the tangents at Q and R, to represent to scale the velocity at Q and R; then UV represents to the same scale the *change in velocity* in passing from Q to R.

Draw UW vertical, and VW parallel to the tangent at P, so as to form the triangle UVW; then, on the assumption that the average resistance of the air acts in the direction of the tangent at P, the triangle of velocity UVW shows that UW represents the change in velocity due to gravity, and WV the change due to the resistance of the air; so that if the shot takes Δt seconds to pass from Q to R, we may put

$$\mathrm{UW} = g\Delta t,$$

$$\mathrm{WV} = r\Delta t,$$

if r denotes the average retardation due to the resistance of the air.

Drawing TYZ and VWX parallel to the tangent at P, and dropping the perpendiculars VY and UZX, then

$$\begin{aligned} UX &= UW \cos i = g\Delta t \cos i \\ &= UZ + ZX = UZ + VY \\ &= (v + \tfrac{1}{2}\Delta v) \sin \tfrac{1}{2}\Delta i + (v - \tfrac{1}{2}\Delta v) \sin \tfrac{1}{2}\Delta i \\ &= 2v \sin \tfrac{1}{2}\Delta i, \end{aligned}$$

or

$$g \cos i = 2v \frac{\sin \tfrac{1}{2}\Delta i}{\Delta t},$$

and when Δt and Δi are indefinitely small, this leads to

$$v \frac{di}{dt} = g \cos i.$$

Also

$$XV = XW + WV = (g \sin i + r)\, \Delta t;$$

also

$$XV = YZ = TZ - TY = (v + \tfrac{1}{2}\Delta v) \cos \frac{\Delta i}{2} - (v - \tfrac{1}{2}\Delta v) \cos \frac{\Delta i}{2} = \Delta v \cos \tfrac{1}{2}\Delta i,$$

$$g \sin i + r = \frac{\Delta v}{\Delta t} \cos \tfrac{1}{2}\Delta i.$$

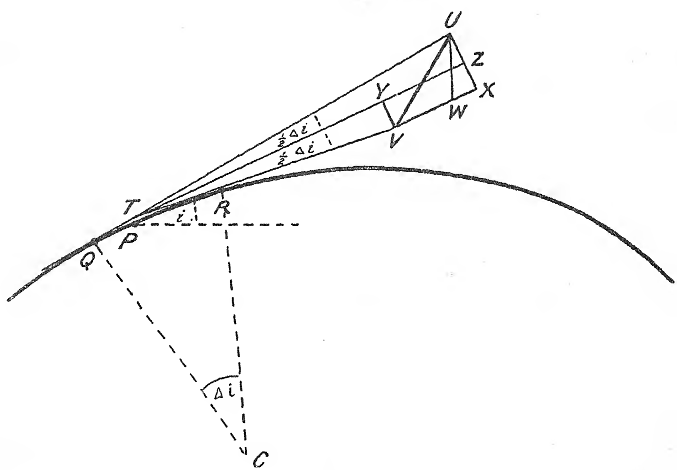


Fig. 4.

In the limit,

$$g \sin i + r = \frac{dv}{dt},$$

the equation obtained by resolving tangentially in the trajectory.

If the trajectory is sufficiently flat for $\cos i$ to be replaced by unity, then, since i diminishes as t increases, the curve QPR being concave downward, (7) becomes

$$v\frac{di}{dt} = -g, \quad \text{or} \quad di = -\frac{g}{v}\,dt \qquad \qquad (8),$$

where v denotes the mean velocity during a very small increment of time dt, during which the direction of motion of the shot changes through di radians.

If the inclination δ, or change of direction $d\delta$, be in degrees, then

$$\frac{\delta}{180} = \frac{i}{\pi} \quad \text{and} \quad d\delta = \frac{180}{\pi}\,di = -\frac{180g}{\pi}\,\frac{g}{v}\,dt.$$

The formula

$$v\frac{di}{dt} = -g$$

may also be proved thus:—

The arc QR is ds feet and the curvature of this arc is di radians, hence the mean curvature per foot is $\frac{di}{ds}$ radians, which may be taken as the curvature at P.

Resolve along the normal at P, where the velocity is v f/s,

$$\frac{v^2}{\rho} = g\cos i,$$

where ρ is the radius of curvature and is equal to $-\frac{ds}{di}$; the negative sign is taken because i decreases as s increases.

Therefore

$$v^2\frac{di}{ds} = -g\cos i,$$

$$v^2\frac{di}{dt}\cdot\frac{dt}{ds} = -g\cos i,$$

giving

$$v\frac{di}{dt} = -g\cos i,$$

and when i is small,

$$di = -\frac{g}{v}\,dt,$$

as before.

Employing this formula with that previously proved

$$\frac{dt}{C} = \frac{dv}{Cr},$$

where C is the ballistic coefficient,

$$\frac{di}{C} = -\frac{g}{v}\,\frac{dv}{Cr} \qquad \qquad (9),$$

hence

$$\int_0^i \frac{di}{C} = \sum_V^v\left(-\frac{g}{v}\,\frac{dv}{Cr}\right) = \int_V^v -\frac{g}{v}\,\frac{dv}{Cr},$$

$$\frac{i}{C} = \int_v^V \frac{g}{v}\cdot\frac{dv}{[A]\,v^n} = \frac{g}{[A]}\int_v^V \frac{dv}{v^{n+1}},$$

giving

$$\frac{i}{C} = I(V) - I(v) = \sum_v^V\left(\frac{g}{v}\cdot\frac{dv}{Cr}\right) \qquad \qquad (10),$$

which is employed for calculating Table IV. If V and v both lie in the same region of velocity, only one integration is necessary, but more than one if these do not lie in the same region of velocity, see p. 44.

TABLE II.

Time t seconds between Velocity V and v f/s. $t = C(T_V - T_v)$.

$$\frac{t}{C} = \sum_v^V \frac{dv}{Cr} = \int_v^V \frac{dv}{[A]v^n} \text{ from p. 33,}$$

hence

$$\frac{t}{C} = \frac{1}{[A]}\left\{\frac{v^{-n+1}}{-n+1}\right\}_v^V$$

$$= \frac{1}{[A](n-1)}\left\{\frac{1}{v^{n-1}} - \frac{1}{V^{n-1}}\right\},$$

where V is the upper limit in each region of velocity. Thus *in Region I*, V is 5000 f/s and v is the variable velocity. Therefore

$$\frac{t}{C} = \frac{1}{(n-1)[A]v^{n-1}} - B_t,$$

where B_t is a constant for any one region of velocity, and *considering Region I* we have $4000 < v < 5000$ f/s.

$$[A] = [A_1] = [\bar{3}\cdot 9790234],$$

$$n = n_1 = 1\cdot 45,$$

$$B_t = B_1 = \frac{1}{0\cdot 45\,[\bar{3}\cdot 9790234]\,(5000)^{0\cdot 45}}.$$

$$\log 0\cdot 45 = \bar{1}\ 6532125,$$

$$[A_1] = \bar{3}\cdot 9790234$$

$$\bar{3}\cdot 6322359,$$

therefore

$$\log \frac{1}{0\cdot 45\,[A_1]} = 2\cdot 3677641.$$

$$0\cdot 45 \log 5000 = 1\cdot 6645365,$$

$$\log B_1 = 0\cdot 7032276,$$

$$B_1 = 5\cdot 0493.$$

Hence in Region I, $4000 < v < 5000$,

$$\frac{t}{C} = [2\cdot 3677641]\,v^{-0\cdot 45} - 5\cdot 0493,$$

where v may have any value from 4000 to 5000 f/s, and so the value of $\frac{t}{C}$ is found for a fall of velocity of 5000 to any other velocity down to 4000 f/s.

Region II, $2600 < v < 4000$.

$$[A] = [A_2] = [\bar{3}\cdot 1865702],$$

$$n = n_2 = 1\cdot 67,$$

$$\frac{t}{C} = \frac{1}{0\cdot 67\,[\bar{3}\cdot 1865702]\,v^{0\cdot 67}} - B_2 \text{ (a constant).}$$

To find B_2, we must have the value of $\frac{t}{C}$ for $v = 4000$ f/s in Region I, the same as that for $v = 4000$ f/s in Region II, hence

$$[2\cdot3677641]\,(4000)^{-0\cdot45} - 5\cdot0493$$

must be equal to

$$\frac{1}{0\cdot67\,[\bar{3}\cdot1865702]\,(4000)^{0\cdot67}} - B_2.$$

$$\begin{aligned} & 2\cdot3677641, \\ 0\cdot45 \log 4000 &= \underline{1\cdot620927} \\ \text{Difference} &= 0\cdot7468371 = \log 5\cdot5826, \end{aligned}$$

and

$$5\cdot5826 - 5\cdot0493 = 0\cdot5333.$$

Again,

$$\begin{aligned} \log A_2 &= \bar{3}\cdot1865702, \\ \log 0\cdot67 &= \underline{\bar{1}\cdot8260748} \\ \text{Sum} &= \bar{3}\cdot0126450, \end{aligned}$$

therefore

$$\begin{aligned} \log \frac{1}{0\cdot67\,[\bar{3}\cdot1865702]} &= 2\cdot9873550 \\ 0\cdot67 \log 4000 &= \underline{2\cdot4133802} \\ \text{Difference} &= 0\cdot5739748, \end{aligned}$$

therefore

$$\frac{1}{0\cdot67\,[\bar{3}\cdot1865702]\,(4000)^{0\cdot67}} = \text{antilog } 0\cdot5739748 = 3\cdot7495.$$

Hence

$$0\cdot5333 = 3\cdot7495 - B_2,$$

$$B_2 = 3\cdot2162,$$

so that *in Region II*, $2600 < v < 4000$,

$$\frac{t}{C} = [2\cdot9873550]\,v^{-0\cdot67} - 3\cdot2162.$$

Region III, $2000 < v < 2600$.

$$[A] = [A_3] = [\bar{3}\cdot7671157],$$

$$n = n_3 = 1\cdot5,$$

and

$$\frac{t}{C} = \frac{1}{0\cdot5\,[\bar{3}\cdot7671157]\,v^{0\cdot5}} - B_3,$$

and B_3 must be found in similar fashion to B_2, the value of $\frac{t}{C}$ for $v = 2600$ f/s in Region II being made equal to that in Region III for a like velocity.

Therefore

$$\frac{1}{0\cdot5\,[\bar{3}\cdot7671157]\,(2600)^{0\cdot5}} - B_3 = [2\cdot9873550]\,(2600)^{-0\cdot67} - 3\cdot2162,$$

from which

$$B_3 = 4\cdot9175.$$

Also

$$\frac{1}{0\cdot5\,[\bar{3}\cdot7671157]} = [2\cdot5329143],$$

hence in Region III, $2000 < v < 2600$ f/s,

$$\frac{t}{C} = [2 \cdot 5339143]\, v^{-0 \cdot 5} - 4 \cdot 9175.$$

For the remaining five regions of velocity, similar calculations are made. The summary for all eight regions is as follows :—

Region.	v	$T_{5000} - T_v = T'_v$
I	5000–4000	$[2 \cdot 3677641]\, v^{-0 \cdot 45} - 5 \cdot 0493$
II	4000–2600	$[2 \cdot 9873550]\, v^{-0 \cdot 67} - 3 \cdot 2162$
III	2600–2000	$[2 \cdot 5339143]\, v^{-0 \cdot 5} - 4 \cdot 9175$
IV	2000–1460	$[3 \cdot 3201033]\, v^{-0 \cdot 8} - 2 \cdot 0505$
V	1460–1190	$[6 \cdot 7193870]\, v^{-2} + 1 \cdot 6373$
VI	1190–1040	$[16 \cdot 8946576]\, v^{-5 \cdot 45} + 3 \cdot 9800$
VII	1040– 840	$[6 \cdot 9212593]\, v^{-2} - 0 \cdot 9022$
VIII	below 840	$[3 \cdot 3501470]\, v^{-0 \cdot 6} - 28 \cdot 4874$

In Table II, T_{5000} is given the value of 113, an arbitrary but convenient number, which ensures that all values of T_v are positive.

Therefore

$$T_v = T_{5000} - T'_v = 113 - T'_v.$$

Examples :—In Region VIII, put $v = 100$ f/s,

$$T'_{100} = [3 \cdot 3501470]\,(100)^{-0 \cdot 6} - 28 \cdot 4874,$$

$$\begin{aligned} & 3 \cdot 3501470 \\ 0 \cdot 6 \log 100 = {} & 1 \cdot 2 \\ \text{Difference} = {} & 2 \cdot 1501470 = \log 141 \cdot 301. \end{aligned}$$

herefore

$$T'_{100} = 141 \cdot 301 - 28 \cdot 4874 = 112 \cdot 814,$$

$$T_{100} = 113 - 112 \cdot 814 = 0 \cdot 186,$$

and this is the actual value for T_{100} in Table II.

From the difference for 1 f/s in the formula for T'_v we have

$$\begin{aligned} T_{100} &= 0 \cdot 186 \\ \Delta &= 0 \cdot 841 \\ T_{101} &= 1 \cdot 027 \\ \Delta &= 0 \cdot 828 \\ T_{102} &= 1 \cdot 855, \text{ and so on,} \end{aligned}$$

and these are the values of T_{101}, T_{102}, ... in the table.

Again, in Region I, put $v = 4000$, then

$$T'_{4000} = [2\cdot367764](4000)^{-0\cdot45} - 5\cdot0493,$$

$$\begin{aligned} & 2\cdot367764 \\ 0\cdot45 \log 4000 = {} & \underline{1\cdot620927} \\ \text{Difference} = {} & 0\cdot746837 = \log 5\cdot58256. \end{aligned}$$

Therefore

$$T'_{4000} = 5\cdot58256 - 5\cdot0493 = 0\cdot53326 = 0\cdot5333,$$

Hence

$$T_{4000} = 113 - 0\cdot533 = 112\cdot467.$$

which is the actual value of T_{4000} found in the table.

So for any other value of v; thus in Region II put $v = 2800$, then from the formula for T'_v we get

$$T'_{2800} = 1\cdot546,$$

giving

$$T_{2800} = 113 - 1\cdot546 = 111\cdot454,$$

the value found in the table.

The Ballistic Table for T'_v, as constructed by the integrals, presents the function as decreasing as the velocity increases. It is more convenient to have T_v increase with the velocity, and this can be done by direct subtraction from *a constant;* this can be done without in any way affecting a Gunnery problem, because the absolute value of T_v is never required, but only the difference such as $T_V - T_v$; hence the suitable value of 113 is given to the constant, from which are subtracted the various values of T'_v.

Table II for T_v might be compiled by the finite difference employed by Bashforth for his Ballistic Tables.

From the formula

$$\frac{dt}{C} = \frac{dv}{pg} = \frac{dv}{Cr},$$

if 10 f/s intervals be taken for the drop of velocity over any interval, then writing

$$\frac{\Delta t}{C} = \frac{\Delta v}{pg} = \frac{10}{pg}, \quad \text{see (3)},$$

the increment Δt for any velocity is found as soon as the value pg is obtained from Table I for the required velocity; thus :—

v	$\bar{v}$	$\Delta T = \frac{10}{pg}$	T
2000			110·272*
	2005	0·01904	
2010			110·291
	2015	0·01890	
2020			110·310
	2025	0·01876	
2030			110·329
	2035	0·01862	
2040			110·347
	2045	0·01849	
2050			110·366

* This value of T_{2000} is quoted from Table II.

TABLE III.

Distance s in feet between velocity V and v f/s, $s = \mathrm{C}(\mathrm{S}_{\mathrm{V}} - \mathrm{S}_v)$.

$$\frac{s}{\mathrm{C}} = \sum_{v}^{\mathrm{V}} \frac{v\,dv}{\mathrm{C}r} = \int_v^{\mathrm{V}} \frac{v\,dv}{[\mathrm{A}]v^n} = \frac{1}{[\mathrm{A}]}\int_v^{\mathrm{V}} \frac{dv}{v^{n-1}},$$

$$\frac{1}{[\mathrm{A}]}\left\{\frac{v^{-n+2}}{-n+2}\right\}_v^{\mathrm{V}} = \frac{+1}{(n-2)[\mathrm{A}]v^{n-2}} - \frac{1}{(n-2)[\mathrm{A}]\mathrm{V}^{n-2}},$$

$$= \frac{+1}{(n-2)[\mathrm{A}]v^{n-2}} - \mathrm{B}_s,$$

where B_s may be taken as constant for any one region of velocity, and V is the upper limit in each region of velocity; thus *in Region I* V is 5000 f/s and v is the variable velocity.

Consider Region I, $4000 < v < 5000$

$$[\mathrm{A}] = [\mathrm{A}_1] = \bar{3}\cdot 9790234],$$

$$n - 2 = -0\cdot 55,$$

$$-\mathrm{B}_s = \frac{+1}{0\cdot 55[\bar{3}\cdot 9790234](5000)^{-0\cdot 55}}.$$

$$\log 0\cdot 55 = \bar{1}\cdot 7403627,$$

$$[\mathrm{A}_1] = \bar{3}\cdot 9790234$$

$$\bar{3}\cdot 7193861.$$

Therefore

$$\log\frac{1}{0\cdot 55[\mathrm{A}_1]} = 2\cdot 2806139,$$

$$0\cdot 55 \log 5000 = 2\cdot 0344335,$$

therefore $\log(-\mathrm{B}_s) = 4\cdot 3150474 = \log 20656\cdot 06,$

therefore

$$-\mathrm{B}_s = 20656\cdot 06.$$

Hence, in Region I, $4000 < v < 5000$,

$$\frac{s}{\mathrm{C}} = 20656\cdot 06 - [2\cdot 2806139]\,v^{0\cdot 55}.$$

Region II, $2600 < v < 4000$.

$$\frac{s}{\mathrm{C}} = \frac{-1}{0\cdot 33[\bar{3}\cdot 1865702]v^{-0\cdot 33}} - \mathrm{B}_s.$$

The value of s_{4000} obtained from this must equal that got from Region I of velocities, hence,

$$\frac{-1}{0\cdot 33[\bar{3}\cdot 1865702](4000)^{-0\cdot 33}} - \mathrm{B}_s = 20656\cdot 06 - [2\cdot 2806139](4000)^{0\cdot 55},$$

$$\log\frac{1}{0\cdot 33[\bar{3}\cdot 1865702]} = 3\cdot 2949159,$$

$$\log(4000)^{0\cdot 33} = 1\cdot 1886798,$$

therefore

$$\log \frac{1}{0\cdot 33\,[A_2]\,(4000)^{-0\cdot 33}} = [4\cdot 4835957] = \log 30450\cdot 58,$$

also

$$0\cdot 55 \log 4000 = 1\cdot 9811330$$
$$\text{add } \underline{2\cdot 2806139}$$
$$4\cdot 2617469 = \log 18270\cdot 35,$$

therefore

$$-30450\cdot 58 - B_s = 20656\cdot 06 - 18270\cdot 35, \qquad -B_s = 32836\cdot 29,$$

and for Region II,

$$\frac{s}{C} = 32836\cdot 29 - [3\cdot 2949159]\,v^{0\cdot 33}.$$

For Region III, $2000 < v < 2600$.

$$\frac{s}{C} = \frac{1}{-0\cdot 5\,[\bar{3}\cdot 7671157]\,v^{-0\cdot 5}} - B_s$$
$$= 23855\cdot 06 - [2\cdot 5339143]\,v^{0\cdot 5},$$

and so on.

The results in tabular form are :—

Region.	v	$S_{5000} - S_v = S'_v$
I	5000–4000	$20656\cdot 06 - [2\cdot 2806139]\,v^{0\cdot 55}$
II	4000–2600	$32836\cdot 29 - [3\cdot 2949159]\,v^{0\cdot 33}$
III	2600–2000	$23855\cdot 06 - [2\cdot 5339143]\,v^{0\cdot 5}$
IV	2000–1460	$46791\cdot 21 - [3\cdot 9221633]\,v^{0\cdot 2}$
V	1460–1190	$[7\cdot 0204170]\,v^{-1} + 3717\cdot 23$
VI	1190–1040	$[16\cdot 9826941]\,v^{-4\cdot 45} + 10545\cdot 79$
VII	1040–840	$[7\cdot 2222893]\,v^{-1} - 1891\cdot 20$
VIII	below 840	$67623\cdot 71 - [3\cdot 5262383]\,v^{0\cdot 4}$

In Table III, S_{5000} is given the value of 50000, and

$$S_v = S_{5000} - S'_v.$$

Examples :—In Region VIII, for $v < 840$ f/s, put

$$v = 100,$$
$$S'_{100} = 67623\cdot 71 - [3\cdot 5262383]\,(100)^{0\cdot 4},$$
$$= 46428\cdot 5,$$

therefore

$$S_{100} = 50000 - 46428\cdot 5 = 3571\cdot 5,$$

which is the value found in the Table.

So

$$S_{101} = 3656 \cdot 1$$

$$\text{Difference for 1 f/s} = \underline{\quad 84 \cdot 0}$$

$$S_{102} = 3740 \cdot 1, \text{ \&c.}$$

$$S_{2800} \text{ (Region II)} = 50000 - 32836 \cdot 29 + [3 \cdot 2949159] (2800)^{\cdot 33}$$

$$= 50000 - 5766 \cdot 9 = 44233 \cdot 1,$$

and so on.

By means of this process Table III for S_v gives a function which increases as the velocity increases; and since in Gunnery the absolute value of S_v is not required, but only the difference such as $S_V - S_v$, the subtracting of S'_v from the constant 50000 presents the table for S_v in the more convenient form.

In a similar manner to Bashforth's, the method of a finite difference of 10 f/s drop in the velocity over any interval could be employed for the construction of Table III for S_v thus:—

Taking the value of pg for any required velocity from Table I, also see (5).

v	$\bar{v}$	$\frac{10}{pg}$	$\Delta s = \frac{v \, \Delta v}{pg} = \frac{10 v}{pg}$	S_v
2000				41435·7*
	2005	0·01904	38·180	
2010				41473·9
	2015	0·01890	38·084	
2020				41512·0
	2025	0·01876	37·990	
2030				41550·0
	2035	0·01862	37·897	
2040				41587·9
	2045	0·01849	37·804	
2050				41625·7

* This value of S_{2000} is quoted from Table III.

TABLE IV.—THE I (v) TABLE.

$$\frac{i}{C} = \frac{\phi^\circ - \theta^\circ}{57 \cdot 3\,C} = I(V) - I(v) \quad \text{or} \quad \tan\phi - \tan\theta = C[I(V) - I(v)].$$

From (10)

$$I(V) - I(v) = \frac{g}{[A]}\int_v^V \frac{dv}{v^{n+1}} = \frac{g}{n[A]v^n} - \frac{g}{n[A]V^n}.$$

In Region I of velocities, $4000 < v < 5000$,

$$n = n_1 = 1 \cdot 45$$

and

$$[A] = [A_1] = [\bar{3} \cdot 9790234],$$

$V = 5000$ f/s, v is the variable velocity; put

$$I'(v) = \frac{g}{n_1[A_1]v^{1 \cdot 45}} - B_i,$$

where B_i is a constant, which disappears in $I(v) - I(v')$.

In Region I take $B_i = 0$, therefore

$$I'(v) = \frac{g}{1 \cdot 45[\bar{3} \cdot 9790234]v^{1 \cdot 45}}.$$

For the computation of tables such as the double entry table, the values of $2I(v)$ are required; for this reason it is found to involve less labour to calculate $2I(v)$ at first, and to halve this for the table giving $\frac{i}{C} = I(V) - I(v)$.

For Region I of velocities,

$$2I'(v) = \frac{2g}{1 \cdot 45[\bar{3} \cdot 9790234]v^{1 \cdot 45}},$$

$$\log[A_1] = \bar{3}\ 9790234,$$

$$\log 1 \cdot 45 = 0 \cdot 1613680$$

$$\bar{2} \cdot 1403914,$$

$$\log 2g = 1 \cdot 8087615,$$

$$\log\frac{2g}{n_1[A_1]} = 3 \cdot 6683701,$$

therefore

$$2I'(v) = [3 \cdot 6683701]v^{-1 \cdot 45}.$$

Region II. of velocities, $2600 < v < 4000$.

$$[A_2] = [\bar{3} \cdot 1865702],$$

$$n = n_2 = 1 \cdot 67,$$

and

$$\frac{2g}{n_2[A_2]} = [4 \cdot 3994748],$$

hence for $v = 4000$ f/s

$$\frac{2g}{n_1 [A_1] (4000)^{1\cdot45}} = \frac{2g}{n_2 [A_2] (4000)^{1\cdot67}} - B_i,$$

giving

$$0\cdot027886 = 0\cdot024212 - B_i$$

$$-B_i = 0\cdot003674,$$

hence

$$2I'(v) = [4\cdot3994748]\, v^{-1\cdot67} + 0\cdot003674.$$

The value of $2I'(v)$ for each region of velocity is as follows:—

Region.	v	$2I'(v)$
I	5000–4000	$[3\cdot6683701]\, v^{-1\cdot45}$
II	4000–2600	$[4\cdot3994748]\, v^{-1\cdot67} + 0\cdot003674$
III	2600–2000	$[3\cdot8655545]\, v^{-1\cdot5} - 0\cdot001960$
IV	2000–1460	$[4\cdot7766823]\, v^{-1\cdot8} + 0\cdot011713$
V	1460–1190	$[8\cdot3520572]\, v^{-3} + 0\cdot059897$
VI	1190–1040	$[18\cdot6302559]\, v^{-6\cdot45} + 0\cdot131293$
VII	1040– 840	$[8\cdot5539295]\, v^{-3} - 0\cdot038958$
VIII	below 840	$[4\cdot7329398]\, v^{-1\cdot6} - 0\cdot567525$

From Region VIII, for $v = 500$,

$$\begin{aligned} 2I'_{500} &= [4\cdot7329398]\,(500)^{-1\cdot6} - 0\cdot567525 \\ &= [0\cdot4145878] - 0\cdot567525 \\ &= 2\cdot59769 - 0\cdot567525 \\ &= 2\cdot030165 \equiv 2\cdot03017, \\ I'_{500} &= 1\cdot015085, \\ I'_{501} &= 1\cdot01094, \\ I'_{502} &= 1\cdot00681, \\ I'_{503} &= 1\cdot00271, \end{aligned}$$

and so on.

From Region I,

$$I'_{4000} = 0\cdot013943.$$

Since differences of $I(v)$, such as $I(v) - I(u)$, only are required in practice, the value of I_{500} is taken in Table IV as zero, and for other velocities we have

$$\begin{aligned} I_{500} &= 0, \\ I_{501} &= 1\cdot015085 - 1\cdot01094 = 0\cdot00415, \\ I_{502} &= 1\cdot015085 - 1\cdot00681 = 0\cdot00827, \\ I_{503} &= 0\cdot01238, \\ &\ldots\ldots\ldots\ldots\ldots \\ I_{4000} &= 1\cdot015085 - 0\cdot013943 = 1\cdot00114. \end{aligned}$$

These are the values found in the Table IV.

The above process, put into the form of a formula, is

$$I_v = \tfrac{1}{2}[2\cdot03017 - 2I'(v)] = 1\cdot015085 - I'v$$

or

$$I_v = k - I'(v),$$

and by means of this Table IV the I function increases with the velocity.

By taking intervals in which the drop of velocity is 10 f/s, as was made by Bashforth in compiling his tables, the I_v table can be calculated thus: taking $\log g = 1\cdot50773$ or $g = 32\cdot19$.

v	$\bar{v}$	$\frac{\Delta t}{C} = \frac{10}{pg}$	$\frac{\Delta i}{C} = \frac{g}{v}\frac{10}{pg}$	I_v
2000				0·97505*
	2005	0·01904	0·000306	
2010				0·975356
	2015	0·01890	0·000302	
2020				0·975658
	2025	0·01876	0·000298	
2030				0·975956
	2035	0·01862	0·000295	
2040				0·97625
	2045	0·01849	0·000291	
2050				0·97654

* This value of I_{2000} is quoted from Table IV.

TABLE V FOR "A," THE ALTITUDE FUNCTION.

By definition (see Chapter IV, p. 64)

$$d\{A(v)\} = \frac{vI(v)\,dv}{Cr} = I_v\,dSv.$$

Therefore

$$2A(v) = \int_v^V -\frac{v}{[A]v^n}\left(\frac{2g}{n[A]v^n} - B_i\right)dv,$$

since

$$Cr = [A]v^n,$$

and

$$2I(v) = \frac{2g}{n[A]v^n} - B_i,$$

where B_i is a constant which has already been found for each of the eight regions of velocity

Therefore

$$2A(v) = 2\frac{-g}{n[A]^2}\int_v^V \frac{dv}{v^{2n-1}} + \frac{B_i}{[A]}\int_v^V \frac{dv}{v^{n-1}} - B_a$$

$$= \frac{2g}{2n(n-1)[A]^2v^{2(n-1)}} - \frac{B_i}{(n-2)[A]v^{n-2}} - B_a.$$

Therefore

$$2A(v) = \frac{g}{n(n-1)[A]^2v^{2(n-1)}} - \frac{B_i}{(n-2)[A]v^{n-2}} - B_a,$$

where B_a is a constant to be found for each region of velocity.

The explanation of the altitude function $A(v)$ is left for a later chapter; the actual value or definition is stated above, and from such definition the compilation only of the A table can be proceeded with.

"A" *Table for Region I,* $4000 < v < 5000$.

For this region B_i is zero, and

$$n = n_1 = 1\cdot45, \quad [A] = [A_1] = [\bar{3}\cdot9790234],$$

hence the term containing B_i disappears and

$$2A_v = \frac{g}{1\cdot45\times0\cdot45[\bar{3}\cdot9790234]^2v^{0\cdot9}} - B_a.$$

Take $2A_{5000}$ equal to zero, then, since

$$\left.\begin{aligned}\log\frac{g}{1\cdot45\times0\cdot45[A_1]^2} &= 5\cdot7351042\\ 0\cdot9\log5000 &= 3\cdot32907300\end{aligned}\right\}\text{difference} = 2\cdot4060312$$

therefore

$$\log B_a = 2\cdot4060312,$$

$$B_a = 254\cdot701,$$

and for Region I of velocities,

$$2A_v = [5\cdot7351042]v^{-0\cdot9} - 254\cdot701.$$

"A" *Table for Region II*, $2600 < v < 4000$.

$$[A] = [A_2] = [\bar{3}\cdot1865702], \quad n = n_2 = 1\cdot67,$$

$$2A_v = \frac{g}{n_2(n_2-1)[A_2]^2 v^{2(n-1)}} - \frac{B_i}{(n_2-2)[A_2]v^{n_2-2}} - B_a,$$

$$= \frac{g}{1\cdot67 \times 0\cdot67\,[A_2]^2 v^{1\cdot34}} - \frac{B_i}{(-0\cdot33)[A_2]v^{-0\cdot33}} - B_a,$$

$$\left.\begin{aligned} \log 1\cdot67 &= 0\cdot2227165 \\ \log 0\cdot67 &= \bar{1}\cdot8260748 \\ 2\log A_2 &= \bar{6}\cdot3731404 \\ & \overline{\bar{6}\cdot4219317} \end{aligned}\right\}$$

$$\left.\begin{aligned} \log g &= 1\cdot5077315 \\ \text{Difference} &= 7\cdot0857998 \end{aligned}\right\}.$$

Hence first term is $[7\cdot0857998]\,v^{-1\cdot34}$.

therefore

$$\left.\begin{aligned} \log 0\cdot33[A_2] &= \bar{4}\cdot7050841 \\ \log B_i = \log 0\cdot003674 &= \bar{3}\cdot5651392 \\ \log \frac{B_i}{0\cdot33[A_2]} &= 0\cdot8600551 \end{aligned}\right\}.$$

Hence second term is $[0\cdot8600551]\,v^{0\cdot33}$, therefore

$$2A_v = [7\cdot0857998]\,v^{-1\cdot34} - [0\cdot8600551]\,v^{0\cdot33} + B_a.$$

To find B_a, the value of $2A_{4000}$ must be the same whether obtained from Region I or from Region II, hence, putting $v = 4000$,

$$[5\cdot7351042](4000)^{-0\cdot9} - 254\cdot701 = [7\cdot0857998](4000)^{-1\cdot34} - [0\cdot8600551](4000)^{0\cdot33} - B_a.$$

$$\left.\begin{aligned} 0\cdot9\log 4000 &= 3\cdot24185400 \\ & 5\cdot7351042 \\ \log 311\cdot351 &= \overline{2\cdot49325020} \end{aligned}\right\}$$

$$\begin{aligned} & 311\cdot351 \\ & 254\cdot701 \\ & \overline{56\cdot650} \end{aligned}$$

$$\left.\begin{aligned} & 7\cdot0857998 \\ 1\cdot34\log 4000 &= 4\cdot8267604 \\ \log 181\cdot568 &= \overline{2\cdot2590394} \end{aligned}\right\}$$

$$\left.\begin{aligned} 0\cdot33\log 4000 &= 1\cdot1886798 \\ & 0\cdot8600551 \\ \log 111\cdot876 &= \overline{2\cdot0487349} \end{aligned}\right\}$$

$$\left.\begin{aligned} & 181\cdot568 \\ & 111\cdot876 \\ & \overline{69\cdot692} \end{aligned}\right\}$$

so that

$$56\cdot650 = 69\cdot692 - B_a,$$

$$B_a = 13\cdot042,$$

and the law for Region II of velocities is

$$2A(v) = [7\cdot0857998]\,v^{-1\cdot34} - [0\cdot8600551]\,v^{0\cdot33} - 13\cdot042.$$

The law for the other regions is found in a similar manner :—

Region.	v	$2A(v)$
I	5000–4000	$[5\cdot7351042]\,v^{-0\cdot9} - 254\cdot701$
II	4000–2600	$[7\cdot0857998]\,v^{-1\cdot34} - [0\cdot8600551]\,v^{0\cdot33} - 13\cdot042$
III	2600–2000	$[6\cdot0984388]\,v^{-1} + [\bar{1}\cdot8261704]\,v^{0\cdot5} - 303\cdot331$
IV	2000–1460	$[7\cdot7957556]\,v^{-1\cdot6} - [1\cdot9908314]\,v^{0\cdot2} + 474\cdot926$
V	1460–1190	$[14\cdot7704142]\,v^{-4} + [5\cdot7978221]\,v^{-1} + 35\cdot262$
VI	1190–1040	$[35\cdot2238835]\,v^{-10\cdot9} + [16\cdot1009356]\,v^{-4\cdot45} + 546\cdot710$
VII	1040– 840	$[15\cdot1741588]\,v^{-4} - [5\cdot8128860]\,v^{-1} + 586\cdot340$
VIII	below 840	$[7\cdot7820568]\,v^{-1\cdot2} + [3\cdot2802233]\,v^{0\cdot4} - 44114\cdot15$

The value of $A(v)$ obtained from these laws decreases as the velocity increases. Table V is made out for a function A_v, which increases as the velocity increases, and also $A_{500} = 0$.

The direct conversion cannot be carried out by the same simple method employed by I_v, because $A_V - A_v$ is a direct function, not of $I_V - I_v$ or ΔI, but of the absolute value of I_v at any point.

Let S'_v, I'_v denote absolute values as obtained from the integration formulæ on pp. 42, 45; let S_v, I_v denote the values got from Tables III and IV.

Let

$$d(A'_v) = I_v\,dS_v.$$

Now

$$I_v = k - I'_v,$$

where

$$k = 1\cdot015085, \text{ see p. } 46.$$

Therefore

$$d(A'_v) = k\,d(S'_v) - I'_v\,d(S'_v).$$

Integrate

$$A'_{500} - A'_v = k(S'_{500} - S'_v) - \{A_{(500)} - A_{(v)}\},$$

where $2A_{(500)}$ and $2A_{(v)}$ are the values got direct from the integration formulæ given just above.

Put

$$A_v = A'_{500} - A'_v,$$

then

$$A_v = \tfrac{1}{2}\left[2k(S'_{500} - S'_v) - (2A_{(500)} - 2A_{(v)})\right],$$

in which S'_{500}, S'_v, $2A_{(500)}$, $2A_{(v)}$, are got direct from the tabular formulæ on pp. 42, 49, and $2k = 2\cdot03017$, since $S'_{500} - S'_v = S_{500} - S_v$, this difference can also be got direct from Table III. The values of A_v found from this formula are those in Table V.

Example 1.—To calculate A_{600} for Table V:—

$$S'_{500} = 27275 \cdot 31 \qquad 2A_{(500)} = 13722 \cdot 16$$
$$S'_{600} = 24222 \cdot 80 \qquad 2A_{(600)} = 8589 \cdot 07$$
$$\Delta S' = 3052 \cdot 51 \qquad \text{Difference} = 5133 \cdot 09$$

$$\log \Delta S' = 3 \cdot 4846571$$
$$\log 2k = 0 \cdot 3075324$$
$$\log (2k \,.\, \Delta S') = 3 \cdot 7921895$$
$$\left.\begin{aligned} 2k \,.\, \Delta S' &= 6197 \cdot 11 \\ 2A_{(500)} - 2A_{(600)} &= 5133 \cdot 09 \end{aligned}\right\}$$
$$2A_{600} = 1064 \cdot 02$$
$$A_{600} = 532 \cdot 01$$

Example 2.—To calculate A_{800}:—

$$S'_{500} = 27275 \cdot 31 \qquad 2A_{(500)} = 13722 \cdot 16$$
$$S'_{800} = 18929 \cdot 84 \qquad 2A_{(800)} = 3397 \cdot 75$$
$$\Delta S' = 8345 \cdot 47 \qquad 2\,\Delta A_{(v)} = 10324 \cdot 41$$

$$\log \Delta S' = 3 \cdot 9214508$$
$$\log 2k = 0 \cdot 3075324$$
$$\log (2k \,.\, \Delta S') = 4 \cdot 2289832$$
$$\left.\begin{aligned} 2k \,.\, \Delta S' &= 16942 \cdot 72 \\ 2\,\Delta A_{(v)} &= 10324 \cdot 41 \end{aligned}\right\}$$
$$2A_v = 6618 \cdot 31$$
$$A_v = 3309 \cdot 16$$

Example 3.—To calculate A_{1200}:—

$$S'_{500} = 27275 \cdot 31 \qquad 2A_{(500)} = 13722 \cdot 16$$
$$S'_{1200} = 12451 \cdot 70 \qquad 2A_{(1200)} = 842 \cdot 67$$
$$\Delta S' = 14823 \cdot 61 \qquad 2\,\Delta A_v = 12879 \cdot 49$$

$$\log \Delta S' = 4 \cdot 1709539$$
$$\log 2k = 0 \cdot 3075324$$
$$\log (2k \,.\, \Delta S') = 4 \cdot 4784863$$
$$\left.\begin{aligned} 2k\Delta S' &= 30094 \cdot 45 \\ 2\,\Delta A_{(v)} &= 12879 \cdot 49 \end{aligned}\right\}$$
$$2A_v = 17214 \cdot 96$$
$$A_v = 8607 \cdot 48$$

To save labour in dealing with large numbers, a change is made at certain intervals and smaller numbers have in consequence to be dealt with. The intervals selected are at velocities of 800, 1000, 1600, and 2800; a slight smoothing is made before entry.

As an example, calculate A_{1200}:—

$$A_v = \tfrac{1}{2}[2k(S'_{500} - S'_v) - \{2A_{(500)} - 2A_{(v)}\}],$$
$$= \tfrac{1}{2}[2k(S'_{1000} - S'_v) - \{2A_{(1000)} - 2A_{(v)}\}],$$
$$+ \tfrac{1}{2}[2k(S'_{500} - S'_{1000}) - \{2A_{(500)} - 2A_{(1000)}\}],$$

which may be written

$$A_v = A'_v + A_{1000},$$

where

$$A'_v = \tfrac{1}{2}[2k(S'_{1000} - S'_v) - \{2A_{(1000)} - 2A_{(v)}\}],$$

$$\begin{array}{rlrl} S_{1000} = & 14792\cdot38 & 2A_{(1000)} = & 1429\cdot72 \\ S'_{1200} = & 12451\cdot68 & 2A_{(1200)} = & 842\cdot67 \\ \hline \Delta S' = & 2340\cdot70 & 2\,\Delta A_{(v)} = & 587\cdot05 \end{array}$$

$$\log \Delta S' = 3\cdot3693458$$
$$\log 2k = 0\cdot3075324$$
$$\log(2k\,.\,\Delta S') = 3\cdot6768782$$

$$\left.\begin{array}{r} 2k\,.\,\Delta S' = 4752\cdot02 \\ 2\,\Delta A_{(v)} = 587\cdot05 \end{array}\right\}$$

Therefore

$$2k\,\Delta S' - 2\,\Delta A_{(v)} = 2A'_v = 4164\cdot97,$$

$$\left.\begin{array}{r} A'_v = 2082\cdot49 \\ A_{1000} = 6525\cdot02 \end{array}\right\}$$

$$A_{1200} = 8607\cdot51,$$

which is practically as found before.

Table V for A_v could be calculated by means of Tables I and IV, by taking intervals of 10 f/s drop of velocity, as was done by Mr. Hadcock, who calculated the altitude function from the formula

$$\Delta A = \frac{v I(v)}{F(v)}\Delta_v = \frac{v\,\Delta_v}{pg} I_v = I_v\,\Delta S_v.$$

v	$\bar{v}$	$I(\bar{v})$	ΔS_v*	$\Delta A = I_{\bar{v}}\,\Delta S_v$	A_v
2000					12309·13†
	2005	0·97520	38·180	37·232	
2010					12346·362
	2015	0·97551	38·084	37·151	
2020					12383·513
	2025	0·97581	37·990	37·071	
2030					12420·584
	2035	0·97610	37·897	36·992	
2040					12457·576
	2045	0·97639	37·804	36·912	
2050					12494·478

* These values of ΔS_v are obtained from p. 43, in calculating S_v by finite differences method.

† This value of A_{2000} is quoted from Table V.

NOTE.—For calculation of the double-entry Table VIII, see p. 79.

CHAPTER III.

THE UNRESISTED MOTION OF A PROJECTILE.

IN ordinary problems of direct fire, the attraction of gravity is a force which is usually small in comparison with the resistance of the air, and may therefore be left out of account in a first approximation to the solution of these problems. In high-angle fire with low velocities, the reverse conditions hold good, and the force of gravity is much greater than the resistance of the air, and the latter may (as an approximation) be left out of account when compared with the force of gravity. As an example, take a projectile weighing 100 lbs. and 6 inches calibre, fired under standard conditions of air density, shape and steadiness; then from Table I, p. 149.

v	$\mathrm{R} = pd^2$ lbs.	Weight in lbs.
400	$0 \cdot 337 \times 36 = 12 \cdot 1$	100
520	$0 \cdot 512 \times 36 = 18 \cdot 4$	100
800	$1 \cdot 02 \times 36 = 36 \cdot 7$	100
1000	$1 \cdot 86 \times 36 = 67$	100
2000	$16 \cdot 25 \times 36 = 585$	100
2400	$21 \cdot 36 \times 36 = 770$	100
2800	$27 \cdot 26 \times 36 = 980$	100

On the assumption that the resistance of the air may be disregarded for high-angle fire with low velocities, a fair approximation to the trajectory is obtained at short ranges, as with howitzer and mortar fire.

Supposing R, the resistance of the air, and therefore also r, the retardation it produces, to be zero,

$$\frac{d^2x}{dt^2} = 0 \quad \ldots \ldots \quad (1),$$

$$\frac{d^2y}{dt^2} = -g \quad \ldots \ldots \quad (2).$$

Integrating these equations with respect to t, supposing the shot is projected from the origin O with velocity V f/s at an elevation α,

$$\frac{dx}{dt} = \text{a constant} \qquad = \mathrm{V}\cos\alpha \quad \ldots \ldots \quad (3),$$

$$\frac{dy}{dt} = \text{a constant} - gt = \mathrm{V}\sin\alpha - gt \quad \ldots \ldots \quad (4).$$

Integrating the equation (3) and (4) again with respect to t,

$$x = \mathrm{V}t\cos\alpha \quad \ldots \ldots \quad (5),$$

$$y = \mathrm{V}t\sin\alpha - \tfrac{1}{2}gt^2 \quad \ldots \ldots \quad (6),$$

no constant of integration being required if the time of flight, t, is reckoned from the instant the shot leaves the point of projection O; these are the equations employed in Chapter II, Part I.

From (5)

$$t = \frac{x}{V \cos \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

and, substituting this value of t in (6),

$$y = x \tan \alpha - \frac{gx^2}{2V^2 \cos^2 \alpha} = TM - TP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Treating this equation as a quadratic in x, by writing it

$$x^2 - 2x \frac{V^2}{g} \sin \alpha \cos \alpha = -\frac{2V^2}{g} y \cos^2 \alpha,$$

and completing the square in x, then

$$\left(x - \frac{V^2}{g} \sin \alpha \cos \alpha\right)^2 = -\frac{2V^2}{g} \cos^2 \alpha \left(y - \frac{V^2}{2g} \sin^2 \alpha\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

which is of the form

$$(x - h)^2 = -p (y - k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

the equation of a parabola (fig. 1) whose axis is vertical, and vertex A at the highest point (h, k), where

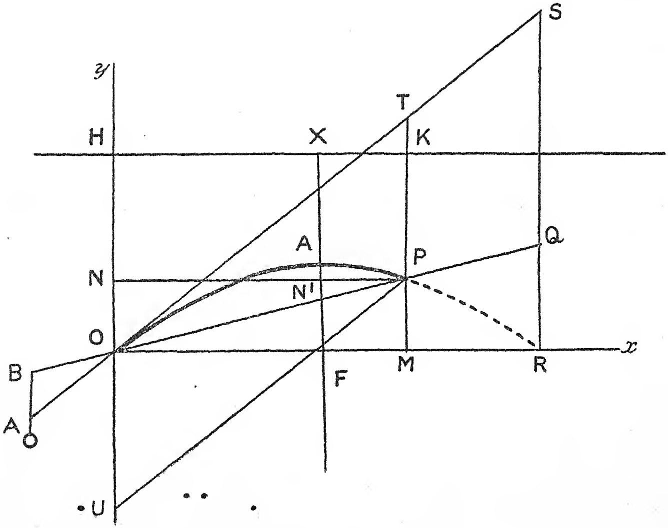
$$h = \frac{V^2}{2g} \sin 2\alpha, \quad k = \frac{V^2}{2g} \sin^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11);$$

also the latus rectum or parameter is

$$p = \frac{2V^2}{g} \cos^2 \alpha = 4AX \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12);$$

the trajectory is therefore a parabola, as first pointed out by Galileo, in 1638.

Fig. 1.



The co-ordinates of the focus F are

$$h = \mathrm{NN}' = \frac{V^2}{2g} \sin 2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

$$k - \tfrac{1}{4}p = \frac{V^2}{2g} \sin^2 \alpha - \frac{V^2}{2g} \cos^2 \alpha = - \frac{V^2}{2g} \cos 2\alpha ;$$

and the height of the directrix HK is

$$\mathrm{OH} = k + \tfrac{1}{4}p = \frac{V^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14);$$

and

$$\mathrm{AN}' = k - y, \quad \mathrm{N'P} = x - h,$$

so that

$$\frac{\mathrm{N'P}^2}{\mathrm{AN}'} = \text{latus rectum} = \frac{2V^2}{g} \cos^2 \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

the property of a parabola.

Denoting by v the velocity at any point (x, y) of the parabolic trajectory,

$$\begin{aligned} v^2 &= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \\ &= V^2 \cos^2 \alpha + (V \sin \alpha - gt)^2 \\ &= V^2 - 2g\,(Vt \sin \alpha - \tfrac{1}{2}gt^2) \\ &= V^2 - 2gy \\ v^2 &= 2g\,(\mathrm{OH} - \mathrm{MP}) = 2g\,.\,\mathrm{PK} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16). \end{aligned}$$

So that the velocity v is that which would be due to falling freely from the level of the directrix, the depth PK below the directrix being called the *head* or *impetus* of the velocity v.

Denoting by X the range in feet, and T the time of flight in seconds, over a horizontal line Ox through O, obtained by putting $y = 0$ in (6) and (8), then

$$T = \frac{2V \sin \alpha}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

$$X = \frac{2V^2 \sin \alpha \cos \alpha}{g} = \frac{V^2 \sin 2\alpha}{g} = p \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

Thus for a given value of V, the range X is a maximum when

$$\sin 2\alpha = 1, \quad \text{or} \quad \alpha = 45^\circ.$$

Generally

$$\sin 2\alpha = \frac{gX}{V^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19),$$

giving the elevation α required for a range X; or

$$V^2 = gX \operatorname{cosec} 2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20),$$

giving the initial velocity V required for a range X with elevation α, as in Chapter II, page 68 (Part I).

From Equation (8), since $y = 0$, when $x = 0$ or X,

$$y = x \tan \alpha \left(1 - \frac{x}{X}\right),$$

$$\tan \alpha = \frac{y}{x\left(1 - \dfrac{x}{X}\right)} = \frac{y}{x} + \frac{y}{X - x},$$

giving

$$\tan \alpha = \tan \theta + \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21),$$

where θ, ϕ are the angular elevations of the point P, as seen from O and R, the beginning and end of the range; this theorem is useful in determining the elevation required with a given range X, so as to clear an obstacle, a wall or rampart, of height y at a distance x from O (see fig. 1).

Denoting, as before, the whole time of flight over the range on a horizontal plane through O by T, and the time of flight from O to P by t; denoting also the time of flight from P down to the ground again by t', then

$$t + t' = \mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and

$$\mathrm{V} \sin \alpha = \tfrac{1}{2} g \mathrm{T};$$

so that (6) may be written

$$y = \tfrac{1}{2} g \mathrm{T} t - \tfrac{1}{2} g t^2$$

$$= \tfrac{1}{2} g t (\mathrm{T} - t) = \tfrac{1}{2} g t t' \quad . \quad . \quad . \quad . \quad . \quad (23);$$

Colonel Sladen's formula, useful in plotting approximately points on a trajectory in direct fire, even when the resistance of the air is taken into account, but where the vertical component of the resistance is insensible.

At the vertex A, $t = t' = \tfrac{1}{2}\mathrm{T}$, and the height of the vertex

$$\mathrm{H} = \tfrac{1}{8} g \mathrm{T}^2 = 4\mathrm{T}^2 = (2\mathrm{T})^2 \quad . \quad . \quad . \quad . \quad . \quad (24),$$

taking $g = 32$; hence the practical rule:—

"The square of twice the time of flight in seconds is the height of the vertex of the trajectory in feet."

Thus if the time of flight is 5 seconds, the height of the vertex is 100 feet; if

T = 0·1 sec., H = $\frac{1}{25}$ foot, less than $\frac{1}{2}$ inch; V = 3000 f/s.

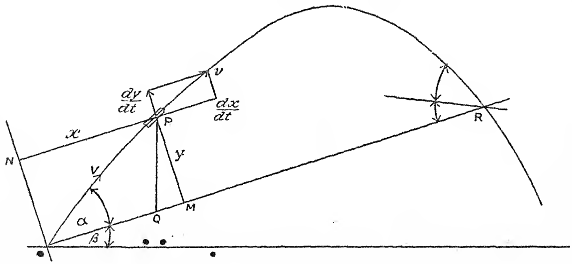
T = 60 secs., H = 14,400 feet.

When firing up or down a slope Ox, at an inclination of β to the horizon, the equations of motion are

$$\frac{d^2x}{dt^2} = -g \sin \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (25);$$

$$\frac{d^2y}{dt^2} = -g \cos \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (26);$$

Fig. 2.



and, integrating twice,

$$\frac{dx}{dt} = \mathrm{V} \cos \alpha - gt \sin \beta;$$

$$\frac{dy}{dt} = \mathrm{V} \sin \alpha - gt \cos \beta;$$

$$x = \mathrm{V}t \cos \alpha - \tfrac{1}{2}gt^2 \sin \beta \quad . \quad . \quad . \quad (27);$$

$$y = \mathrm{V}t \sin \alpha - \tfrac{1}{2}gt^2 \cos \beta \quad . \quad . \quad . \quad (28);$$

and α now denotes the *tangent* elevation of the gun, the *quadrant* elevation being $\alpha + \beta$.

Then, with the preceding notation,

$$\mathrm{T} = \frac{2\mathrm{V} \sin \alpha}{g \cos \beta} \quad . \quad . \quad . \quad (29);$$

$$\mathrm{X} = \frac{2\mathrm{V}^2 \sin \alpha \cos \alpha}{g \cos \beta} - \frac{2\mathrm{V}^2 \sin^2 \alpha \sin \beta}{g \cos^2 \beta}$$

$$= 4a \frac{\sin \alpha \cos (\alpha + \beta)}{\cos^2 \beta}$$

$$= 2a \frac{\sin (2\alpha + \beta) - \sin \beta}{\cos^2 \beta} \quad . \quad . \quad . \quad (30),$$

if a denotes $\frac{1}{2}\mathrm{V}^2/g$, *head* or *impetus* of the velocity V.

Thus for given V or a, and a given slope β, the range X is a maximum when

$$\sin (2\alpha + \beta) = 1,$$

$$2\alpha + \beta = 90^\circ,$$

$$\alpha = 45^\circ - \tfrac{1}{2}\beta,$$

a direction which bisects the angle between the slope and the vertical.

Also, as before,

$$y = \tfrac{1}{2}gtt' \cos \beta \quad . \quad . \quad . \quad (31),$$

so that the distance from the slope Ox, measured vertically, is $\frac{1}{2}gtt'$.

The parabolic theory is sometimes useful in assigning limits within which the real trajectory in a resisting medium must lie; an example showing this is given in Part I, p. 71.

The area of the parabolic segment OPR (fig. 1), where the equation $x^2 = py$ represents the curve with origin at A, and the axis of y downwards is

$$2 \int_0^y x \, dy,$$

where the upper limit is the height of A above OR.

Therefore

$$\text{area of segment} = 2 \int_0^y p^{\frac{1}{2}} y^{\frac{1}{2}} \, dy$$

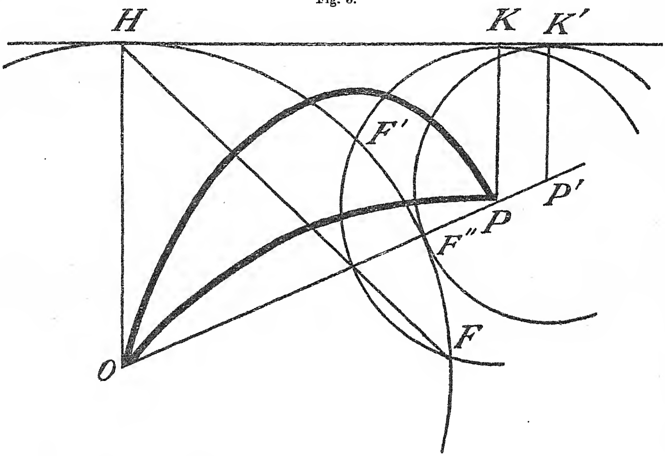
$$= 2p^{\frac{1}{2}} \tfrac{2}{3} y^{\frac{3}{2}} = \tfrac{4}{3} xy.$$

So that the area of the parabolic segment is two-thirds that of the circumscribing rectangle, and therefore the mean height of the ordinates of the arc is two-thirds of the maximum ordinate; hence the average height of a projectile in a parabolic trajectory is two-thirds of the height of the vertex. Captain James M. Ingalls, U.S.A., has pointed out the

practical use of this result for the tenuity of the air at great altitudes in a long trajectory, as showing that a good approximation is obtained to the average density of the air traversed by the projectile at a height in the atmosphere of two-thirds of the estimated height of the vertex.

To determine geometrically the directions of projection from a point O, with given velocity due to the head OH, so as to strike a point P, describe a circle with centre P and radius PK, touching the horizontal line HK through H in K; then if this circle cuts the circle with centre O and radius OH in F and F′, the required directions are perpendicular to HF and HF′, or bisect the angles HOF and HOF′; and these directions, therefore, are equally inclined to the bisector of the angle POH, which is the direction of projection for maximum range on the plane OP; and if the circles do not intersect, the point P is out of range.

Fig. 3.



These results follow because

$$FO = OH, \qquad FP = PK;$$
$$F'O = OH, \qquad F'P = PK;$$

so that F and F′ are the foci of the parabolas that can be drawn passing through O and P, and having the common directrix HK.

The lower parabola with the smaller angle of projection is that required for *direct fire*, and the upper parabola for *high angle* or *mortar* fire.

When the points F and F′ coalesce in F″ the circles touch, and the point P is out of the range attainable from O in the direction OP, when it is beyond P′, where

$$OP' = OH + P'K' = P'K'',$$

and

$$K''K' = OH,$$

so that the locus of points just within range is the parabola whose focus is O and vertex H, and the space inside the paraboloid generated by the revolution of this parabola about its axis is the space which can be covered from O with the given velocity of projection; points outside this paraboloid being out of range from O.

Suppose, for instance, that OP is the trace of an inclined plane through O, this plane will cut the paraboloid in an ellipse with focus at O, and this ellipse will be the area covered on the inclined plane OP by a gun at O.

The section of the paraboloid made by a vertical plane PK will be a parabola; this will be, for instance, the area covered on a vertical wall PK by a fire engine at O, supposing OH is the greatest height to which the engine can send the jet; and to attain the boundary of the area, the jet must be aimed at points on the wall lying on the horizontal straight line at a height 2OH, twice the *impetus* or *head* of the velocity.

The following treatment of a parabolic arc will be found useful in the next chapter on high angle fire.

Let OP, fig. 4, be an arc of a parabola, vertex at A, the direction of a projectile's trajectory at O is ϕ and at P it is θ.

Let the equation of the parabola of which OP is an arc be $x^2 = py$.

Any point on this is $x = \lambda p$, $y = \lambda^2 p$, where λ is the variable; at any point P,

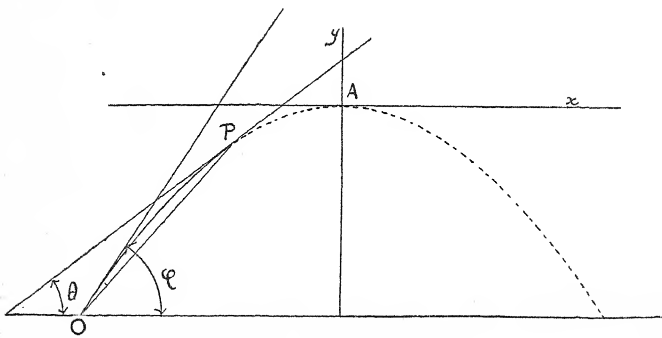
$$\frac{dy}{dx} = \tan\theta = \frac{2\lambda p}{p} = 2\lambda,$$

also

$$\frac{dx}{d\theta} = \frac{dx}{d\lambda}\cdot\frac{d\lambda}{d\theta} = \frac{p\,.\sec^2\theta}{2},$$

$$\frac{ds}{d\theta} = \frac{ds}{dx}\cdot\frac{dx}{d\theta} = \sec\theta\,.\frac{p}{2}\sec^2\theta = \frac{p}{2}\sec^3\theta.$$

Fig. 4.



so that the mean value from ϕ to θ of the secant of the angle of direction is

$$\sec\eta = \frac{s}{x} = \frac{\int_\theta^\phi \frac{ds}{d\theta}d\theta}{\int_\theta^\phi \frac{dx}{d\theta}.d\theta} = \frac{\int_\theta^\phi \sec^3\theta\,d\theta}{\int_\theta^\phi \sec^2\theta\,d\theta}.$$

Integrating by parts,

$$\int \sec^3\theta\,d\theta = \tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\int \sec\theta\,d\theta;$$

and

$$\int \sec\theta\, d\theta = \int \frac{d\theta}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}} = \int \frac{\sec^2\frac{\theta}{2}\, d\theta}{1 - \tan^2\frac{\theta}{2}},$$

put

$$x = \tan\frac{\theta}{2}, \qquad dx = \tfrac{1}{2}\sec^2\frac{\theta}{2}\, d\theta,$$

and

$$\therefore \int \sec\theta\, d\theta = \log\frac{1+x}{1-x} = \log\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \log(\sec\theta + \tan\theta).$$

Therefore

$$\sec\eta = \frac{i(\phi) - i(\theta)}{\tan\phi - \tan\theta},$$

where

$$i(\phi) = \tfrac{1}{2}\tan\phi\sec\phi + \tfrac{1}{2}\log(\sec\phi + \tan\phi).$$

This function is tabulated in Table VI; it is useful also in the calculation of a trajectory when the quadratic law of resistance is assumed.

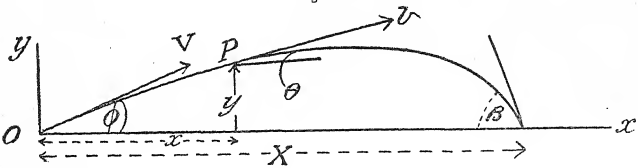
CHAPTER IV.

HIGH ANGLE FIRE.

WHEN the curvature of the trajectory becomes considerable, as in high angle and curved fire, the methods of Chapter I, Part II, for direct fire require modification. We proceed then to consider the equations of motion of a projectile in a resisting medium, when projected with given velocity in a given direction; and to show how these equations, where otherwise intractable, can be slightly modified so as to give tangible practical results.

The motion is referred to two co-ordinate axes, Ox and Oy, drawn horizontally and vertically in the plane of fire through O, the muzzle of the gun; the resistance of the air is taken to act in the opposite direction to the motion of the centre of gravity of the projectile, so that there is no cause tending to draw the shot out of its original plane of fire, and to cause drift or deviation: this subsidiary effect must be considered separately.

Fig. 1.



Let the velocity at O be V f/s and the direction of the shot be ϕ (in radians of circular measure); let x, y denote (in feet) the co-ordinates of the C.G. of the shot at P after a time of flight of t seconds, and θ the inclination (radians) at P, so that θ is the angle which the tangent to the trajectory at P makes with the horizontal Ox; the velocity at P being v f/s, and the retardation r feet per second per second,

$$\frac{d^2x}{dt^2} = \frac{d\,(v\cos\theta)}{dt} = -r\cos\theta \quad \text{. . . . .} \quad (1),$$

$$\frac{d^2y}{dt^2} = \frac{d\,(v\sin\theta)}{dt} = -g - r\sin\theta \quad \text{. . . . .} \quad (2).$$

Eliminating r from (1) and (2)

$$\cos\theta\, d\,(v\sin\theta) - \sin\theta\, d\,(v\cos\theta) = -g\cos\theta\, dt,$$

multiply both sides by $\dfrac{\sec^2\theta}{v}$,

$$\frac{1}{v\cos\theta}\, d\,(v\sin\theta) - \frac{\sin\theta}{v\cos^2\theta}\, d\,(v\cos\theta) = -\frac{g\,dt}{v\cos\theta},$$

$$d\left(\frac{v\sin\theta}{v\cos\theta}\right) = -g\,\frac{dt}{v\cos\theta},$$

$$g\,dt = -v\,\frac{d\theta}{\cos\theta} \quad \text{. . . . .} \quad (3).$$

This formula has already been proved in Chapter II, page 35, where it was shown that $v\frac{d\theta}{dt} = -g\cos\theta$; *see* also *Notes on Dynamics* (1908), by Sir G. Greenhill, F.R.S., pp. 123, 124.

To obtain dx, dy, $d(v\cos\theta)$, ds in terms of $d\theta$, substitute the value of dt from (3) in

$$\frac{dx}{dt} = v\cos\theta, \qquad \frac{dy}{dt} = v\sin\theta, \qquad \frac{ds}{dt} = v,$$

and in (1) where $\frac{d(v\cos\theta)}{dt} = -r\cos\theta$.

From (3)

$$g\,dt = -v\frac{d\theta}{\cos\theta} \quad . \quad . \quad . \quad (4),$$

and

$$g\,dx = -v^2\,d\theta \quad . \quad . \quad . \quad (5),$$

$$g\,dy = -v^2\tan\theta\,d\theta \quad . \quad . \quad . \quad (6),$$

$$g\,d(v\cos\theta) = +v\frac{\mathrm{F}(v)}{\mathrm{C}}\,d\theta \quad . \quad . \quad . \quad (7);$$

also

$$g\,d(\tan\theta) = g\sec^2\theta\,d\theta \quad . \quad . \quad . \quad (8).$$

In (7) $\frac{\mathrm{F}(v)}{\mathrm{C}}$ has been substituted for r, and is obtained thus:—

$$\frac{r}{g} = \frac{\mathrm{R}}{w} = \frac{n\,d^2p}{w} = \frac{p}{\mathrm{C}} \text{ (\textit{see} p. 11)};$$

also

$$pg = \mathrm{C}r = \mathrm{K}\left(\frac{v}{1000}\right)^3 = \mathrm{F}(v),$$

so that $\frac{\mathrm{F}(v)}{g} = p =$ resistance of the air in pounds to a 1-inch projectile moving with velocity v f/s, under standard conditions, also

$$\frac{\mathrm{F}(v)}{\mathrm{C}} = r.$$

Equations (4) to (8) have θ as the independent variable; to turn these so that the independent variable be the horizontal component of velocity, $v\cos\theta = v_1$, make use of (7), from which,

$$d\theta = \frac{\mathrm{C}g}{v\mathrm{F}(v)}\,dv_1 \quad . \quad . \quad . \quad (9),$$

then equations (4) to (8) become

$$dt = -\frac{\mathrm{C}\sec\theta}{\mathrm{F}(v_1\sec\theta)}\,.\,dv_1 \quad . \quad . \quad . \quad (10),$$

$$dx = -\frac{\mathrm{C}v_1\sec\theta}{\mathrm{F}(v_1\sec\theta)}\,dv_1 \quad . \quad . \quad . \quad (11),$$

$$dy = -\frac{\mathrm{C}v_1\sec\theta\tan\theta}{\mathrm{F}(v_1\sec\theta)}\,.\,dv_1 \quad . \quad . \quad . \quad (12),$$

$$\theta = +\frac{\mathrm{C}g}{v_1\sec\theta\mathrm{F}(v_1\sec\theta)}\,.\,dv_1 \quad . \quad . \quad . \quad (13),$$

$$d(\tan\theta) = +\frac{\mathrm{C}g\sec\theta}{v_1\mathrm{F}(v_1\sec\theta)}\,.\,dv_1 \quad . \quad . \quad . \quad (14).$$

Equations (10) to (14) have v_1 ($= v \cos\theta$) as the independent variable; and integrating these, supposing V_1 the initial value of v_1 and making V_1 the upper limit so as to cancel the negative sign,

$$t = C\int_{v_1}^{V_1} \frac{\sec\theta}{F(v_1\sec\theta)}\,dv_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

$$x = C\int_{v_1}^{V_1} \frac{v_1\sec\theta}{F(v_1\sec\theta)}\,dv_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16),$$

$$y = C\int_{v_1}^{V_1} \frac{v_1\sec\theta\tan\theta}{F(v_1\sec\theta)}\,dv_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

$$\phi - \theta = C\int_{v_1}^{V_1} \frac{g}{v_1\sec\theta F(v_1\sec\theta)}\,.\,dv_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18),$$

$$\tan\phi - \tan\theta = C\int_{v_1}^{V_1} \frac{g\sec\theta}{v_1 F(v_1\sec\theta)}\,dv_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

Equations (15) to (17) cannot be integrated as they stand, because the relation between θ and v_1 is not known.

But, as originally pointed out by Euler, these difficulties can be turned if we notice that in the ordinary trajectories in practice the quantities θ, $\cos\theta$, and $\sec\theta$ vary so slowly that they may be replaced by their *mean* values η, $\cos\eta$, and $\sec\eta$; especially if in the calculations the trajectory, when considerable, is divided up into arcs of small curvature (the *curvature* of an arc is defined as the angle between the tangents or normals at the ends of the arc).

In equations (15), (16), (18), (19) the mean angle η enters only in the form of $\sec\eta$ or $\cos\eta$, slowly varying quantities for moderate values of η, so that η need not be determined with great accuracy.

According to Didion (*Traité de Balistique*, p. 119), the mean value of $\sec\eta$ is obtained by supposing the arc from ϕ to θ a portion of a parabola, with a vertical axis, and that

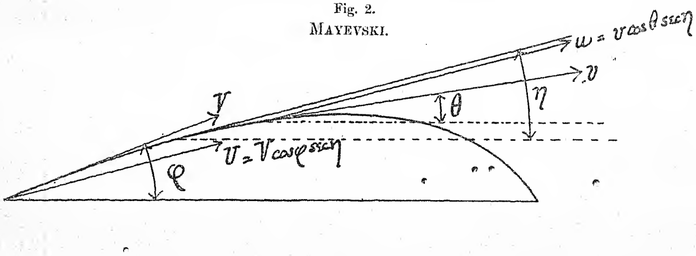
$$\sec\eta = \frac{s}{x} = \frac{\displaystyle\int_\theta^\phi \frac{ds}{d\theta}\,d\theta}{\displaystyle\int_\theta^\phi \frac{dx}{d\theta}\,d\theta} = \frac{i(\phi) - i(\theta)}{\tan\phi - \tan\theta} \text{ (see p. 59)} \quad . \quad . \quad . \quad . \quad . \quad (20),$$

where $i(\phi) = \frac{1}{2}\tan\phi\sec\phi + \frac{1}{2}\log(\sec\phi + \tan\phi)$, a function tabulated in Table VI. Replacing the variable angle θ by some mean angle η, and introducing the Mayevski *pseudo*-velocities U and u, defined by

$$\left.\begin{aligned} U &= V_1\sec\eta = V\cos\phi\sec\eta, \\ u &= v_1\sec\eta = v\cos\theta\sec\eta, \\ dv_1 &= \cos\eta\,du, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21).$$

so that

Fig. 2.
MAYEVSKI.



the equations (15), (16), (18), (19) may now be written (u being the independent variable)

$$t = C\int_u^U \frac{1}{F(u)}\,du \quad . \quad . \quad . \quad (22),$$

$$x = C \cos \eta \int_u^U \frac{u}{F(u)}\,du \quad . \quad . \quad . \quad (23),$$

$$\phi - \theta = C \cos \eta \int_u^U \frac{g}{uF(u)}\,du \quad . \quad . \quad . \quad (24),$$

$$\tan \phi - \tan \theta = C \sec \eta \int_u^U \frac{g}{uF(u)}\,du \quad . \quad . \quad . \quad (25).$$

But $F(u) = Cr$, *see* p. 61 of this chapter, and from Chapter II Cr is of the form $[A]\,u^n$, or, shortly, Au^n (*see* p. 32); hence, as in Chapter II for direct fire, these integrals are the same as those which gave the functions T, S, and I, but with the pseudo-velocity u as the argument instead of the real velocity v.

Equations (22) to (25) become

$$\frac{t}{C} = \int_u^U \frac{1}{Au^n}\,du = T(U) - T(u) \quad . \quad . \quad . \quad (26),$$

$$\frac{x}{C} = \cos \eta \int_u^U \frac{1}{Au^{n-1}}\,du = \cos \eta\,\{S(U) - S(u)\} \quad . \quad . \quad . \quad (27),$$

$$\frac{\phi - \theta}{C} = \cos \eta \int_u^U \frac{g}{Au^{n+1}}\,du = \cos \eta\,\{I(U) - I(u)\} \quad . \quad . \quad . \quad (28),$$

$$\frac{\tan \phi - \tan \theta}{C} = \sec \eta \int_u^U \frac{g}{Au^{n+1}}\,du = \sec \eta\,\{I(U) - I(u) \quad . \quad . \quad . \quad (29).$$

If ϕ and θ are expressed in degrees, then

$$\frac{\phi^\circ - \theta^\circ}{C} = \frac{180}{\pi} \cos \eta\,\{I(U) - I(u)\} \quad . \quad . \quad . \quad (30).$$

It will be noticed that η cannot be exactly the same mean angle in equations (27), (28), and (29); thus it is obviously different in equations (28) and (29); but, when dealing with arcs of small curvature, the discrepancies due to using the same η throughout will be insensible.

Equations (26) to (29) are in the form employed by General Mayevski, who modified Siacci's original equations by introducing Euler's mean angle η; the direction of the pseudo-velocity "u" is also different to Siacci's u, as will be seen when his method is demonstrated.

The Altitude Function A (u).

Replacing $\tan \theta$ by $\frac{dy}{dx}$ in equation (29),

$$\tan \phi - \frac{dy}{dx} = C \sec \eta\,\{I(U) - I(u)\} \quad . \quad . \quad . \quad (31).$$

Integrate with respect to x over the arc considered,

$$x \tan \phi - y = C \sec \eta \left\{ xI(U) - \int_0^x I(u)\,dx \right\}.$$

But from equation (23)

$$\frac{dx}{du} = -\mathrm{C}\cos\eta\,\frac{u}{\mathrm{F}(u)},$$

the negative sign is taken because x increases as u decreases, and making U the upper limit,

$$x\tan\phi - y = \mathrm{C}x\sec\eta\mathrm{I}(\mathrm{U}) - \mathrm{C}^2\int_u^{\mathrm{U}}\frac{u\mathrm{I}(u)}{\mathrm{F}(u)}\,du \quad \ldots\ldots\ldots \quad (32).$$

In Siacci's notation

$$\int_u^{\mathrm{U}}\frac{u\mathrm{I}(u)}{\mathrm{F}(u)}\,du = \mathrm{A}(\mathrm{U}) - \mathrm{A}(u),$$

where $\mathrm{A}(u)$ is called the *altitude function.*

Dividing (32) by x,

$$\tan\phi - \frac{y}{x} = \mathrm{C}\sec\eta\mathrm{I}(\mathrm{U}) - \mathrm{C}^2\frac{\mathrm{A}(\mathrm{U}) - \mathrm{A}(u)}{x}.$$

But from (27),

$$x = \mathrm{C}\cos\eta\,\{\mathrm{S}(\mathrm{U}) - \mathrm{S}(u)\},$$

therefore

$$\frac{y}{x} = \tan\phi - \mathrm{C}\sec\eta\left\{\mathrm{I}(\mathrm{U}) - \frac{\mathrm{A}(\mathrm{U}) - \mathrm{A}(u)}{\mathrm{S}(\mathrm{U}) - \mathrm{S}(u)}\right\} \quad \ldots\ldots\ldots \quad (33).$$

This is Mayevski's equation with the altitude function A, in which $\sec\eta$ and the initial and final pseudo-velocities U and u for any arc depend upon ϕ and θ, the initial and final inclinations to the horizontal of the tangents to the arc.

By means of Mayevski's method high-angle trajectories can be calculated with great exactness by breaking such trajectories into small arcs.

The equations are

$$\mathrm{C} = f\,\frac{w}{\kappa\sigma\tau_0 d^2},$$

where τ_0 is the tenuity factor at the earth's surface, f is the factor which corrects for the mean height of the projectile above the ground in any particular arc of the trajectory,

$$\left.\begin{aligned} x\ (\text{in feet}) &= \mathrm{C}\cos\eta\,[\mathrm{S}(\mathrm{U}) - \mathrm{S}(u)],\\ t\ (\text{in seconds}) &= \mathrm{C}\,[\mathrm{T}(\mathrm{U}) - \mathrm{T}(u)],\\ \tan\theta &= \tan\phi - \mathrm{C}\sec\eta\,[\mathrm{I}(\mathrm{U}) - \mathrm{I}(u)],\\ \frac{y}{x} &= \tan\phi - \mathrm{C}\sec\eta\left[\mathrm{I}(\mathrm{U}) - \frac{\mathrm{A}(\mathrm{U}) - \mathrm{A}(u)}{\mathrm{S}(\mathrm{U}) - \mathrm{S}(u)}\right]\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (33\mathrm{A})$$

U and u are the pseudo-velocities at the commencement and end of any arc, so that

$$\left.\begin{aligned} \mathrm{U} &= \mathrm{V}\cos\phi\sec\eta\\ u &= v\cos\theta\sec\eta\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (33\mathrm{B}).$$

For very small arcs, no matter what the elevation be, these equations are accurate. It is, however, found that the work of calculating a high-angle trajectory by means of arcs in which the total change of direction is as small as 1° is much simplified, for the following reasons :—

1. The equations are strictly accurate.
2. The mean angle η may be taken as $\frac{\phi+\theta}{2}$ without error.

3. After obtaining three values of the altitude factor f, the succeeding values of f can be found by plotting a curve of f in conjunction with θ, thus obviating the necessity of calculating the arc more than once. The first three or four arcs are re-calculated until no change in the elements is obtained; a second re-calculation is always necessary.

4. A general table can be constructed for

$$\frac{\tan\phi - \tan\theta}{\sec\eta} = \text{E, say,}$$

this simplifies the calculations; thus

Arc.	log E.
80°–79°	$\bar{2}$ 98222
79°–78°	$\bar{2}$ 94303
78°–77°	$\bar{2}$·90723
&c.	

5. The altitude formula

$$\frac{y}{x} = \tan\phi - \mathrm{C}\sec\eta\left[\mathrm{I}(\mathrm{U}) - \frac{\mathrm{A}(\mathrm{U}) - \mathrm{A}(u)}{\mathrm{S}(\mathrm{U}) - \mathrm{S}(u)}\right]$$

can be replaced with great exactness by

$$\frac{y}{x} = \tan\eta.$$

The example of a high-angle trajectory for the 18-pr. Q.F., which is worked out below in tabular form, was calculated by Captain J. F. R. N. Maitland-Addison, R.A., and an explanation of his method now follows:—

Operations for Calculating a High-Angle Trajectory by means of small arcs, employing Mayevski's Equations.

Before commencing calculations a curve of f should be drawn for all the heights likely to be obtained, the arguments being the barometer and thermometer values at the ground. Range tables are compiled for barometer 30 inches, thermometer 60° F. See fig. 1, p. 7 of Chapter I, for a curve of f, selecting the middle one or else the table on p. 7.

1. $C_0 = \frac{w}{\kappa\sigma d^2}$, ignoring f.
2. Calculate the muzzle pseudo-velocity, U.
3. Calculate the remaining pseudo-velocity, u.
4. Calculate the horizontal distance $= x$.
5. Calculate the height, $y = x\tan\eta$.
6. Find f from the curve and the value got for y.
7. Include f in ballistic coefficient, and thus $C_1 = f\frac{w}{\kappa\sigma . d^2}$.
8. Re-calculate arc as before.

Example:—It will be observed that for the arc 80°–79° the value of f obtained in the re-calculation of the arc is the same as that first obtained, so that the elements obtained in the second calculation will not be altered by a third calculation, which is therefore unnecessary.

9. Carry out the re-calculation for the first three or four arcs as above.
10. Plot the values of f thus obtained; values of f for other arcs can be obtained with

accuracy by extrapolation for a change of 1° of arcs, thus rendering a re-calculation of any arc unnecessary.

Example:—Arc 76°–75°;

f by extrapolation $= 1 \cdot 444$;

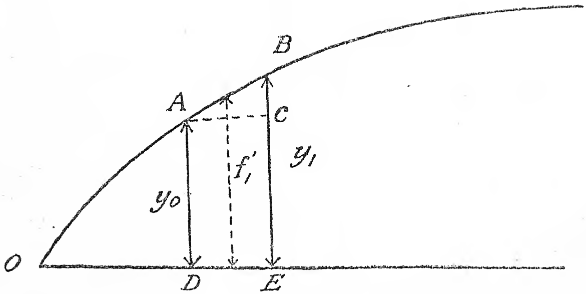
f as obtained from the calculated value of $y = 1 \cdot 448$.

There is no accumulation of error in f, as this latter value is now plotted instead of the former and the curve extrapolated for the next arc.

The value of f for succeeding arcs is taken for a height at the half-way point of the arc, thus if

OA represents 1st arc,
AB ,, 2nd ,,

Fig. 3.



then f_1 for the 2nd arc is that obtained for a value of $y = \mathrm{AD} + \frac{\mathrm{BC}}{2}$, and so on for other arcs.

Proceeding as above shown, the calculations are very quickly performed for any number of arcs.

11. The time of flight for each arc is then calculated.

12. For balloon firing the angles of sight to the target are given by

$$\theta = \tan^{-1} \frac{y}{x}.$$

13. The tangent elevations required are given by

$$\text{T.E.} = \phi - \theta.$$

14. The ranges along the lines of sight are given by

$$\text{X (feet)} = \sqrt{x^2 + y^2}.$$

Table of $\dfrac{\tan\phi - \tan\theta}{\sec\eta} = E.$

Arc.	log cos η.	log E.	Arc.	log cos η.	log E.
ϕ θ			ϕ θ		
80–79	$\bar{1}$·26063	$\bar{2}$·98222	40–39	$\bar{1}$·88741	$\bar{2}$·35457
79–78	$\bar{1}$·29966	$\bar{2}$·94303	39–38	$\bar{1}$·89354	$\bar{2}$·34823
78–77	$\bar{1}$·33534	$\bar{2}$·90722	38–37	$\bar{1}$·89947	$\bar{2}$·34258
77–76	$\bar{1}$·36819	$\bar{2}$·87429	37–36	$\bar{1}$·90518	$\bar{2}$·33670
76–75	$\bar{1}$·39860	$\bar{2}$·84378	36–35	$\bar{1}$·91069	$\bar{2}$·33131
75–74	$\bar{1}$·42690	$\bar{2}$·81543	35–34	$\bar{1}$·91599	$\bar{2}$·32592
74–73	$\bar{1}$·45334	$\bar{2}$·78892	34–33	$\bar{1}$·92111	$\bar{2}$·32078
73–72	$\bar{1}$·47814	$\bar{2}$·76408	33–32	$\bar{1}$·92603	$\bar{2}$·31590
72–71	$\bar{1}$·50148	$\bar{2}$·74070	32–31	$\bar{1}$·93077	$\bar{2}$·31116
71–70	$\bar{1}$·52350	$\bar{2}$·71865	31–30	$\bar{1}$·93532	$\bar{2}$·30657
70–69	$\bar{1}$·54433	$\bar{2}$·69781	30–29	$\bar{1}$·93970	$\bar{2}$·30218
69–68	$\bar{1}$·56408	$\bar{2}$·67802	29–28	$\bar{1}$·94390	$\bar{2}$·29801
68–67	$\bar{1}$·58284	$\bar{2}$·65926	28–27	$\bar{1}$·94793	$\bar{2}$·29389
67–66	$\bar{1}$·60070	$\bar{2}$·64134	27–26	$\bar{1}$·95179	$\bar{2}$·29025
66–65	$\bar{1}$·61773	$\bar{2}$·62432	26–25	$\bar{1}$·95549	$\bar{2}$·28631
65–64	$\bar{1}$·63398	$\bar{2}$·60808	25–24	$\bar{1}$·95902	$\bar{2}$·28289
64–63	$\bar{1}$·64953	$\bar{2}$·59248	24–23	$\bar{1}$·96240	$\bar{2}$·27963
63–62	$\bar{1}$·66441	$\bar{2}$·57759	23–22	$\bar{1}$·96562	$\bar{2}$·27610
62–61	$\bar{1}$·67866	$\bar{2}$·56334	22–21	$\bar{1}$·96868	$\bar{2}$·27339
61–60	$\bar{1}$·69234	$\bar{2}$·54967	21–20	$\bar{1}$·97159	$\bar{2}$·27022
60–59	$\bar{1}$·70547	$\bar{2}$·53651	20–19	$\bar{1}$·97435	$\bar{2}$·26749
59–58	$\bar{1}$·71809	$\bar{2}$·52393	19–18	$\bar{1}$·97696	$\bar{2}$·26499
58–57	$\bar{1}$·73022	$\bar{2}$·51169	18–17	$\bar{1}$·97942	$\bar{2}$·26250
57–56	$\bar{1}$·74189	$\bar{2}$·50012	17–16	$\bar{1}$·98174	$\bar{2}$·26004
56–55	$\bar{1}$·75313	$\bar{2}$·48881	16–15	$\bar{1}$·98391	$\bar{2}$·25807
55–54	$\bar{1}$·76395	$\bar{2}$·47803	15–14	$\bar{1}$·98594	$\bar{2}$·25592
54–53	$\bar{1}$·77439	$\bar{2}$·46759	14–13	$\bar{1}$·98783	$\bar{2}$·25406
53–52	$\bar{1}$·78445	$\bar{2}$·45747	13–12	$\bar{1}$·98958	$\bar{2}$·25227
52–51	$\bar{1}$·79415	$\bar{2}$·44775	12–11	$\bar{1}$·99119	$\bar{2}$·25078
51–50	$\bar{1}$·80351	$\bar{2}$·43849	11–10	$\bar{1}$·99267	$\bar{2}$·24915
50–49	$\bar{1}$·81254	$\bar{2}$·42933	10–9	$\bar{1}$·99410	$\bar{2}$·24806
49–48	$\bar{1}$·82126	$\bar{2}$·42071	9–8	$\bar{1}$·99520	$\bar{2}$·24659
48–47	$\bar{1}$·82968	$\bar{2}$·41220	8–7	$\bar{1}$·99627	$\bar{2}$·24571
47–46	$\bar{1}$·83781	$\bar{2}$·40410	7–6	$\bar{1}$·99720	$\bar{2}$·24468
46–45	$\bar{1}$·84566	$\bar{2}$·39626	6–5	$\bar{1}$·99800	$\bar{2}$·24376
45–44	$\bar{1}$·85324	$\bar{2}$·38866	5–4	$\bar{1}$·99866	$\bar{2}$·24318
44–43	$\bar{1}$·86056	$\bar{2}$·38131	4–3	$\bar{1}$·99919	$\bar{2}$·24272
43–42	$\bar{1}$·86763	$\bar{2}$·37441	3–2	$\bar{1}$·99959	$\bar{2}$·24238
42–41	$\bar{1}$·87446	$\bar{2}$·36736	2–1	$\bar{1}$·99985	$\bar{2}$·24189
41–40	$\bar{1}$·88105	$\bar{2}$·36091	1–0	$\bar{1}$·99998	$\bar{2}$·24202

18-pr. Q.F.

Angle of Projection $\phi = 80°$; M.V. = 1590 f/s; $\tau_0 = 1$ and $\kappa\sigma$ is Unity, giving $C_0 = \frac{18 \cdot 5}{(3 \cdot 3)^2}$

	Arc . . .	80°–79°		79°–78°		78°–77°		77°–76°		76°–75°	75°–74°	74°–73°	73°–72°.
$C = \frac{w}{nd^2}$	log C_0 f log f log C	0·23015 .... 0·0 0·23015	0·23015 .... 0·02857 0·25872	0·23015 .... 0·02898 0·25913	0·23015 .... 0·07882 0·30897	0·23015 .... 0·07882 0·30897	0·23015 .... 0·11227 0·34242	0·23015 .... 0·11261 0·34276	0·23015 0·13862 0·36877	0·23015 1·444* 0·15957 0·38972	0·23015 1·505* 0·17754 0·40769	0·23015 1·555* 0·19173 0·42188	0·23015 1·605* 0·20548 0·43563
$U = V \cos\phi \sec\eta$ $\eta = \frac{\phi+\theta}{2}$	log V log cos ϕ log sec η log U U	3·20140 $\bar{1}$·23967 0·73937 3·18044 1515	 1515	3·02408 $\bar{1}$·28060 0·70034 3·00502 1012	 1012	2·95472 $\bar{1}$·31788 0·66466 2·93726 865	 865	2·90233 $\bar{1}$·35209 0·63181 2·88623 770	 770	2·85783 $\bar{1}$·38368 0·60140 2·84291 696	2·81811 $\bar{1}$·41300 0·57310 2·80421 637	2·78313 $\bar{1}$·44034 0·54666 2·77013 589	2·75158 $\bar{1}$·46594 0·52186 2·73938 549
$I(u) = I(U) - \frac{E}{C}$	log E log C log (ΔI) ΔI I (U) I (u) u	$\bar{2}$·98222 0·23015 $\bar{2}$·75207 0·05650 0·95288 0·89638 1095	$\bar{2}$·98222 0·25872 $\bar{2}$·72350 0·05291 0·95288 0·89997 1107	$\bar{2}$·94303 0·25913 $\bar{2}$·68390 0·04829 0·86184 0·81355 932	$\bar{2}$·94303 0·30897 $\bar{2}$·63406 0·04306 0·86184 0·81878 940	$\bar{2}$·90722 0·30897 $\bar{2}$·59825 0·03965 0·75997 0·71832 827	$\bar{2}$·90722 0·34242 $\bar{2}$·56480 0·03671 0·75797 0·72126 830	$\bar{2}$·87429 0·34276 $\bar{2}$·53153 0·03400 0·64793 0·61393 746	$\bar{2}$·87429 0·36877 $\bar{2}$·50552 0·03203 0·64793 0·61590 747	$\bar{2}$·84378 0·38972 $\bar{2}$·45406 0·02845 0·53372 0·50527 680	$\bar{2}$·81543 0·40769 $\bar{2}$·40774 0·02557 0·41722 0·39165 626	$\bar{2}$·78892 0·42188 $\bar{2}$·36704 0·02328 0·29948 0·27620 581	$\bar{2}$·76408 0·43563 $\bar{2}$·32845 0·02130 0·18046 0·15916 543
$x = C \cos\eta$ $[S(U) - S(u)]$	S (U) S (u) Δ (S) log Δ (S) log C log cos η log x x	39370·2 36588·0 2782·2 3·44439 0·23015 $\bar{1}$·26063 2·93517 861	39370·2 36723·7 2646·5 3·42267 0·25872 $\bar{1}$·26063 2·94202 875	35405·4 33990·3 1415·1 3·15079 0·25913 $\bar{1}$·29966 2·70958 512	35405·4 34142·7 1262·7 3·10130 0·30897 $\bar{1}$·29966 2·70993 513	32603·8 31721·0 882·8 2·94586 0·30897 $\bar{1}$·33534 2·59017 389	32603·8 31792·5 811·3 2·90918 0·34242 $\bar{1}$·33534 2·58694 386	30331·4 29727·8 603·6 2·78075 0·34276 $\bar{1}$·36819 2·49170 310	30331·4 29753·2 578·2 2·76208 0·36877 $\bar{1}$·36819 2·49904 316	28431·8 28005·4 426·4 2·62982 0·38972 $\bar{1}$·39860 2·41814 262	26828·6 26519·9 308·7 2·48954 0·40769 $\bar{1}$·42690 2·32413 211	25457·1 25222·1 235·0 2·37107 0·42188 $\bar{1}$·45334 2·24629 176	24262·1 24078·4 183·7 2·26411 0·43563 $\bar{1}$·47814 2·17788 151
$y = x \tan\eta$	log x log tan η log y y $h = y_0 + y_{1/2}$ f	2·93517 0·73203 3·66720 4647 2324 1·068	2·94202 ·73203 3·67405 *4721* 2360 1·069	2·70958 0·69154 3·40112 2518 5980 1·199	2·70993 0·69154 3·40147 *2520* 5981 1·199	2·59017 0·65424 3·24441 1756 8119 1·295	2·58694 0·65424 3·24118 *1743* 8112 1·296	2·49170 0·61975 3·11145 1293 9631 1·376	2·49904 0·61965 3·11869 *1314* 9641 1·378	2·41814 0·58734 3·00548 *1013* 10805 1·448	2·32413 0·55701 2·88114 *761* 11692 1·509	2·24629 0·52840 2·77469 *595* 12370 1·560	2·17788 0·50128 2·67916 *478* 12907 1·604
$v = u \sec\theta \cos\eta$	log u log cos η log sec θ log v v	....	3·04415 $\bar{1}$·26063 0·71940 3·02418 1057	....	2·97313 $\bar{1}$·29966 0·68212 2·95491 901	....	2·91908 $\bar{1}$·33534 0·64791 2·90233 799	....	2·87332 $\bar{1}$·36819 0·61632 2·85783 721	2·8351 $\bar{1}$·39860 0·58700 2·81811 658	2·79657 $\bar{1}$·42690 0·55966 2·78313 607	2·76418 $\bar{1}$·45334 0·53406 2·75158 564	2·73480 $\bar{1}$·47814 0·51002 2·72296 528

* These values of f are obtained by extrapolation, making use of figures given by previous arcs.

Calculation for Time of Flight.

Arc.	80°–79°.	79°–78°.	78°–77°.	77°–76°.
U	1515	1012	865	770
u	1107	940	830	747
T (U)	109·083	105·757	102·754	99·968
T (u)	107·006	104·461	101·796	99·205
T (U) – T (u)	2·077	1·296	0·958	0·763
log [T (U) – T (u)]	0·31744	0·11261	$\bar{1}$·98137	$\bar{1}$·88252
log C	0·25872	0·30897	0·34242	0·36877
log t	0·57616	0·42158	0·32379	0·25129
t	3·77	2·64	2·11	1·78

Calculation of the Angle of Sight, Tangent Elevation, and Range along the Line of Sight.

Arc.	80°–79°.	78°–77°.	76°–75°.	73°–72°.
log y	3·67403	3·95347	4·05354	4·11879
log x	2·94201	3·24895	3·37144	3·46090
log tan θ	0·73202	0·70452	0·68210	0·65789
θ	79° 30′	78° 50′	78° 16′	77° 36′
$\phi - \theta$	0° 30′	1° 10′	1° 44′	2° 24′
$(x^2 + y^2)^{1/2}$	4802	9158	11554	13460
Also t	3·77	8·52	11·82	15·05
Remaining velocity	1057	799	658	528

18-PR. Q.F. Q.E. = 80°.

TABULATED RESULTS (TABLE I).

Range in yards along line of sight.	Angle of sight, θ.	Height in feet.	Time of flight.	Remaining velocity, f/s.
	° ′			
1601	79 30	4721	3·77	1057
3053	78 50	8984	8·52	799
3851	78 16	11312	11·82	658
4487	77 36	13146	15·05	528

Plotting these ranges and angles of sight and other functions with range, the following results are obtained :—

TABLE II FOR 18-PR. Q.E. = 80°.

Range in yards along line of sight.	Angle of sight.	Height in feet.	Time of flight.	Remaining velocity.	Tangent elevation.
	° ′				° ′
0	80 0	—	—	1590	0 0
500	79 52	1480	1·01	1382	0 8
1000	79 43	2975	2·18	1216	0 17
2000	79 20	5880	4·95	976	0 40
3000	78 52	8815	8·32	809	1 8

For the construction of a complete range table for firing at balloons or such like targets it is sufficient to calculate trajectories in arcs for $\phi = 80°, 60°, 40°, 20°, 0°$, and then all the intermediate elements can be found by plotting.

Thus Tables I and II for $\phi = 80°$ are also calculated for $\phi = 60°, 40°, 20°$, and $0°$, and then the final range table is obtained by selecting the elements for any particular range as argument from each of the five tables of Table II; plotting the elements and reading for even angles of sight. For other ranges and angles of sight the elements can be obtained by further plotting until the intervals are sufficiently close for interpolating by eye.

SIACCI'S METHOD.

Starting with the general equations of motion (4), (5), (7), and (8),

$$\left.\begin{aligned} g\,dt &= -v\frac{d\theta}{\cos\theta} \quad \ldots \quad (4),\\ g\,dx &= -v^2\,d\theta \quad \ldots \quad (5),\\ g\,d(v\cos\theta) &= v\frac{F(v)}{C}\,d\theta \quad \ldots \quad (7),\\ g\,d(\tan\theta) &= g\sec^2\theta\,d\theta \quad \ldots \quad (8). \end{aligned}\right\}$$

Writing v_1 for $v\cos\theta$ then as before from (7), $d\theta = \frac{Cg}{vF(v)}\,dv_1$, substitute in (4), (5), and (8), then

$$dt = -C\frac{dv_1}{\cos\theta F(v)} \quad \ldots \quad (34),$$

$$dx = -C\frac{v}{F(v)}\,dv_1 \quad \ldots \quad (35),$$

$$d(\tan\theta) = Cg\frac{\sec^2\theta}{vF(v)}\,dv_1 \quad \ldots \quad (36).$$

Now $F(v) = Cr = [A]v^n$, or writing A in short for [A], $F(v) = Av^n$, hence

$$F(v) = A(v_1 \sec\theta)^n = Av_1^n \sec^n\theta$$

The equations now become

$$dt = -C\frac{\cos^{n-1}\theta}{Av_1^n}\,dv_1 \quad \ldots \quad (37),$$

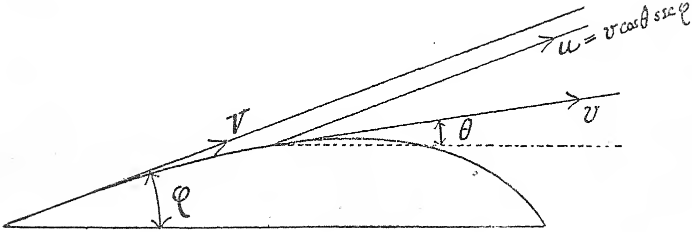
$$dx = -C\frac{\cos^{n-1}\theta}{Av_1^{n-1}}\,dv_1 \quad \ldots \quad (38),$$

$$d(\tan\theta) = Cg\frac{\cos^{n-1}\theta}{Av_1^{n+1}}\,dv_1 \quad \ldots \quad (39).$$

These three equations are not integrable, unless $\cos\theta$ is made equal to unity as is done in direct fire, or unless some approximate value is given to $\cos^{n-1}\theta$ in high-angle fire.

Fig. 4.

SIACCI.



Siacci's approximation is

$$\cos^{n-1}\theta = \cos^{n-2}\phi.$$

Siacci also makes use of a pseudo-velocity (see fig. 4) u, defined thus:—

$$v\cos\theta\sec\phi = v_1\sec\phi = u.$$

The initial value of u being

$$\left.\begin{array}{l} v\cos\theta\sec\phi = v_1\sec\phi = u. \\ V_1\sec\phi = V\cos\phi\sec\phi = V. \end{array}\right\}$$

It is seen from this that $v\cos\theta = u\cos\phi$, and u for any arc or for the whole trajectory is the component of v (the velocity at any point) parallel to the line of departure.

Converting the three general equations of motion so that the independent variable is Siacci's pseudo-velocity u, as defined above, and making use of the approximation $\cos^{n-1}\theta = \cos^{n-2}\phi$, then, since $v_1\sec\phi = u$, we have

$$dt = -C\frac{\cos^{n-2}\phi}{A(v_1\sec\phi)^n}\cdot\sec^n\phi\cdot\frac{d(v_1\sec\phi)}{\sec\phi} = -\frac{C}{A\cos\phi}\cdot\frac{du}{u^n} \quad \ldots \quad (40),$$

$$dx = -C\frac{\cos^{n-2}\phi}{A(v_1\sec\phi)^{n-1}}\cdot\sec^{n-1}\phi\cdot\frac{d(v_1\sec\phi)}{\sec\phi} = -\frac{C}{A}\cdot\frac{du}{u^{n-1}} \quad \ldots \quad (41),$$

$$d(\tan\phi) = \frac{Cg}{A}\cdot\frac{\cos^{n-2}\phi}{(v_1\sec\phi)^{n+1}}\cdot\sec^{n+1}\phi\cdot\frac{d(v_1\sec\phi)}{\sec\phi} = \frac{Cg}{A\cos^2\phi}\cdot\frac{du}{u^{n+1}} \quad \ldots \quad (42),$$

from which

$$t = \frac{C}{(n-1)\,A\cos\phi}\left(\frac{1}{u^{n-1}} - \frac{1}{V^{n-1}}\right) = \frac{C}{\cos\phi}\{T(V) - T(u)\} \quad . \quad . \quad . \quad . \quad (43),$$

$$x = \frac{C}{(n-2)\,A}\left(\frac{1}{u^{n-2}} - \frac{1}{V^{n-2}}\right) = C\{S(V) - S(u)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44),$$

$$\tan\phi - \tan\theta = \frac{C}{\cos^2\phi}\{I(V) - I(u)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

In this last equation put $\tan\theta = \frac{dy}{dx}$, integrate as before, then

$$\frac{y}{x} = \tan\phi - \frac{C}{\cos^2\phi}\left\{I(V) - \frac{A(V) - A(u)}{S(V) - S(u)}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46).$$

In this, the ballistic coefficient C is defined by Siacci thus,

$$C = \frac{1}{\beta}\,\frac{w}{\kappa\sigma\left(\frac{\tau_0}{f}\right)d^2} = \frac{1}{\beta}\,\frac{w}{nd^2},$$

where κ, σ, τ_0, f, w, d have the usual meaning as given in Chapter I, but β is a factor which must be found to correct for various values of

$$\beta = \frac{\cos^{n-2}\phi}{\cos^{n-1}\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47).$$

The various values of β are shown in tabulated form below; these have been calculated by Captain J. F. R. N. Maitland-Addison, R.A.

Siacci's Formulas for Calculating in Arcs Summed up.

$$\left.\begin{aligned}
C &= f\,\frac{w}{\beta\,.\,\kappa\sigma\,.\,\tau_0\,.\,d^2} \\
x &= C\,[S(V) - S(u)] \\
t &= C\sec\phi\,[T(V) - T(u)] \\
\tan\theta &= \tan\phi - \frac{C}{\cos^2\phi}[I(V) - I(u)] \\
\frac{y}{x} &= \tan\phi - \frac{C}{\cos^2\phi}\left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)}\right] \\
V &= V\cos\phi\sec\phi = V \\
u &= v\cos\theta\sec\phi
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (48).$$

TABLE OF FUNCTION $\bar{\beta}$.

Degrees.	Range in yards.																				
	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	11,000	12,000	13,000	14,000	15,000	16,000	17,000	18,000	19,000	20,000	21,000
6	1·00	1·00	1·00	1·00																	
7	1·00	1·00	1·00	1·00																	
8	1·00	1·00	1·00	1·00																	
9	1·00	1·00	1·00	1·00																	
10	1·00	1·00	1·00	1·00																	
11	1·00	1·00	1·00	1·00																	
12	1·00	1·00	1·00	1·00	0·96	0·97	1·00	1·00													
13	1·00	1·00	1·00	1·00	0·96	0·97	0·99	0·99													
14	1·01	1·01	1·01	1·01	0·97	0·97	0·99	0·98													
15	1·01	1·01	1·01	1·01	0·97	0·98	0·98	0·96													
16	1·02	1·02	1·02	1·02	0·98	0·98	0·97	0·94													
17	1·02	1·02	1·02	1·02	0·98	0·99	0·96	0·92													
18	1·03	1·03	1·03	1·03	0·99	0·99	0·95	0·90	0·94	0·99	0·99	0·99	0·99								
19	1·03	1·03	1·03	1·03	1·00	0·99	0·95	0·89	0·94	0·97	0·97	0·98	0·99								
20	1·04	1·04	1·04	1·04	1·01	1·00	0·96	0·90	0·93	0·95	0·95	0·97	0·99								
21	1·04	1·04	1·04	1·04	1·02	1·00	0·96	0·91	0·92	0·93	0·93	0·96	0·98								
22	1·05	1·05	1·05	1·05	1·03	1·00	0·97	0·92	0·91	0·90	0·91	0·95	0·98								
23	1·05	1·05	1·05	1·05	1·04	1·01	0·98	0·93	0·90	0·87	0·88	0·94	0·97								
24	1·06	1·06	1·06	1·06	1·06	1·01	0·99	0·94	0·89	0·84	0·85	0·92	0·97								
25	1·06	1·06	1·06	1·06	1·06	1·02	0·99	0·94	0·90	0·85	0·84	0·90	0·95								
26	1·07	1·07	1·07	1·07	1·07	1·03	1·00	0·95	0·91	0·86	0·84	0·88	0·93								
27	1·07	1·07	1·07	1·07	1·07	1·04	1·01	0·96	0·92	0·87	0·83	0·86	0·91								
28	1·08	1·08	1·08	1·08	1·08	1·05	1·02	0·97	0·94	0·89	0·83	0·84	0·88								
29	1·09	1·09	1·09	1·09	1·09	1·07	1·03	0·98	0·96	0·91	0·82	0·82	0·85								
30	1·10	1·10	1·10	1·10	1·10	1·09	1·04	0·99	0·98	0·93	0·82	0·79	0·82	0·86	0·92	0·95	0·97				
31	1·10	1·10	1·10	1·10	1·10	1·10	1·05	1·00	0·98	0·94	0·83	0·80	0·81	0·84	0·89	0·92	0·95				
32	1·11	1·11	1·11	1·11	1·11	1·11	1·06	1·01	0·99	0·95	0·84	0·80	0·80	0·82	0·86	0·89	0·93				
33	1·12	1·12	1·12	1·12	1·12	1·12	1·07	1·02	0·99	0·96	0·86	0·81	0·79	0·80	0·83	0·87	0·91				
34	1·13	1·13	1·13	1·13	1·13	1·13	1·09	1·03	1·00	0·97	0·88	0·81	0·78	0·78	0·81	0·85	0·89				
35	1·14	1·14	1·14	1·14	1·14	1·14	1·11	1·04	1·00	0·98	0·90	0·82	0·77	0·76	0·79	0·83	0·88				
36	1·15	1·15	1·15	1·15	1·15	1·15	1·13	1·06	1·01	0·99	0·92	0·83	0·77	0·75	0·77	0·81	0·87	0·91	0·94	0·97	0·99
37	1·16	1·16	1·16	1·16	1·16	1·16	1·14	1·07	1·02	1·00	0·93	0·84	0·77	0·75	0·76	0·79	0·85	0·89	0·92	0·95	0·97
38	1·17	1·17	1·17	1·17	1·17	1·17	1·15	1·08	1·03	1·01	0·94	0·85	0·77	0·74	0·75	0·77	0·83	0·87	0·90	0·93	0·95
39	1·18	1·18	1·18	1·18	1·18	1·18	1·16	1·09	1·04	1·02	0·95	0·86	0·78	0·74	0·74	0·76	0·81	0·86	0·89	0·91	0·93
40	1·19	1·19	1·19	1·19	1·19	1·19	1·18	1·11	1·05	1·03	0·96	0·87	0·78	0·73	0·73	0·75	0·79	0·85	0·88	0·90	0·92
41	1·20	1·20	1·20	1·20	1·20	1·20	1·20	1·13	1·06	1·04	0·98	0·88	0·78	0·72	0·72	0·74	0·78	0·84	0·87	0·89	0·91
42	1·22	1·22	1·22	1·22	1·22	1·22	1·22	1·15	1·08	1·05	1·00	0·90	0·78	0·72	0·71	0·73	0·77	0·83	0·86	0·88	0·90
43	1·23	1·23	1·23	1·23	1·23	1·23	1·23	1·17	1·09	1·06	1·02	0·93	0·80	0·73	0·72	0·73	0·77	0·81	0·85	0·88	0·90
44	1·25	1·25	1·25	1·25	1·25	1·25	1·24	1·18	1·11	1·06	1·00	0·91	0·80	0·74	0·73	0·74	0·77	0·80	0·84	0·87	0·90
45	1·26	1·26	1·26	1·26	1·26	1·26	1·26	1·19	1·12	1·07	1·01	0·91	0·80	0·74	0·74	0·76	0·78	0·81	0·83	0·86	0·89

TABLE OF FUNCTION $\bar{\beta}$ FOR QUADRATIC LAW.

V = 0 to 800 f.s.

For all ranges within the Quadratic Law.

Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.
1	1·00	16	1·02	31	1·07	46	1·19
2	1·00	17	1·02	32	1·07	47	1·20
3	1·00	18	1·02	33	1·08	48	1·21
4	1·00	19	1·02	34	1·09	49	1·23
5	1·00	20	1·03	35	1·09	50	1·24
6	1·00	21	1·03	36	1·10	51	1·26
7	1·00	22	1·03	37	1·11	52	1·28
8	1·00	23	1·03	38	1·11	53	1·30
9	1·00	24	1·04	39	1·12	54	1·32
10	1·00	25	1·04	40	1·13	55	1·34
11	1·01	26	1·05	41	1·14	56	1·36
12	1·01	27	1·05	42	1·15	57	1·38
13	1·01	28	1·05	43	1·16	58	1·40
14	1·01	29	1·06	44	1·17	59	1·42
15	1·01	30	1·06	45	1·18	60	1·45

Captain J. F. R. N. Maitland-Addison, R.A., points out that for very SMALL arcs

$$\bar{\beta} = \frac{\cos^{n-2}\phi}{\cos^{n-1}\theta} = \frac{\cos^{n-2}\phi}{\cos^{n-1}\phi} = \sec\phi.$$

It is still more accurate, however, when working in very SMALL arcs, to put

$$\bar{\beta} = \sec\eta, \quad \text{where} \quad \eta = \frac{\phi+\theta}{2},$$

and then the formulas are of the same form as Mayevski s but the pseudo-velocities V and u are, of course, different from Mayevski's both in magnitude and *direction*.

HIGH-ANGLE TRAJECTORY IN ONE ARC BY THE USE OF SIACCI'S β FUNCTION.

Strictly speaking, all high-angle trajectories should be calculated in a series of arcs, the extent of any arc depending on its inclination with the horizontal. For this purpose Mayevski's equations can be used with advantage.

But with a very fair approximation the calculations for range, &c., can be made *in one arc*, care being taken to give to the compensating factor β its proper value.

In (48), the direction of the pseudo-velocity u is always parallel to that of V, and considering the trajectory in one arc, $y = 0$ at the striking point when this is on the same level as the firing point, so that

$$0 = \tan\phi - \frac{C}{\cos^2\phi}\left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)}\right],$$

which gives

$$\sin 2\phi = Ca \quad \ldots \ldots \ldots \ldots \quad (49),$$

where

$$C = f\frac{w}{\beta\kappa\sigma\tau_0 d^2},$$

$$a = 2\left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)}\right].$$

The following example has been worked out by Captain J. F. R. N. Maitland-Addison, R.A.:—

Example.

Find the range of a projectile fired from a 9·2-inch high-angle gun, with a muzzle velocity of 2065 f/s at an angle of 30°, the weight of the projectile being 290 lbs.

It is reasonable to assume that, over such a curved trajectory, the projectile will not have the same steadiness as it would have in direct fire. Taking a 10 per cent. of unsteadiness gives $\sigma = 1\cdot1$, the first step is to determine a first approximation of the range, with

$$\tau = 1\cdot00 \text{ for } \begin{cases}\text{barometer of 30 inches,}\\ \text{thermometer of } 60^\circ \text{ F.,}\\ \text{air } \tfrac{2}{3}\text{rds saturated.}\end{cases}$$

$$\sigma = 1\cdot1, \quad \kappa = 1\cdot0, \quad \tau = 1\cdot0, \quad \beta = 1\cdot0.$$

$$w = 290, \qquad \log w = 2\cdot46240$$

$$d = 9\cdot2, \qquad 2\log d = 1\cdot92758$$

$$0\cdot53482$$

$$n = \kappa\sigma\tau = 1\cdot1, \qquad \log n = 0\cdot04139$$

$$\log C = 0\cdot49343$$

$$\sin 2\phi = C\,.\,a.$$

$$\phi = 30, \qquad \log\sin 2\phi = \bar{1}\cdot93753$$

$$\log C = 0\cdot49343$$

$$\log a = \bar{1}\cdot44410$$

$$a = 0\cdot27804$$

Looking out Table VIII, for a velocity of 2065 f/s, we see that for

$$\frac{R}{C} = 4100, \quad a = 0\cdot27533. \qquad \frac{R}{C} = 4200, \quad a = 0\cdot28745.$$

Hence, for $a = 0 \cdot 27804$, the value of

$$\frac{R}{C} \text{ is } = 4100 + \tfrac{271}{1212} 100 = 4122 \cdot 4.$$

$$\log \frac{R}{C} = 3 \cdot 61515$$

$$\log C = 0 \cdot 49343$$

$$\log R = 4 \cdot 10858$$

$$R = 12840 \text{ yards.}$$

Knowing this, it now remains to determine the values of f and β.

As an approximation, the height of the trajectory can be determined from the time of flight, and therefore the remaining pseudo-velocity must first be ascertained from

$$X = C\{S_V - S_u\}.$$

$R = 12{,}840$ yards,

$X = 38{,}520$ feet,

$$\log X = 4 \cdot 58570$$

$$\log C = 0 \cdot 49343$$

$$\log \frac{X}{C} = 4 \cdot 09227$$

$$\frac{X}{C} = 12367 \cdot 2$$

$$S_V = 41682 \cdot 2$$

$$S_u = 29315 \cdot 0$$

$$u = 730 \text{ f/s,}$$

and then

$$t = C \sec \phi \{T_V - T_u\}.$$

$$T_{2065} = 110 \cdot 393$$

$$T_{730} = 98 \cdot 617$$

$$\Delta T = 1 \cdot 07100$$

$$\log \sec \phi = 0 \cdot 06247$$

$$\log C = 0 \cdot 49343$$

$$\log t = 1 \cdot 62690$$

$$t = 42 \cdot 35 \text{ seconds.}$$

Then, approximately, $y = 4t^2 = 7174$ feet, and the mean height $= \frac{2}{3}y = 4782$ feet. From (fig. 1) p. 7, $f = 1 \cdot 16$ approximately. From the β function table it can be seen that the value of β corresponding to a range of 12,840 yards and an angle of projection of 30° is about $0 \cdot 8$. It is evident that this value, combined with the value of $f = 1 \cdot 16$, when inserted in the ballistic coefficient, will materially increase the range, so that before proceeding to make a re-calculation, it is advisable to take different values for β and f to those above. On looking in the β table, it is seen that at the point in question β increases rapidly with the range. Hence we will take a value of say $0 \cdot 9$, corresponding to a range of about 14,500 yards, and since the height of the trajectory will be greater than 7174 feet, we will take $f = 1 \cdot 2$: and now the ballistic coefficient must be re-constructed and the range calculated *de novo*.

As a second approximation

$$\log \frac{w}{nd^2} = 0 \cdot 49343$$

$$f = 1 \cdot 2, \log f = 0 \cdot 07918$$

$$0 \cdot 57261$$

$$\beta = 0 \cdot 9, \log \beta = \bar{1} \cdot 95424$$

$$\log C = 0 \cdot 61837$$

$$\log \sin 2\phi = \bar{1} \cdot 93753$$

$$\log C = 0 \cdot 61837$$

$$\log a = \bar{1} \cdot 31916$$

$$a = 0 \cdot 20853$$

From Table VIII, the value of $\frac{R}{C}$ corresponding to this value of "a" is

$$\frac{R}{C} = 3500$$

$$\log \frac{R}{C} = 3 \cdot 54407$$

$$\log C = 0 \cdot 61837$$

$$\log R = 4 \cdot 16244$$

$$R = 14{,}536 \text{ yards.}$$

It will be seen that the value of $\beta = 0 \cdot 9$ was a good prediction, and a further approximation is not necessary.

Actually at practice, the mean of 5 rounds fired under the conditions in this example was 14,755 yards, or about 200 yards more than the calculated range, so that the estimation of the steadiness as $1 \cdot 1$ was rather too large, but the error involved cannot be said to be large.

The remaining pseudo-velocity at the vertex can be calculated from :—

$$\tan \phi - \tan \theta = \frac{C}{\cos^2\phi} \{I_V - I_u\}$$

and since $\tan \theta = 0$, at the vertex

$$\sin 2\phi = 2C \{I_V - I_{u_0}\}$$

$$I_{u_0} = I_V - \frac{\sin 2\phi}{2C}$$

$$= I_V - \frac{a}{2}$$

and

$$u = 1034 \text{ f/s.}$$

The range to the vertex is 8204 yards.

The height to the vertex is given by

$$\frac{y}{x} = \tan \phi - \frac{C}{\cos^2\phi} \left\{ I_V - \frac{A_V - A_u}{S_V - S_u} \right\}$$

where x is the range (horizontal) to the vertex. In this case $y = 8923$ feet, so that the mean height would be 5948 feet, giving f as about $1 \cdot 2$; the prediction made for f was therefore good. Only experience will show how to make such predictions ; and until such be acquired, several approximations must be made until no further alterations in the calculations take place.

Direct Fire.

In direct fire the complete trajectory is calculated as one arc, the angle of projection ϕ is small, β may be taken as unity, also the tenuity of the air throughout is considered to be the same as at the firing point, so that $f = 1$; the value of C is therefore

$$C = \frac{w}{\kappa\sigma\tau_0 d^2}.$$

Suppose X feet denote the range in feet on a horizontal plane, obtained with an initial velocity V f/s, elevation $\phi°$, and that v denotes the actual velocity at any point of the trajectory, then the pseudo-velocity u is replaced by v.

Let ω denote the angle of descent, which on a horizontal plane is the same as the angle of arrival, then the formulas for a direct-fire trajectory become (see (48)):—

$$\left.\begin{aligned} C &= \frac{w}{\kappa\sigma\tau_0 d^2} \\ X &= C\{S(V) - S(v)\} \\ T &= C \sec\phi \{T(V) - T(v)\} \\ \tan\theta &= \tan\phi - C\sec^2\phi\{I(V) - I(v)\} \\ \frac{y}{x} &= \tan\phi - C\sec^2\phi\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (50).$$

But when ϕ is very small, as for flat trajectories, or when the projectile is considered as moving horizontally between screens, such as in the experiments carried out for finding the resistance of the air (Chapter I) for the compilation of the Ballistic Tables (Chapter II), then the 3rd and 4th equations in equation (50) become

$$T = C\{T(V) - T(v)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

$$\tan\theta - \tan\phi = -C.\{I(V) - I(v)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52).$$

At the striking point of the trajectory $y = 0$, $\theta = -\omega$, and v becomes the striking velocity; from (50), putting $y = 0$, then

$$\tan\phi = C\sec^2\phi\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\},$$

or

$$\sin 2\phi = 2C\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\},$$

$$\sin 2\phi = Ca \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53),$$

where

$$a = 2\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54).$$

Also from (50), when $\theta = -\omega$,

$$\begin{aligned} \tan\omega &= \tan\phi + C\sec^2\phi\{I(V) - I(v)\} \\ &= -C\sec^2\phi\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\} + C\sec^2\phi\{I(V) - I(v)\} \\ &= C\sec^2\phi\left\{\frac{A(V) - A(v)}{S(V) - S(v)} - I(v)\right\}. \end{aligned}$$

For flat trajectories, $\dfrac{\cos\phi}{\cos\omega}$ is approximately unity, and the last equation becomes

$$\sin 2\omega = 2C\left\{\frac{A(V) - A(v)}{S(V) - S(v)} - I(v)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55).$$

Let v_0 denote the velocity of the projectile at the vertex of the trajectory, where it moves horizontally for an instant and $\theta = 0$, then from (50)

$$\tan\phi = C\sec^2\phi\,\{I(V) - I(v_0)\}$$

or

$$\sin 2\phi = 2C\,\{I(V) - I(v_0)\} \quad \ldots\ldots \quad (56).$$

From (53) and (56)

$$I(v_0) = \frac{A(V) - A(v)}{S(V) - S(v)} \quad \ldots\ldots \quad (57),$$

therefore from (55)

$$\sin 2\omega = 2C\,\{I(v_0) - I(v)\} \quad \ldots\ldots \quad (58).$$

Then (52), putting $\theta = -\omega$,

$$\tan\omega = -\tan\phi + C\,\{I(V) - I(v)\} \quad \ldots\ldots \quad (59).$$

From (52) the direction of the projectile to the horizontal can be found at any point of the trajectory when the remaining velocity v is known and *vice versâ.*

In equations (53), (54), (55), (57), (58), (59) the v refers to the striking velocity at the end of the trajectory.

Summary of Formulas for Direct Fire with FLAT Trajectories.

$$C = \frac{W}{\kappa\sigma\tau_0 d^2},$$

$$X = C\,\{S(V) - S(v)\},$$

$$T = C\,\{T(V) - T(v)\},$$

$$\tan\theta = \tan\phi - C\,\{I(V) - I(v)\},$$

$$\sin 2\phi = Ca,$$

$$^{*}a = 2\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\};$$

also

$$\sin 2\phi = 2C\,\{I(V) - I(v_0)\},$$

$$I(v_0) = \frac{A(V) - A(v)}{S(V) - S(v)},$$

$$\sin 2\omega = 2C\left\{\frac{A(V) - A(v)}{S(V) - S(v)} - I(v)\right\} = 2C\,\{I(v_0) - I(v)\},$$

$$\tan\omega = -\tan\phi + C\,\{I(V) - I(v)\}.$$

---

* The double-entry Table VIII gives the values of $a = 2\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\}$ in terms of V and the reduced range $\frac{R}{C}$ where R is in yards, and $\frac{R}{C} = \frac{3X}{C} = S(V) - S(v)$; hence by giving to V different values, v is calculated for different ranges, supposing $C = 1$; by means of Tables IV, V, a series of values of a can be obtained for these ranges direct.

Example:—A 9·2-inch gun has a muzzle velocity $V = 2600$ f/s; $C = 5$; find the value of a for $V = 2600$, range $R = 5000$ yards.

The reduced range $\frac{R}{C} = 1000$ yards $= 3000$ feet, therefore

$$3000 = S_{2600} - S_v,$$

hence

$$v = 1785{\cdot}7 \text{ f/s},$$

$$a = 2\left\{I_{2600} - \frac{A(2600) - A(1785{\cdot}7)}{3000}\right\} = 0{\cdot}01845.$$

This is the figure found in Table VIII for values of $V = 2600$ f/s, $\frac{R}{C} = 1000$ yards.

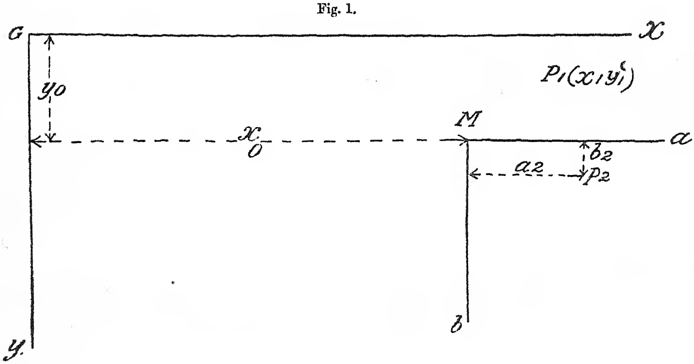
CHAPTER V.

ACCURACY OF FIRE.

IN Part I, Chapter III, examples were given for finding 50 per cent. zones from the results of firing a number of rounds; the practical use to be made of the Table of Probability factors was also shown, not only when the mean point of impact is the centre of the target, but also when it is not so situated.

In this chapter the theoretical basis of the rules employed will be given. Take as co-ordinate axes the line drawn from the gun G to the centre of the target for axis of x, and for axis of y the horizontal line through the gun at right angles to the x axis.

Fig. 1.



The abscissæ of the points of impact, P_1, P_2, ..., give the ranges actually obtained, and the arithmetic mean of the abscissæ, or the average abscissa, yields the abscissa of the centre of impact.

Similarly, the arithmetic mean or average of the ordinates gives the ordinate of the centre of impact.

Let M be the centre of impact and let the co-ordinates of the n points of impact $P_1, P_2 \ldots P_n$ be

$$x_1,\ x_2,\ x_3 \ldots x_n,$$
$$y_1,\ y_2,\ y_3 \ldots y_n.$$

Let (x_0, y_0) represent the co-ordinates of M, then

$$x_0 = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\Sigma x}{n},$$

$$y_0 = \frac{y_1 + y_2 + \ldots + y_n}{n} = \frac{\Sigma y}{n}.$$

Hence the position of the centre of impact is determined.

The choice of co-ordinate axes is quite arbitrary. It may be convenient sometimes to choose an origin of co-ordinates on the target itself; this is frequently done, and is, of course, done when the target is vertical. Occasionally, however, it is useful to put the successive ranges in evidence as has been done above, so that the ordinate of the centre of impact gives the mean range of the gun as fired.

Now transfer the origin to the centre of impact without altering the directions of the axes.

Let

$$a_1,\ a_2,\ a_3 \ldots a_n,$$

$$b_1,\ b_2,\ b_3 \ldots b_n$$

denote respectively the abscissæ and ordinates of the points of impact referred to the new axes.

Since the centre of impact is now at the origin,

$$0 = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = \frac{\Sigma a}{n},$$

$$0 = \frac{b_1 + b_2 + b_3 + \ldots + b_n}{n} = \frac{\Sigma b}{n},$$

and

$$\Sigma a = \Sigma b = 0.$$

The numbers a_1, $a_2 \ldots a_n$ are called the *longitudinal deviations* of the points of impact, and the numbers b_1, $b_2 \ldots b_n$ the *horizontal* or *lateral* deviations of the points of impact.

Observe that *deviations* have always reference to the centre of impact.

The result reached indicates that the algebraic sum of the horizontal or longitudinal deviations is zero, so that the sum of the positive deviations (in either direction) is equal to the sum of the negative deviations, numerical value being alone attended to.

When the position of the centre of impact on a horizontal plane is known, then the angle of descent (assumed to be the same for all the shots) determines the position of the centre of impact and of all the points of impact upon a vertical target, z denoting the vertical axis through the centre of the target.

Suppose any one shot strike the horizontal target at a distance l_1 yards from the vertical one *and beyond it,* then this vertical target will be struck at a height $l_1 \tan \beta$ yards above the ground, so that (neglecting ricochets) the centre of impact is at a height

$$z = \frac{l_1 \tan \beta + l_2 \tan \beta + \ldots + l_1 \tan \beta}{n} = \frac{l_1 + l_2 + \ldots l_n \tan \beta}{n} = l \tan \beta.$$

The centre of impact has an important property connected with what is known as the "theory of least squares."

The sum of the squares of the longitudinal (or horizontal) deviations with reference to the centre of impact is a minimum; that is, less than if a point, other than the centre of impact, were taken as origin of the co-ordinate axes with reference to which the deviations are measured.

This can easily be proved, because

$$a_1 = x_1 - x_0; \qquad a_2 = x_2 - x_0; \qquad a_3 = x_3 - x_0 \text{ and so on,}$$

therefore

$$a_1^2 + a_2^2 + \ldots + a_n^2 = (x_1 - x_0)^2 + (x_2 - x_0)^2 + \ldots (x_n - x_0)^2,$$

$$\therefore \Sigma a^2 = \Sigma x^2 - 2x_0 \Sigma x + n x_0^2.$$

But

$$\Sigma x = n x_0,$$

therefore

$$\Sigma a^2 = \Sigma x^2 - n x_0^2,$$

showing that Σa^2 is always less than Σx^2, the defect being $n x_0^2$, an essentially positive quantity, unless $x_0 = 0$, when, obviously, there must be equality.

Hence Σx^2 is a minimum when the origin is the centre of impact.

It follows that the sum of the squares of the absolute deviations has the minimum value

$$\Sigma a^2 + \Sigma b^2,$$

when the deviations are taken with respect to the centre of impact.

Certain definitions are now necessary in order that we may connect the dispersion of the points of impact with the accuracy and precision of the weapon.

The *mean longitudinal deviation* is the arithmetical mean of the absolute values of the longitudinal deviations. By absolute value is meant numerical value with abstraction of algebraic sign.

This is calculated either by dividing the sum of the absolute values by the number of shots or by dividing the sum of the values of the positive deviations by half the number of shots.

With abstraction of sign, the expression is

$$\frac{\Sigma a}{n}.$$

The *mean longitudinal quadratic deviation*, as found by theory, is (*see* p. 93)

$$\sqrt{\frac{\Sigma a^2}{n-1}},$$

which, when n is not very small, so that $n-1$ may be replaced by n, is practically the square root of the arithmetic mean of the squares of the longitudinal deviations.

The *probable longitudinal deviation* is that, with respect to which the probabilities of obtaining greater and less deviations are equal; that is to say, in the results of a large number of shots of the same series, half of the longitudinal deviations should be less than the probable deviation and the other half greater; and the probability of obtaining a deviation less than the probable deviation from any particular shot would be one-half. The deviation of a single shot is evidently just as likely to exceed as to fall short of the probable deviation.

The same definitions apply, *mutatis mutandis*, to horizontal, vertical, and absolute deviations.

Similar definitions are employed with regard to "errors" in the theory of observation. The following notation will be used:—

The mean longitudinal deviation : $\epsilon(x) = \frac{\Sigma(a)}{n}$;

" " horizontal deviation : $\epsilon(y) = \frac{\Sigma(b)}{n}$;

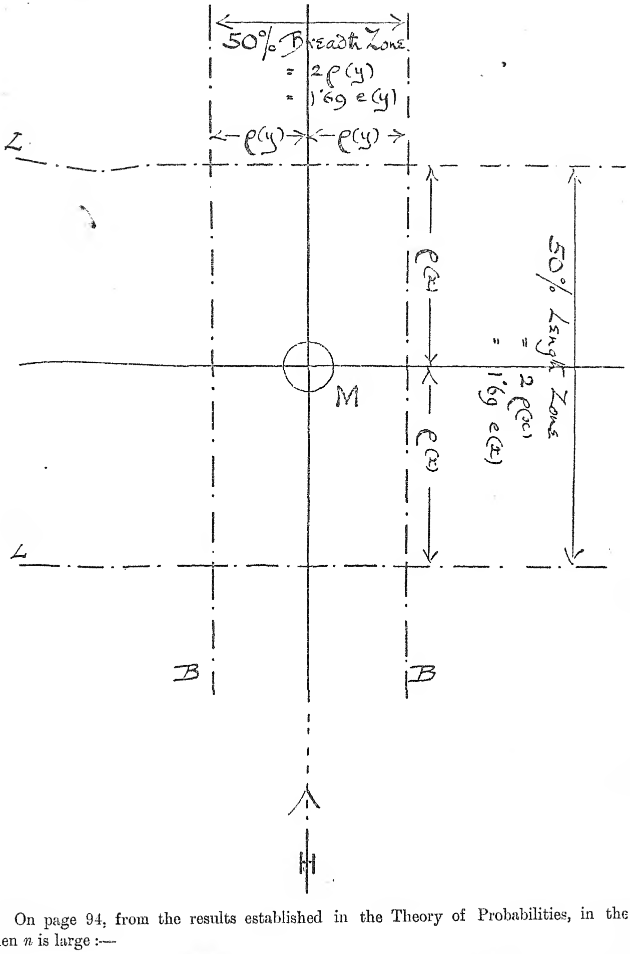
" " longitudinal quadratic deviation : $\mathrm{E}(x) = \sqrt{\frac{\Sigma(a^2)}{(n-1)}}$;

" " horizontal quadratic deviation : $\mathrm{E}(y) = \sqrt{\frac{\Sigma(b^2)}{(n-1)}}$.

The probable longitudinal deviation : $\rho(x)$;

" " horizontal deviation : $\rho(y)$.

Fig. 2.



On page 94, from the results established in the Theory of Probabilities, in the case when n is large :—

$$\rho = 0\cdot6745\mathrm{E}\,; \qquad \mathrm{E} = 1\cdot4826\rho\,;$$

$$\rho = 0\cdot8453e\,; \qquad \epsilon = 1\cdot1829\rho\,;$$

$$\frac{\mathrm{E}}{e} = \sqrt{\frac{\pi}{2}}\,; \qquad \mathrm{E} = 1\cdot2533e\,; \qquad \epsilon = 0\cdot7978\mathrm{E}\,;$$

where all the letters may refer to either x or y. Of the three quantities e, E, and ρ, the probable deviation ρ is usually chosen as a means of comparison of different guns or different series of shots with the same gun.

From the results of a series of shots both e and E may be calculated by measurements connected with the group of impacts, and from either or both of these quantities ρ may be deduced by multiplication by a numerical coefficient. The calculation of e being more simple than that of E, ρ is deduced with greater facility from e than from E; but unless the number of shots is very great, the calculation of ρ from E has a greater guarantee of accuracy than from e.

Suppose that lines are drawn parallel to the line joining the gun G with the centre of impact M, and distant ρ_y to the right and left of M; also through M, and on either side of it at a distance ρ_x, draw lines at right angles to MG.

Then, looking to the definition of ρ_x, in the zone LL, of length $2\rho_x$ and of indefinite width, 50 °/。 of the shots (the number being large) will probably fall; this is termed the 50 °/。 length zone.

Hence the 50 °/。 *length zone* has a length

$$2\rho_x = 1{\cdot}69e_x = 1{\cdot}349E_x.$$

So in the breadth zone BB of indefinite length but of width $2\rho_x$, 50 °/。 of the shots will probably fall; this is called the 50 °/。 *breadth zone*, its width is

$$2\rho_y = 1{\cdot}69e_y = 1{\cdot}349E_y.$$

The 50°/。 height zone is similarly constructed, its height on a vertical target is

$$2\rho_z = 2\rho_x \tan\beta,$$

where β is the angle of descent for the range, and the mean point of impact on the vertical target is in the middle of this 50 °/。 height zone.

If the 50 °/。 breadth and length zones be superposed we obtain a rectangle which must contain 50 °/。 of 50 °/。 or 25 °/。 of the total number of hits. This is called a 25 °/。 *rectangle*.

In a similar manner there is a 25 °/。 rectangle on a vertical target derived from the 50 °/。 breadth and height zones.

The relative accuracy of different guns at different ranges is frequently estimated by the dimensions of this rectangle.

Example 1.—From data obtained at Sandy Hook with a M.L. rifled mortar at a mean range of 3357 yards the following values of x and y were obtained, the origin being on the horizontal target at the shortest range ("Handbook of Problems of Direct Fire," by Captain James M. Ingalls):—

No. of round.	Range.	x.	y.	a.	b.
		yards.	yards.		
178	3264	0	4	− 93·11	−4·67
179	3348	84	16	− 9·11	+7·33
180	3296	32	9	− 61·11	+0·33
181	3427	163	12	+ 69·89	+3·33
182	3473	209	0	+115·89	−8·67
183	3318	54	6	− 39·11	−2·67
184	3320	56	10	− 37·11	+1·33
185	3408	144	12	+ 50·89	+3·33
186	3360	96	9	+ 2·89	+0·33

Here

$$\Sigma x = 838, \qquad \Sigma y = 78.$$

Therefore

$$X_0 = \frac{1}{9}\Sigma x = 93\cdot 11 \quad \text{and} \quad Y_0 = \frac{1}{9}\Sigma y = 8\cdot 67,$$

giving the position of the centre of impact.

Since

$$a_1 = x_1 - X_0, \text{ \&c.,} \quad \text{and} \quad b_1 = y_1 - Y_0, \text{ \&c.,}$$

we calculate the a and b columns which give the co-ordinates of the points of impact referred to the centre of impact as origin.

The sum of the absolute values of the deviations a is $479\cdot 11$, and that of the deviations b is $31\cdot 99$.

Hence

$$\epsilon(x) = \frac{479\cdot 11}{9} = 53\cdot 23; \qquad \epsilon(y) = \frac{31\cdot 99}{9} = 3\cdot 55;$$

and from the numerical formulas

$$\rho(x) = 0\cdot 845\epsilon(x) = 44\cdot 98 \text{ yards}; \qquad 2\rho(x) = 1\cdot 69\epsilon(x) = 89\cdot 96 \text{ yards};$$

$$\rho(y) = 0\cdot 845\epsilon(y) = 2\cdot 99 \text{ yards}; \qquad 2\rho(y) = 1\cdot 69\epsilon(y) = 5\cdot 99 \text{ yards};$$

giving the probable longitudinal and horizontal deviations and the width of the 50 °/。 length and breadth zones as computed from the mean deviations.

Also

$$\Sigma a^2 = 36306\cdot 9; \quad \text{hence} \quad E(x) = \sqrt{\frac{\Sigma a^2}{8}} = 67\cdot 37;$$

$$\Sigma b^2 = 182; \qquad E(y) = \sqrt{\frac{\Sigma b^2}{8}} = 4\cdot 77.$$

Therefore

$$\rho(x) = 0\cdot 6745E(x) = 45\cdot 44 \text{ yards} \quad \text{and} \quad 2\rho(x) = 1\cdot 349E(x) = 90\cdot 88 \text{ yards};$$

$$\rho(y) = 0\cdot 6745E(y) = 3\cdot 215 \text{ yards} \quad \text{and} \quad 2\rho(y) = 1\cdot 349E(y) = 6\cdot 43 \text{ yards}:$$

the similar results computed from the mean quadratic deviations, and it will be seen that they differ but slightly from those obtained from the mean deviations.

A 25 °/。 rectangle made by the overlapping of the 50 °/。 zones is $90\cdot 88$ yards by $6\cdot 43$ yards.

The diagrams given herewith are extracted from Krupp's *Artillerie*, a handbook issued at Düsseldorf, 1902; they will serve as further exercises for determining 50 °/。 zones from practice.

Fig. a.

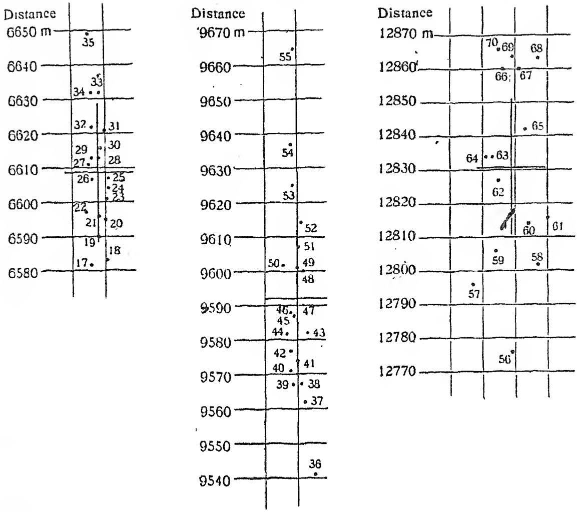


Fig. b.

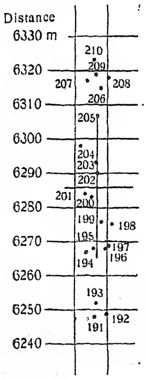
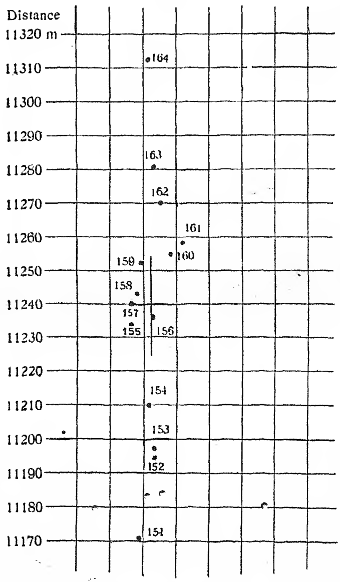


Fig. c.



The percentage of hits in other zones, which are symmetrical about the centre of impact in the direction of either axis, may be determined, and also the width of zone that may be expected to include a given percentage of hits.

To do this, we require a table of probability factors deduced from theoretical considerations, explained on p. 94.

Table of Probability Factors.

The following gives the proportional width of other zones (containing a different percentage of hits) to one of 50 °/o as unity.

Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.
1	0·02	21	0·40	41	0·80	61	1·27	81	1·94
2	0·04	22	0·41	42	0·82	62	1·30	82	1·98
3	0·06	23	0·43	43	0·84	63	1·33	83	2·03
4	0·07	24	0·45	44	0·86	64	1·36	84	2·08
5	0·09	25	0·47	45	0·89	65	1·39	85	2·13
6	0·11	26	0·49	46	0·91	66	1·42	86	2·18
7	0·13	27	0·51	47	0·93	67	1·45	87	2·24
8	0·15	28	0·53	48	0·95	68	1·48	88	2·30
9	0·17	29	0·55	49	0·98	69	1·51	89	2·37
10	0·18	30	0·57	50	1·00	70	1·54	90	2·44
11	0·20	31	0·59	51	1·02	71	1·57	91	2·52
12	0·22	32	0·61	52	1·04	72	1·60	92	2·60
13	0·24	33	0·63	53	1·07	73	1·64	93	2·69
14	0·26	34	0·65	54	1·09	74	1·67	94	2·78
15	0·28	35	0·67	55	1·12	75	1·71	95	2·91
16	0·30	36	0·70	56	1·14	76	1·74	96	3·04
17	0·32	37	0·72	57	1·17	77	1·78	97	3·32
18	0·34	38	0·74	58	1·19	78	1·82	98	3·45
19	0·36	39	0·76	59	1·22	79	1·86	99	3·82
20	0·38	40	0·78	60	1·25	80	1·90	100	Infinite*

* As a factor of 4 gives more than 99 °/o of the rounds fired, it may be taken for practical purposes to give the total of 100 °/o.

In the first column will be found numbers representing the percentages of hits that may be expected in the zones; the corresponding factors represent the multiples that the widths of the zones are of the width of the 50 °/o zone.

To find the width of the length zone that will contain 75 °/o of the hits, we enter the table at the number 75 in the column headed "Per cent.," and find the corresponding factor to be 1·71. We deduce, therefore, that the width of the required zone is 1·71 times the width of the 50 °/o length zone.

Also to find the percentage of hits that will be included in breadth zone 1·25 times the width of the 50 °/o breadth zone, we enter the table at the number 1·25 in the column headed "Factor," and find the corresponding percentage to be 60. We conclude that 60 °/o of hits will be found in the given breadth zone.

Intermediate results can be obtained from the table by interpolation.

Rectangles containing a given percentage of hits can be obtained, and conversely we can determine the percentage of hits that will be found in any given rectangle which is symmetrical about the centre of impact.

Suppose a rectangle to be obtained by superposition of a length zone of p °/。 and a breadth zone of q °/。, then the rectangle will contain

$$p \text{ °/。 of } q \text{ °/。, or } \frac{pq}{100} \text{ °/。 of the hits.}$$

For the design of a rectangle to contain R °/。 of hits we have the relation

$$\frac{pq}{100} = R$$

for the determination of p and q. The equation has an infinite number of solutions, so that we can design an infinite number of rectangles containing the given percentage R of hits. We may give q any value we please, and thence determine p from the equation

$$q = \frac{100R}{p}.$$

We look out p and $\frac{100R}{p}$ in the column of the table headed "Per cent.," and thence find the widths of the length and breadth zones, which, by superposition, give an R °/。 rectangle. These widths are the longitudinal and horizontal sides of the rectangle.

The 25 °/。 rectangle already met with is thus only one of an infinite number of 25 °/。 rectangles. For its design we excluded 50 °/。 of hits for horizontal deviations and 50 °/。 for longitudinal deviations.

It is frequently desired, as in this case, to exclude the same number of hits for horizontal as for longitudinal deviations, and then the determination of the rectangle rests upon the equation

$$q^2 = 100R,$$

or

$$q = 10\sqrt{R}.$$

An example will make the subject clearer.

Example 2.—Find a rectangle containing 50 °/。 of hits such that the same number of hits may be excluded for horizontal as for longitudinal deviations. Here R = 50, and if q be the percentage of hits in the breadth and length zones which, by superposition, give the rectangle

$$q = p = 10\sqrt{50} = 70 \cdot 7;$$

entering the table we find, by interpolation, the factor 1·56, so that the widths of the zones are 1·56 times the widths of the corresponding 50 °/。 zones. Hence the sides of the rectangle are,

$$1 \cdot 56 \times 2\rho_x = 3 \cdot 12\rho_x,$$

and

$$1 \cdot 56 \times 2\rho_y = 3 \cdot 12\rho_y.$$

A study of the table shows that a zone four times the width of the 50 °/。 zone practically contains the whole of the hits. This zone is termed the "*enveloping zone.*" By superposition of the enveloping breadth and length zones we obtain the *enveloping rectangle*, which may be shown to comprise 98·6 °/。 (practically all) of the hits.

It is obvious that in many cases the horizontal deviations will not be of so much importance as those in the longitudinal direction, and that it will be useful to calculate rectangles which give relatively small importance to the horizontal deviations. In the extreme case of a gun which shoots practically perfectly as to line we need only consider the length zones which are the extreme cases of the rectangles.

Example 3.—The numbers of hits excluded for horizontal and longitudinal deviations respectively being in the ratio of 2 to 3, determine the dimensions of the 50 °/° rectangle.

Let the longitudinal zone be one of p °/° and the width zone be one of q °/°.

Then

$$100 - q = \frac{2}{3}(100 - p),$$

or

$$3q = 2p + 100 \quad \ldots \ldots \ldots \ldots \quad (1).$$

Also p °/° of q °/° gives the 50 °/° rectangle, therefore

$$\frac{p}{100} \times \frac{q}{100} = \frac{50}{100},$$

$$pq = 5000 \quad \ldots \ldots \ldots \ldots \quad (2),$$

from (1) and (2)

$$p = 65 \cdot 14,$$

$$q = 76 \cdot 76.$$

From the table the factors are found to be 1·40 and 1·77.

Hence the sides of the rectangle are

$$1 \cdot 40 \times 2\rho(x) = 2 \cdot 80\rho(x)$$

$$1 \cdot 77 \times 2\rho(y) = 3 \cdot 54\rho(y)$$

The actual number of hits obtained upon a *given* target depends upon the position of the mean point of impact relative to the target. For examples showing the method of calculating the actual number of hits, *see* Part I, pp. 96–109.

The co-ordinates of the centre of impact, M (fig. 1), have been denoted by X_0, Y_0. If the target is horizontal and the origin at the firing point, X_0 is the arithmetic mean of the several ranges actually obtained; only when the number of rounds is increased indefinitely does X_0 represent the exact range appertaining to the gun as laid.

The probable deviation of a single point of impact has been denoted by $\rho(x)$; this also is deduced from the rounds fired, and is only exact when the number of rounds increases without limit. The probable deviation of the centre of impact, deduced from a series of n rounds, from the true centre of impact is found by dividing the probable deviation of a single shot, deduced from the series, by the square root of the number of shots.

Thus if $\rho(x)$ be the probable deviation in range of a single point of impact,

$$\frac{\rho(x)}{\sqrt{n}}$$

is the probable deviation in range of the centre of impact of a group of n shots.

As an example take the data of Example 1. From 9 rounds a mean range of 3,357 yards was obtained, and the probable deviation in range of a single shot was found to be 45·44 yards. The range that might be expected to be obtained from a single shot would be denoted by

$$3357 \pm 45 \cdot 44 \text{ yards};$$

and it is an even chance that if another 9 shots were fired their arithmetic mean would fall within the limits :—

$$3357 \pm \frac{45 \cdot 44}{\sqrt{9}},$$

or

$$3357 \pm 15 \cdot 15 \text{ yards}.$$

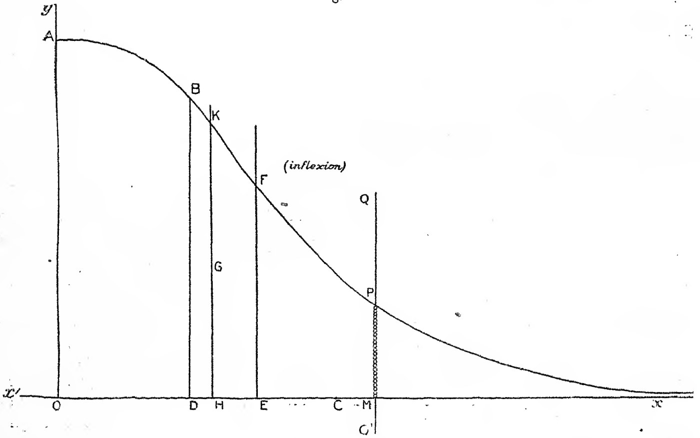
Probability of Fire

According to the theory of Probability, a certain curve, called the *error curve*, of the shape in fig. 3, can be drawn representing graphically by its area the percentage (°/。) of shots which in the long run can be expected on a target of given dimensions.

Suppose, for instance, that $x'Ox$ in fig. 3 is drawn along the line of mean direction so that O is at the point of mean impact, and that in a very large series of practice all the shot which struck on the line QQ′ are arranged in contact along the ordinate MP; if this is done with all the shot, they will be found arranged in a certain area, bounded by the straight line $x'Ox$ and the error curve $x'Ax$.

This curve of error can be realised experimentally by an instrument invented by

Fig. 3.



Mr. Francis Galton, which he calls the *Quincunx*, from the Latin word describing the arrangement of trees in an orchard (*see* Part I, p. 96).

A charge of small shot is allowed to pour through the funnel at the top; the shot knock against pins arranged like trees, and are scattered thereby in an arbitrary manner; but it is found that the shot always group themselves in the stall at the bottom in a manner which imitates closely the profile of the error curve.

The error curve (fig. 3) is redrawn here to a larger scale and as accurately as possible, the right-hand half alone being given, and in accordance with abstruse theoretical principles, the error curve can be best represented by an equation of the form

$$y = ae^{-\frac{x^2}{c^2}} \text{ or } a \exp\left(-\frac{x^2}{c^2}\right).$$

So that when $x = O$, $y = a = OA$, and "a" represents the number of shots (out of a large number) which struck on the line OA.

The curve $y = ae^{-\frac{x^2}{c^2}}$ is symmetrical to the right and left of QA, where O is the mean point of impact, thus stating that plus and minus deviations are equally likely to occur. Again,

as x increases the ordinate y decreases, which is the same as stating that small deviations (from O) are more frequent than large ones. The curve rapidly approaches the axis $x'Ox$, on either side of OA, which includes the statement that large deviations are rare, and, beyond a certain limit, practically do not occur.

All the shots fall within the area $x'OxAx'$. Denote this area by 2A, then

$$2A = a\int_{-\infty}^{\infty} e^{-\frac{x^2}{c^2}}\,dx = 2a\int_0^{\infty} e^{-\frac{x^2}{c^2}}\,dx$$

$$= 2ac\int_0^{\infty} e^{-t^2}\,dt \quad \text{where} \quad t = \frac{x}{c}.$$

But $\int_0^{\infty} e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$, a well-known definite integral.

Hence

$$2A = ac\sqrt{\pi}.$$

But $a\int_{-\infty}^{\infty} e^{-\frac{x^2}{c^2}}\,dx$ is the probability of a shot falling anywhere along $x'Ox$ from $x = -\infty$ to $x = +\infty$, hence it is a certainty, and $2A = 1$, giving

$$a\sqrt{\pi} = \frac{1}{c};$$

denoting $\frac{1}{c}$ by h, then

$$\frac{1}{c} = h = a\sqrt{\pi}.$$

The probability curve can now be written

$$y = \frac{h}{\sqrt{\pi}}e^{-h^2x^2}.$$

The total area of this curve is $2A = 1$, and twice the area OMPA therefore represents the probability that the error will be within $\pm x$. Denote the area OMPA by P, then

$$P = \frac{2h}{\sqrt{\pi}}\int_0^x e^{-h^2x^2}\,dx.$$

Putting $hx = t$,

$$P = \frac{2}{\sqrt{\pi}}\int_0^{t\,(=hx)} e^{-t^2}\,dt = \frac{2}{\sqrt{\pi}}\int_0^{t\,(=hx)} \exp(-t^2)\,dt.$$

This integral must be evaluated by approximate numerical computation.

Integrating by parts, and putting

$$\int_0^t e^{-t^2}\,dt = \int_0^{\infty} e^{-t^2}\,dt - \int_t^{\infty} e^{-t^2}\,dt.$$

But

$$\int_0^{\infty} e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$$

and

$$\int_0^t e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2} - \frac{e^{-t^2}}{2t}\left\{1 - \frac{1}{2t^2} + \frac{1.3}{(2t^2)^2} - \frac{1.3.5}{(2t^2)^3} + \dots\right\},$$

so that

$$P = \frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}\,dt = 1 - \frac{e^{-t^2}}{t\sqrt{\pi}}\left\{1 - \frac{1}{2t^2} + \frac{1.3}{(2t^2)^2} - \frac{1.3.5}{(2t^2)^3} + \dots\right\}.$$

The tabulation of P for various values of t is as follows :—

$t = hx.$	$P = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2}\,dt.$									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0·0	0·0000	0·0113	0·0226	0·0338	0·0451	0·0564	0·0676	0·0789	0·0901	0·1013
0·1	·1125	·1236	·1318	·1459	·1569	·1680	·1790	·1900	·2009	·2118
0·2	·2227	·2335	·2443	·2550	·2657	·2763	·2869	·2974	·3079	·3183
0·3	·3286	·3389	·3491	·3593	·3694	·3794	·3893	·3992	·4090	·4187
0·4	·4284	·4380	·4475	·4569	·4662	·4755	·4847	·4937	·5027	·5117
0·5	0·5205	0·5292	0·5379	0·5465	0·5549	0·5633	0·5716	0·5798	0·5879	0·5959
0·6	·6039	·6117	·6194	·6270	·6346	·6420	·6494	·6566	·6638	·6708
0·7	·6778	·6847	·6914	·6981	·7047	·7112	·7175	·7238	·7300	·7361
0·8	·7421	·7480	·7538	·7595	·7651	·7707	·7761	·7814	·7867	·7918
0·9	·7969	·8019	·8068	·8116	·8163	·8209	·8254	·8299	·8342	·8385
1·0	0·8427	0·8468	0·8508	0·8548	0·8586	0·8624	0·8661	0·8698	0·8733	0·8768
1·1	·8802	·8835	·8868	·8900	·8931	·8961	·8991	·9020	·9048	·9076
1·2	·9103	·9130	·9155	·9181	·9205	·9229	·9252	·9275	·9297	·9319
1·3	·9340	·9361	·9381	·9400	·9419	·9438	·9456	·9473	·9490	·9507
1·4	·9523	·9539	·9554	·9569	·9583	·9597	·9611	·9624	·9637	·9649
1·5	0·9661	0·9673	0·9684	0·9695	0·9706	0·9716	0·9726	0·9736	0·9745	0·9755
1·6	·9763	·9772	·9780	·9788	·9796	·9804	·9811	·9818	·9825	·9832
1·7	·9838	·9844	·9850	·9856	·9861	·9867	·9872	·9877	·9882	·9886
1·8	·9891	·9895	·9899	·9903	·9907	·9911	·9915	·9918	·9922	·9925
1·9	·9928	·9931	·9934	·9937	·9939	·9942	·9944	·9947	·9949	·9951
2·0	0·9953	0·9955	0·9957	0·9959	0·9961	0·9963	0·9964	0·9966	0·9967	0·9969
2·1	·9970	·9972	·9973	·9974	·9975	·9976	·9977	·9979	·9980	·9980
2·2	·9981	·9982	·9983	·9984	·9985	·9985	·9986	·9987	·9987	·9988
2·3	·9989	·9989	·9990	·9990	·9991	·9991	·9992	·9992	·9992	·9993
2·4	·9993	·9993	·9994	·9994	·9994	·9995	·9995	·9995	·9995	·9996
2·5	0·9996	0·9996	0·9996	0·9997	0·9997	0·9997	0·9997	0·9997	0·0998	0·9998
2·6	·9998	·9998	·9998	·9998	·9998	·0998	·9998	·9998	·9998	·9999
∞	1·0000									

And by interpolation we have—

$hx = t.$	P.	$hx = t.$	P.
0·04434	0·05	0·5342	0·55
0·08888	0·10	0·5951	0·60
0·1337	0·15	0·6609	0·65
0·1791	0·20	0·7329	0·70
0·2253	0·25	0·8135	0·75
0·2724	0·30	0·9062	0·80
0·3208	0·35	1·0179	0·85
0·3708	0·40	1·1631	0·90
0·4227	0·45	1·3859	0·95
0·4769	0·50	1·8215	0·99

In the long run, the mean arithmetic error ϵ, is the abscissa OH of the C.G. of the area $\mathrm{OA}x = \mathrm{A}_{(\infty)}$, and the equation to AKP is $y = \dfrac{h}{\sqrt{\pi}} \exp(-h^2x^2)$, so that

$$\epsilon \,.\, \mathrm{A}_{(\infty)} = \int_0^\infty x \frac{h}{\sqrt{\pi}} e^{-h^2x^2}\,dx\,.$$

$$= \frac{h}{\sqrt{\pi}}\left(-\frac{1}{2h^2}\right)\left[e^{-h^2x^2}\right]_0^\infty;$$

$$\epsilon \,.\, \tfrac{1}{2} = +\frac{1}{2h\sqrt{\pi}}, \quad \text{since} \quad \mathrm{A}_{(\infty)} = \tfrac{1}{2}.$$

Therefore

$$\epsilon = \frac{1}{h\sqrt{\pi}}\,.$$

Now, let E denote the mean quadratic error, so that E is the radius of gyration of the area $\mathrm{AO}x$ about $\mathrm{OA}y$.

$$\mathrm{E}^2 \,.\, \mathrm{A}_{(\infty)} = \int_0^\infty x^2 \,.\, y\,dx$$

$$= \frac{h}{\sqrt{\pi}}\int_0^\infty x^2 e^{-h^2x^2}\,dx$$

$$= \frac{h}{\sqrt{\pi}} \,.\, \left(-\frac{1}{2h^2}\right)\int_0^\infty x\,\frac{d\,(e^{-h^2x^2})}{dx}\,dx.$$

Since $\mathrm{A}\,(\infty) = \frac{1}{2}$,

$$(\mathrm{E})^2 \,.\, \tfrac{1}{2} = \frac{h}{\sqrt{\pi}}\left(-\frac{1}{2h^2}\right)\left[xe^{-h^2x^2}\right]_0^\infty + \frac{h}{\sqrt{\pi}} \cdot \frac{1}{2h^2}\int_0^\infty e^{-h^2x^2}\,dx\,.$$

$$= -\frac{1}{2h\sqrt{\pi}}\left[\frac{x}{1 + (hx)^2 + \dfrac{(hx)^4}{2!} + \ldots}\right]_0^\infty + \frac{h}{\sqrt{\pi}} \cdot \frac{1}{2h^2} \cdot \frac{\sqrt{\pi}}{2h}$$

$$= \frac{1}{4h^2},$$

therefore

$$(\mathrm{E})^2 = \frac{1}{2h^2} \quad \text{and} \quad \mathrm{E} = \frac{1}{h\sqrt{2}};$$

so that the point F on the curve ABFP is a point of inflexion, as is seen by putting

$$\frac{d^2y}{dx^2} = 0, \quad \text{where} \quad y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2};$$

so that

$$\frac{\mathrm{E}}{\epsilon} = \sqrt{\frac{\pi}{2}},$$

and

$$h = \frac{t}{x} = \frac{1}{\epsilon\sqrt{\pi}} = \frac{1}{\mathrm{E}\sqrt{2}}\,.$$

The abscissa ρ = OD of the ordinate BD, which cuts the area $\mathrm{AO}x$ in half, is called the *probable error*, because in the long run half the shots have a greater error and the other half a less error than ρ. The line BD and the parallel symmetrical line B′D′ cut out the middle half of the whole area $x'\mathrm{B'AB}x$ of the error curve, so that the probability is that half the total

number of shots fall within the zone DD′ and the other half fall outside DD′; hence the zone D′D will probably catch 50 °/₀ of the shots, and of the remainder 25 °/₀ fall in the zone D′x' and 25 °/₀ in the zone Dx.

The zone D′D is called the 50 °/₀ zone.

To find the length D′D $= 2\rho$.

The value of $t\,(= hx)$ corresponding to P $= 0\cdot5$ is $\cdot4769$, hence

$$h\,.\,\rho = \cdot4769.$$

But

$$h = \frac{1}{\epsilon\sqrt{\pi}} = \frac{1}{\mathrm{E}\sqrt{2}},$$

hence

$$\rho = \cdot4769\sqrt{\pi}\,.\,\epsilon = 0\cdot8453\epsilon,$$

$$2\rho = 1\cdot6906\epsilon,$$

also

$$\rho = \cdot4769\sqrt{2}\mathrm{E} = 0\cdot6745\mathrm{E},$$

$$2\rho = 1\cdot359\mathrm{E}.$$

So that a 50 °/₀ zone breadth is equal to the mean arithmetical error obtained from the analysis of all available practice, multiplied by $1\cdot69$.

Other zones are found in a similar manner, thus the 25 °/₀ zone:—

When P $= 0\cdot25$, then $t = hx = 0\cdot2253$, and the 25 °/₀ zone is

$$2x = 0\cdot2253\times\frac{2}{h},$$

$$25\ °/_{\circ}\ \text{zone} = \cdot2253\times2\sqrt{\pi}\epsilon = 0\cdot9024\epsilon,$$

also

$$\frac{\text{breadth of } 25\ °/_{\circ}\ \text{zone}}{\text{breadth of } 50\ °/_{\circ}\ \text{zone}} = \frac{\cdot2253}{\cdot4769} = 0\cdot4725,$$

similarly

$$\frac{\text{breadth of } 75\ °/_{\circ}\ \text{zone}}{\text{breadth of } 50\ °/_{\circ}\ \text{zone}} = \frac{\cdot8135}{\cdot4769} = 1\cdot7058,$$

and so on for any zone.

To determine the °/₀ of hits to be expected in a zone bounded by any ordinate MP and its symmetrical ordinate M′P′, the ratio of the breadth MM′ of this zone to D′D, the breadth of the 50 °/₀ zone, is calculated, and called the *probability factor*, and a *Table of Probability Factors* is calculated, giving the °/₀ which the MPP′M′ of the error curve bears to the whole area and the corresponding probability factor.

When the ordinates PM and P′M′ which limit the zone occupied by the target are not symmetrical with respect to the line of mean impact AO, the °/₀ of hits to be expected on each part AOMP and AOM′P′ must be calculated separately, and these °/₀ are added or subtracted according as PM and P′M′ are on opposite sides of AO or on the same side.

Thus, if the number of hits on a zone bounded by PM and P′M′ is less than what should be expected, the inference is that the gun is not laid properly so as to bring the line of mean impact AO midway between PM and P′M′.

The probability table, on p. 87, has the factor $0\cdot47$ opposite the 25 °/₀ and $1\cdot7$ is opposite the 75 °/₀; the factor for other percentages is worked out as shown above. It is understood that the centre of impact is at the centre of the target.

To four places of decimals, the factors for the following percentages are now given :—

Per cent.	Factor.	Per cent.	Factor.
5	0·0930	55	1·1201
10	0·1863	60	1·2479
15	0·2804	65	1·3857
20	0·3756	70	1·5368
25	0·4725	75	1·7058
30	0·5713	80	1·9002
35	0·6727	85	2·1344
40	0·7775	90	2·4389
45	0·8863	95	2·9061
50	1	99	3·8195

If the target fired at is limited by two dimensions, say, length and breadth, or breadth and height, it is treated as the overlapping of two such unlimited zones, for which the separate °/ₒ of hits is calculated, and the product of these gives the required percentage.

Modern range tables contain three columns, giving at each range the size of the 50 °/ₒ zone for errors in range, direction, and vertical deviation; and now the probability factor enables us to calculate the °/ₒ of hits to be expected on a zone of given depth or length in range, or of breadth in direction, or of given vertical height; thence we infer the number of shots required to make an assigned number of hits, and can decide whether the object is worth the ammunition to be expended.

The theory of Probability is also useful in the design of match targets, and in comparing the results of competitive artillery practice carried out under different conditions.

In designing a vertical target for rifle shooting, the breadth and height may be taken as four times that of the 50 °/ₒ zones, as more than 99 °/ₒ of the shots should now be caught by the target, if the rifle is properly aimed.

The overlapping of the two 50 °/ₒ zones will give a 25 °/ₒ rectangle, which may be taken as appropriate for the *bull's-eye;* two 70·7 °/ₒ zones will enclose a 50 °/ₒ rectangle, which will serve as the boundary of the *inner;* while two 86·6 °/ₒ zones will enclose a 75 °/ₒ rectangle, appropriate for the *magpie*, the space between this and the enveloping rectangle being the *outer*.

On a circular target the radius of the bull's-eye, centre, inner, and outer would be obtained by the revolution of the error curve round Oy, and determining the radius of the cylinder which cuts out 25 °/ₒ 50 °/ₒ, 75 °/ₒ, and 99 °/ₒ of the total volume enclosed by the surface generated.

Then if r is the radius of the circle, with centre at the point of mean impact on the target which catches 100 P °/ₒ of the shots,

$$P = \frac{V(r)}{V(\infty)}.$$

The equation of the probability curve is

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2}, \quad \text{or} \quad \frac{h}{\sqrt{\pi}} \exp(-h^2x^2),$$

therefore

$$V(r) = \frac{h}{\sqrt{\pi}} \int_0^r e^{-h^2x^2} . 2\pi x . dx = \frac{h}{\sqrt{\pi}} . 2\pi\left(-\frac{1}{2h^2}\right)\left[e^{-h^2x^2}\right]_0^r,$$

$$V(\infty) = \frac{h}{\sqrt{\pi}} \int_0^\infty e^{-h^2x^2} . 2\pi x . dn = \frac{h}{\sqrt{\pi}} . 2\pi\left(-\frac{1}{2h^2}\right)\left[e^{-h^2x^2}\right]_0^\infty,$$

therefore

$$P = 1 - e^{-h^2 r^2}$$

and

$$\frac{1}{h} = \epsilon \sqrt{\pi} = E \sqrt{2},$$

where ϵ is the mean arithmetical error, and E is the mean quadratic error.

If ρ represent the radius of the probable deviation, $h\rho = \cdot 4769$ (see p. 94), then

$$rh = \sqrt{\left(\log_e \frac{1}{1-P}\right)},$$

and

$$\frac{r}{\rho} = \frac{1}{\cdot 4769} \sqrt{\left(\log_e \frac{1}{1-P}\right)},$$

$$= 2 \cdot 097 \sqrt{2 \cdot 302} \sqrt{\left(\log_{10} \frac{1}{1-P}\right)},$$

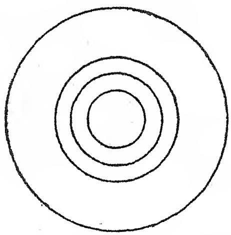
$$= 3 \cdot 181 \sqrt{\left(\log_{10} \frac{1}{1-P}\right)},$$

so that the radii of the circles for the various °/。 are calculated as follows:—

P.	$\frac{1}{1-P}$.	$\log_{10} \frac{1}{1-P}$.	$\frac{r}{\rho}$.
0·25	1·3333	0·1249	1·13
0·50	2	0·3010	1·75
0·75	4	0·6020	2·5
0·99	100	2·000	4·5

Thus with a rifle at 500 yards range the probable deviation ρ might be about 4 inches, thus making the radii of the 25, 50, 75, and 99 °/。 circles about 4·5, 7, 10, and 18 inches, as shown in fig. 4, drawn to a scale of $\frac{1}{20}$. An expert marksman should bring the centre of impact very close to the centre of the target, and then 99 °/。 of the shots should be on the target, and 25, 25, 25, and 24 °/。 in each compartment.

Fig. 4.



If four marks are scored for a bull's-eye and 3, 2, 1 marks for the other compartments, the probable score for 100 shots would be

$$25 \times 4 + 25 \times 3 + 25 \times 2 + 24 = 249.$$

The same thing will hold when the two errors, lateral and vertical, $\rho\,(x)$ and $\rho\,(y)$, are not equal; and now the circular curves must be replaced by similar ellipses.

Determination of the Mean and Probable Errors of Functions of Independent Observed Quantities.

Let ϵ_e denote the probable error in yards at a given range due to the layer, and ϵ_g the probable error in yards at the same range due to the gun, these probable errors are independent of each other, and then

$$E^2 = \epsilon_e^2 + \epsilon_g^2$$

where E denotes the probable error due to the combined probable errors of the layer and the gun (see Chauvenet's *Spherical and Practical Astronomy*, Vol. II, p. 497).

Abnormal or Doubtful Rounds in Analysis.

In the analysis of a number of rounds of a series, one or more rounds may appear to differ abnormally from the rest, the question whether these rounds are due to causes of so exceptional a character that they are not samples of what may be expected and ought therefore to be rejected, should be decided by reference to some criterion which is well established.

In Chauvenet's *Spherical and Practical Astronomy*, Vol. II, pp. 558–566, such a criterion is explained.

Let x denote the error of a doubtful round so that x represents the difference between the range of the round and the mean arithmetic range.

Let r denote the probable longitudinal quadratic deviation or the mean quadratic error of all the rounds, so that (pp. 94)

$$r = \frac{0 \cdot 4769}{h} = 0 \cdot 6745 \sqrt{\frac{\Sigma(a^2)}{n-1}},$$

where $\Sigma(a^2)$ denote the sum of the squares of all the errors from the mean range, n is the number of rounds.

In the following table n denotes the number of rounds fired, and if $\frac{x}{r}$ is less than the number opposite n, the round should *not* be rejected.

If the ratio $\frac{x}{r}$ be greater than the number against n, then the round may be considered doubtful. The neighbouring means, that is to say, the mean longitudinal errors of the groups of rounds fired at the same elevations immediately above and below, may in some cases remove the doubt; in other cases there may be some equally good extraneous evidence. It must be clearly understood that only one doubtful round at a time can ever be cast out by this method.

The Value of $\frac{x}{r}$ corresponding to Different Values of n.

n.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	...	...	...	2·05	2·27	2·44	2·57	2·67	2·76	2·84
1	2·91	2·96	3·02	3·07	3·12	3·16	3·19	3·22	3·26	3·29
2	3·32	3·35	3·38	3·41	3·43	3·45	3·47	3·49	3·51	3·53
3	3·55	3·57	3·58	3·60	3·62	3·64	3·65	3·67	3·68	3·69
4	3·71	3·72	3·73	3·74	3·75	3·77	3·78	3·79	3·80	3·81
5	3·82	3·83	3·84	3·85	3·86	3·87	3·88	3·88	3·89	3·90
6	3·91	3·92	3·93	3·94	3·95	3·95	3·96	3·97	3·97	3·98
7	3·99	3·99	4·00	4·01	4·02	4·02	4·03	4·04	4·05	4·05
8	4·06	4·06	4·06	4·07	4·07	4·08	4·09	4·09	4·10	4·11
9	4·11	4·12	4·13	4·14	4·14	4·15	4·15	4·15	4·16	4·16

If $n = 100$, $\frac{x}{r} = 4 \cdot 16$; $n = 200$, $\frac{x}{r} = 4 \cdot 48$; $n = 500$, $\frac{x}{r} = 4 \cdot 90$.

Example 1 :—

Round.	Range in yards.	Error from mean range, a yards.
1	4011	+ 11
2	4025	+ 25
3	4015	+ 15
4	4050	+ 50
5	4045	+ 45
6	3700	− 300
7	4038	+ 38
8	4008	+ 8
9	4008	+ 8
10	4032	+ 32
11	4025	+ 25
12	4008	+ 8
13	4010	+ 10
14	4025	+ 25

The mean range is 4000 yards for 14 rounds,

$$\Sigma(a^2) = 90{,}000 + 9506 = 99{,}506,$$

$$\sqrt{\frac{\Sigma(a^2)}{n-1}} = \sqrt{\frac{99{,}506}{13}} = 87 \cdot 34.$$

$$r = 0 \cdot 6745 \times 87 \cdot 34.$$

The doubtful round is round 6.

For $n = 14$ the factor from the table gives $3 \cdot 12$, and the criterion is whether

$$\frac{x}{r} \text{ is greater or not than } 3 \cdot 12,$$

that is, if

$$x \quad ,, \quad ,, \quad ,, \quad 3 \cdot 12r,$$

$$x \text{ is } 4000 - 3700 = 300,$$

$$3 \cdot 12r = 3 \cdot 12 \times 0 \cdot 6745 \times 87 \cdot 34 = 146 \cdot 0,$$

hence x is greater than $3 \cdot 12r$, and round 6 may be rejected as abnormal.

Example 2.—Take the data of the example on p. 84 ; consider round 182. Here $n = 9$ and $2 \cdot 84$ is the factor from the table,

$$\sqrt{\frac{\Sigma(a^2)}{n-1}} = \sqrt{\frac{36{,}306 \cdot 6}{8}} = \sqrt{4538 \cdot 3} = 67 \cdot 38 ;$$

the criterion for non-rejection is that $x = a = 115 \cdot 89$ should be less than

$$\cdot 6745 \times 67 \cdot 38 \times 2 \cdot 84 = 129 \cdot 0.$$

hence $\frac{x}{r}$ is less than $2 \cdot 84$, and round 182 must not be rejected.

CHAPTER VI.

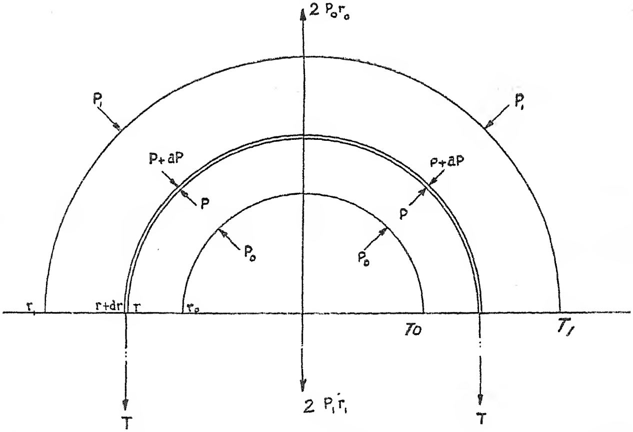
GUN CONSTRUCTION.

CONSIDERING an inch length of the bore of a gun, the hydrostatic thrust of the interior pressure of P_0 tons/in.^{2} acting normally to the diametral plane $r_1 r_0 r_0 r_1$ is

$$\int_0^\pi P_0 r_0 \, d\theta \sin\theta = 2P_0 r_0.$$

The thrust of the exterior pressure of P_1 tons/in., normal to the plane $r_1 r_0 r_0 r_1$ is $2P_1 r_1$, and

Fig. 1.



acts in the opposite direction to $2P_0 r_0$. The difference, $2P_0 r_0 - 2P_1 r_1$, is the resultant tension acting on the metal across the two sections $r_1 r_0$.

Let T represent the tensional stress in tons/in.^{2} at any point, the total stress is $2\int_{r_0}^{r_1} T\,dr$, so that

$$\int_{r_0}^{r_1} T\,dr = P_0 r_0 - P_1 r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

At any point r in the section $r_0 r_1$, let the radial pressure be P and the circumferential tension T; at $(r + dr)$ let these become $P + dP$ and $T + dT$; then the mean tension across the small area dr is

$$\frac{T + T + dT}{2} = T + \frac{dT}{2},$$

and

$$2Pr = 2(P + dP)(r + dr) + 2\left(T + \frac{dT}{2}\right) dr,$$

from which

$$(P+T)\,dr = -r\,dP$$

$$P+T = -r\frac{dP}{dr},$$

$$T = -\frac{d}{dr}(Pr) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The stresses to which the metal of a gun are subjected by the pressure of the powder gases, or by the shrinkage of the tubes in built-up guns, or the tension of the wire in wire-wound guns, must be examined before the principles of gun construction can be understood. A metal subjected to a stress will take a permanent set as soon as the stress reaches the elastic limit (in tons per square inch) of the metal.

For gun steel, the elastic limit of extension and of compression is practically the same.

When a piece of metal is pulled, as, for instance, a test piece of steel in a testing machine, it is found that the *extension*, measured by the ratio of the *elongation* to the original length, is proportional to the *tension*, which we shall measure in tons per square inch of cross-section.

Thus doubling the tension doubles the extension; and so on in proportion, provided the elastic limit is not exceeded.

This experimental law is called "Hooke's Law," and it is the axiomatic foundation of the Mathematical Theory of Elasticity. Expressed in an algebraical form, if a pull of P tons in a bar, K in.^{2} in cross-section, stretches the length from L to $L+l$, then the tension $\frac{P}{K}$ tons/in.^{2}, and the extension $\frac{l}{L}$, are, by "Hooke's Law," connected by the relation

$$\frac{\frac{P}{K}}{\frac{l}{L}} = E, \text{ a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3);$$

where E denotes a number of tons/in.^{2}, called Young's *modulus* of elasticity of the material; thus for steel E is about 12,500 tons/in.^{2}, and the extension,

$$\frac{l}{L} = \frac{P}{K} \cdot \frac{1}{E} = A \cdot \frac{P}{K}.$$

In this case the metal is subject to a single tension, and a certain amount of lateral contraction takes place; experiment shows that, for steel, the strain at right angles to the direction of a stress within the elastic limit of the metal is equal to one-third of the strain in the direction of the stress.

Consider a small brick-shaped piece of the metal of a gun, the dimensions defined as follows:—

(i) By two adjacent concentric cylinders of radii r and $r + dr$;
(ii) By two consecutive radial planes at an angle $d\theta$;
(iii) By two transverse planes at distances x and $x + dx$ from one end.

This gives

Width of lower part of the element	$= r\,d\theta$,
Depth of element	$= dr$,
Thickness of element	$= dx$.

Let the element be acted upon, normally to the surface, by

T, a circumferential stress;
P, a radial stress;
, a longitudinal stress, which is therefore parallel to the axis of the gun.

The piece of metal will be slightly altered in its dimensions, suppose the width $r\,d\theta$ becomes $(r+u)\,d\theta$, due to the increase u of the radius r of the circumferential fibre, so that the fibre is stretched from a length $2\pi r$ to a length $2\pi(r+u)$.

The circumferential extension is

$$e_t = \frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r}.$$

The depth dr becomes $d(r+u)$, so that the radial extension is

$$e_p = \frac{d(r+u) - dr}{dr} = \frac{du}{dr}.$$

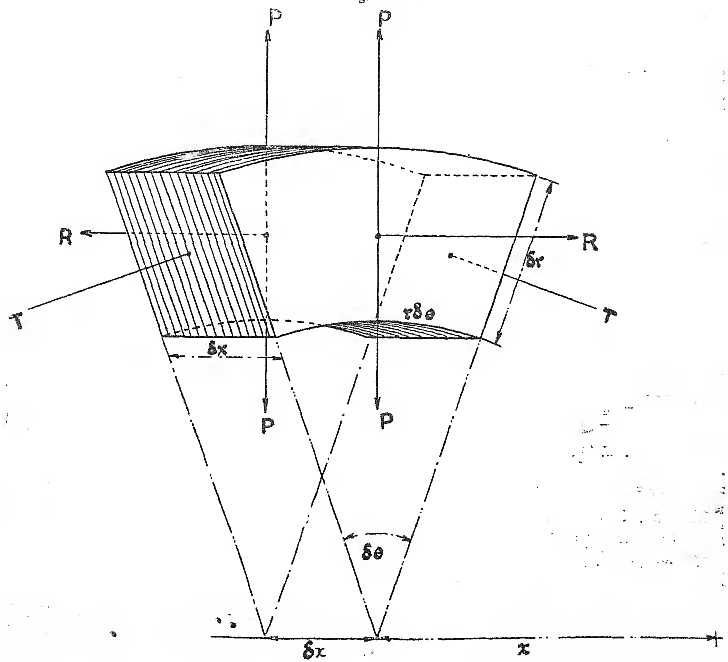
Let the longitudinal extension be denoted by

$$e_r = \frac{dl}{dx}.$$

From Hooke's law, and considering the metal as homogeneous,

$$\left.\begin{aligned} e_t &= \mathrm{AT} - \mathrm{B}(\mathrm{P}+\mathrm{R}) \\ e_p &= \mathrm{AP} - \mathrm{B}(\mathrm{T}+\mathrm{R}) \\ e_r &= \mathrm{AR} - \mathrm{B}(\mathrm{P}+\mathrm{T}) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Fig. 2.



where $A = \frac{1}{E}$ and $\frac{B}{A} = \sigma$ (Poisson's ratio), which is equal to $\frac{1}{3}$ for steel, so that

$$E e_t = T - \sigma (P + R),$$
$$E e_p = P - \sigma (T + R),$$
$$E e_r = R - \sigma (P + T).$$

The interior and exterior radial pressures P act towards each other, and not as shown in the diagram, hence the sign of P must be changed in the last three equations, so that

$$\left.\begin{aligned} E e_t &= T + \sigma (P - R) \quad (a) \\ E e_p &= -P - \sigma (T + R) \quad (b) \\ E e_r &= R + \sigma (P - T) \quad (c) \end{aligned}\right\} \quad \ldots \ldots \quad (5).$$

The values of e_t, e_p, e_r may be positive or negative: if positive, the strain is positive and represents an elongation; if negative, the strain represents a contraction.

It is usually assumed that R is constant, the constant value of R being taken as equal to the total longitudinal thrust $p_0 \pi r_0^2$ tons, divided by the area of the cross-section of the material of the gun, or, in considering the *initial stresses* in a state of repose, R may be put equal to zero. On the assumption that any plane cross-section at right angles to the axis always remains plane under the stresses in the material, then e_r is constant or zero; and from the equation $E . e_r = R + \sigma (P - T)$ the result follows that

$$T - P = \text{constant} = 2b, \text{ say} \quad \ldots \ldots \quad (6).$$

Also from (2)

$$P + T = -r \frac{dP}{dr} \quad \ldots \ldots \quad (7),$$

hence, by subtraction of (6) from (7), and employing the small letters p and t,

$$2p = -r \frac{dp}{dr} - 2b,$$

$$2 (p + b) = -r \frac{dp}{dr},$$

$$\frac{dp}{p + b} = -2 . \frac{dr}{r},$$

$$\log_e (p + b) = \log_e a r^{-2},$$

where a is a constant of integration.

$$p + b = \frac{a}{r^2},$$

$$p = \frac{a}{r^2} - b \quad \ldots \ldots \quad (8),$$

therefore, from (6),

$$t = \frac{a}{r^2} + b \quad \ldots \ldots \quad (9),$$

where a and b are constants whose values depend upon the data in any particular case.

Suppose for a single tube that the internal (radial) pressure p_0 be known, as also the external (radial) pressure p_1, then

$$\left.\begin{aligned} p_0 &= \frac{a}{r_0^2} - b \\ p_1 &= \frac{a}{r_1^2} - b \end{aligned}\right\} \quad \ldots \ldots \quad (10),$$

from which

$$\left.\begin{aligned} a &= \frac{r_1^2 r_0^2}{r_1^2 - r_0^2}(p_0 - p_1) \\ b &= \frac{p_0 r_0^2 - p_1 r_1^2}{r_1^2 - r_0^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (11),$$

and in the interior of the tube, the radial pressure p at a point distant r from the axis of the tube is

$$p = \frac{a}{r^2} - b,$$

and the circumferential tensions are

$$\left.\begin{aligned} t_0 &= \frac{a}{r_0^2} + b \\ t_1 &= \frac{a}{r_1^2} + b \\ t &= \frac{a}{r^2} + b \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (12),$$

When $p_1 = 0$, then

$$\left.\begin{aligned} p &= p_0 \frac{r_0^2}{r^2} \cdot \frac{r_1^2 - r^2}{r_1^2 - r_0^2} \\ t &= p_0 \frac{r_0^2}{r^2} \cdot \frac{r_1^2 + r^2}{r_1^2 - r_0^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (13),$$

also $\frac{a}{r_1^2} = b$ from (10); from (10), (12)

$$t_0 + p_0 = \frac{2a}{r_0^2}$$

and

$$t_0 - p_0 = 2b = \frac{2a}{r_1^2}$$

so that

$$\frac{r_1}{r_0} = \sqrt{\frac{t_0 + p_0}{t_0 - p_0}}.$$

From this it is seen that no thickness is sufficient to stand an internal pressure p_0 greater than t_0 if the exterior of the tube is unsupported, but this drawback is overcome by exterior reinforcing hoops shrunk on to an assigned initial tension.

As in Part I of Text-Book Gunnery, the different kinds of stresses set up in a gun are distinguished by symbols. These are now reproduced.

Powder stresses are those set up by the radial pressures of the powder gas throughout the mass of the gun. The symbols for these are P, T.

The *initial stresses* are those set up by the shrinking of one tube or hoop over another, or by the winding on of wire at considerable tension over a tube. The symbols for these are (p), (t).

The *firing stresses* are those which are created in the made up-gun when the powder charge is fired; it is therefore made up of both the powder and the initial stresses. The symbols for these are p, t.

So that firing stress = powder stress + initial stress.

Summing up, the symbols are

Powder stresses	. . . . . .	P, T.
Firing ,,	. . . . . .	p, t.
Initial ,,	. . . . . .	(p), (t).

In any cross-section of a gun, distances are denoted by r_0, r_1, $r_2 \dots r_n$, where each is the radius in inches of the internal radius of the 1st (or inner tube) of the 2nd, 3rd ... n^{th} hoops respectively, so that r_0 denotes the radius of the surface exposed to the powder pressure, whilst r_3 would denote the common cylindrical surface which is the exterior surface of the 3rd hoop and the interior surface of the 4th hoop.

Powder Stress.

In dealing with powder stresses, all consideration of the initial stresses in the completed gun is left alone, the problem then amounts to this :—

A pressure P_0 is applied to the interior of the inner tube, suppose the gun consist of an inner tube and three hoops, what will be the radial pressure and circumferential tension produced at any part of the gun, due to the powder pressure P_0?

In dealing with this, the whole gun is considered as one homogeneous cylinder of internal radius r_0 and external radius r_3, as though the surfaces of separation between any two hoops did not exist. This holds good, no matter how many hoops the gun may have.

Initial and Firing Stresses.

In these there is not a continuity of circumferential stress at the surfaces of separation between any two hoops, and the hoop tension can (and often does) change suddenly. It is therefore necessary to use different symbols for each surface between two hoops, as follows, for initial stresses :—

(t_0), (t_1), (t_2), (t_3), (t_4) for the inner surface of each hoop,

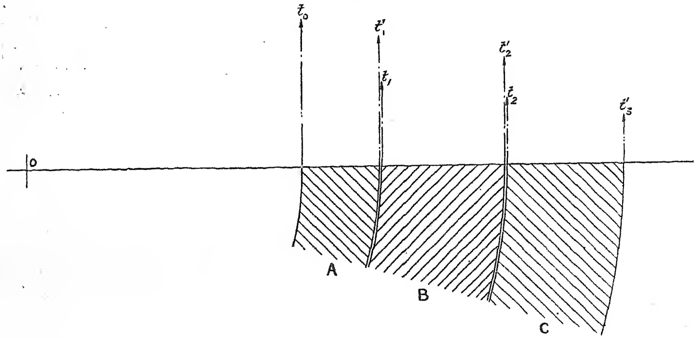
(t'_1), (t'_2), (t'_3), (t'_4) for the outer surface of each hoop.

So, in the firing stresses :—

t_0, t_1, t_2, t_3 for the inner surface of each hoop,

t'_1, t'_2, t'_3 for the outer surface of each hoop.

Fig. 3.



So that

t_0 and t'_1 apply to tube A,

t_1 and t'_2 apply to tube B,

t_2 and t'_3 apply to tube C,

t_3 is outside C, and therefore does not enter with a gun of three tubes only.

Radially the pressure is the same at the surface of separation, so that there is no need of suffixed symbols for the radial stress.

Firing Stresses p, t.

Suppose the values r_0, r_1, r_2, r_3, &c., at any section of the bore or chamber of a gun are known, we have two limits assigned, viz.:—

Carbon steel { Tension of material for inner tube is 15 tons/in.^{2} (maximum permissible), Tension of material for outer tubes is 18 tons/in.^{2} (maximum permissible),

Nickel Steel { Tension of material for inner tube is 20 tons/in.^{2} (maximum permissible), Tension of material for outer tubes is 24 tons/in.^{2} (maximum permissible),

then to find the firing stresses at any part of the section in the various hoops, we start with the outer hoop of the gun, and working inwards are enabled to find the maximum pressure in the bore; this maximum pressure is our maximum powder pressure, and knowing this the various powder stresses are found.

Deducting the powder stresses from the firing stresses gives the initial stresses, and then the amount of shrinkage is calculated which will produce the required initial stresses.

The maximum allowable pressure p_0 inside the gun is theoretical. As a matter of precaution no gun is allowed to be subjected to the full amount calculated, the charges being so arranged that the pressure shall not exceed a normal chamber pressure varying from $\frac{2}{3}$ to $\frac{1}{2}$ of this p_0, so that under ordinary conditions the elastic limit of no part of the material may be reached, or permanent extension take place. Thus if it is found that $p_0 = 20$ tons/in.^{2}, the working pressure would vary from $20 \times \frac{2}{3}$ to $20 \times \frac{1}{2}$, *i.e.*, from 13·3 to 10 tons/in.^{2}, as the case may be.

This working pressure, or normal chamber pressure, is that pressure which should not be exceeded by the ordinary service charge, and in the case of cordite the temperature of the charge is fixed at 80° F. It might be called the specification pressure.

The actual pressure which a charge does produce, as ascertained by means of the crusher gauge and coppers, is frequently below the working pressure, and is dependent on such things as wear of the gun, temperature of charge, &c.

Gunmakers Formula.

Consider any tube, the $(n+1)$th, of a built-up gun; t_n and p_{n+1} are probably known, then from the fundamental formulas (8) and (9),

$$t_n = \frac{a}{r_n^2} + b,$$

$$p_{n+1} = \frac{a}{r^2_{n+1}} - b,$$

$$p_n = \frac{a}{r_n^2} - b,$$

whence

$$p_n - p_{n+1} = \frac{a}{r_n^2 r^2_{n+1}} (r^2_{n+1} - r_n^2),$$

$$t_n + p_{n+1} = \frac{a}{r_n^2 r^2_{n+1}} (r^2_{n+1} + r_n^2),$$

therefore

$$\frac{p_n - p_{n+1}}{t_n + p_{n+1}} = \frac{r^2_{n+1} - r_n^2}{r^2_{n+1} + r_n^2},$$

$$p_n = \frac{r^2_{n+1} - r_n^2}{r^2_{n+1} + r_n^2} (t_n + p_{n+1}) + p_{n+1} .$$

This formula is of great importance, and will be found very useful; it is generally called the gunmaker's formula.

A complete example is worked out in Part I, p. 163; in this example t_0 is taken as 15 and $t_1 = t_2 = 18$ tons/in.^{2}: also

$$\begin{aligned} r_0 &= 4, & r_0^2 &= 16, \\ r_1 &= 5\cdot 6, & r_1^2 &= 31\cdot 36, \\ r_2 &= 8\cdot 7, & r_2^2 &= 75\cdot 69, \\ r_3 &= 11\cdot 8, & r_3^2 &= 139\cdot 24. \end{aligned}$$

The firing stresses are found by working from the outside tube first and making use of the gunmaker's formula; thus

$$p_3 = 0,$$

$$p_2 = \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2}(t_2 + p_3) + p_3 = 5\cdot 32,$$

$$p_1 = \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}(t_1 + p_2) + p_2 = 15\cdot 0,$$

$$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1 = 24\cdot 74.$$

The hoop tensions are found at once from formula

$$t'_n - p_n = t_{n-1} - p_{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

Since in any one hoop (the n^{th})

$$t'_n = \frac{a}{r_n^2} + b; \qquad p_n = \frac{a}{r_n^2} - b. \qquad \text{Therefore} \quad t'_n - p_n = 2b.$$

$$t_{n-1} = \frac{a}{r^2_{n-1}} + b; \qquad p_{n-1} = \frac{a}{r^2_{n-1}} - b. \qquad \text{Therefore} \quad t_{n-1} - p_{n-1} = 2b.$$

Therefore

$$t'_n - p_n = t_{n-1} - p_{n-1}.$$

Applying this,

$$\text{Jacket}\begin{cases} t'_3 = (t_2 - p_2) + p_3 = 18 - 5\cdot 32 + 0 = 12\cdot 68, \\ t_2 = 18, \end{cases}$$

$$\text{Breech-piece}\begin{cases} t'_2 = (t_1 - p_1) + p_2 = 18 - 15 + 5\cdot 32 = 8\cdot 32, \\ t_1 = 18, \end{cases}$$

$$\text{Tube}\begin{cases} t'_1 = (t_0 - p_0) + p_1 = 15 - 24\cdot 74 + 15 = 5\cdot 26, \\ t_0 = 15. \end{cases}$$

Powder Stress: P, T.

The maximum allowable pressure has been found to be $24\cdot 74$ tons/in.^{2}, this therefore is the maximum powder pressure in the interior of the bore.

Therefore

$$P_0 = 24\cdot 74.$$

To obtain the powder stresses at other parts of the gun, we treat the whole gun as one homogeneous mass of internal radius r_0 and external radius r_3.

$$P_0 = 24 \cdot 74 = \frac{a}{r_0^2} - b,$$

$$P_3 = \quad 0 \quad = \frac{a}{r_3^2} - b,$$

at any point

$$P = \frac{a}{r^2} - b,$$

$$T = \frac{a}{r^2} + b,$$

and a, b are the same constants throughout tube, breech-piece, and jacket.

Solving the equations, we have

$$a = \frac{r_0^2 r_3^2}{r_3^2 - r_0^2} p_0; \qquad b = \frac{r_0^2}{r_3^2 - r_0^2} p_0.$$

Therefore

$$P = \frac{r_0^2}{r_3^2 - r_0^2}\left(\frac{r_3^2}{r^2} - 1\right) = \frac{r_0^2}{r^2} \cdot \frac{r_3^2 - r^2}{r_3^2 - r_0^2} p_0,$$

$$T = \frac{r_0^2}{r_3^2 - r_0^2}\left(\frac{r_3^2}{r^2} + 1\right) = \frac{r_0^2}{r^2} \cdot \frac{r_3^2 + r^2}{r_3^2 - r_0^2} p_0$$

Therefore

$$P_0 = 24 \cdot 74,$$

$$P_1 = \frac{16}{31 \cdot 36} \cdot \frac{139 \cdot 24 - 31 \cdot 36}{139 \cdot 24 - 16} 24 \cdot 74 = 11 \cdot 1,$$

$$P_2 = \frac{16}{75 \cdot 69} \cdot \frac{139 \cdot 24 - 75 \cdot 69}{123 \cdot 24} 24 \cdot 74 = \quad 2 \cdot 7,$$

$$P_3 = 0.$$

$$T_0 = \frac{16}{16} \cdot \frac{139 \cdot 24 + 16}{139 \cdot 24 - 16} \cdot 24 \cdot 74 = 31 \cdot 2,$$

$$T_1 = (T_0 - P_0) + P_1 = 17 \cdot 56,$$

$$T_2 = (T_0 - P_0) + P_2 = \quad 9 \cdot 16,$$

$$T_3 = (T_0 - P_0) + P_3 = \quad 6 \cdot 46.$$

Initial Stress: (p), (t).

These are got by subtracting powder from firing stresses.

$$(p_0) = p_0 - P_0 = 24 \cdot 74 - 24 \cdot 74 = \quad 0,$$

$$(p_1) = p_1 - P_1 = 15 \quad - 11 \cdot 1 \quad = \quad 3 \cdot 9,$$

$$(p_2) = p_2 - P_2 = \quad 5 \cdot 32 - \quad 2 \cdot 7 \quad = \quad 2 \cdot 62,$$

$$(p_3) = p_3 - P_3 = \quad 0 \quad - \quad 0 \quad = \quad 0,$$

$$\text{Tube}\begin{cases}(t_0) = t_0 - T_0 = 15 \quad - 31 \cdot 2 = -16 \cdot 2, \\ (t'_1) = t'_1 - T_1 = \quad 5 \cdot 26 - 17 \cdot 56 = -12 \cdot 30,\end{cases}$$

$$\text{Breech-piece}\begin{cases}(t_1) = t_1 - T_1 = 18 \quad - 17 \cdot 56 = \quad 0 \cdot 44, \\ (t'_2) = t'_2 - T_2 = \quad 8 \cdot 32 - \quad 9 \cdot 16 = - \quad 0 \cdot 84.\end{cases}$$

$$\text{Jacket}\begin{cases}(t_2) = t_2 - T_2 = 18 \quad - \quad 9 \cdot 16 = \quad 8 \cdot 84, \\ (t'_3) = t'_3 - T_3 = 12 \cdot 68 - \quad 6 \cdot 46 = \quad 6 \cdot 22.\end{cases}$$

Figs. 8 and 9, facing p. 165 of Part I, 1907, show the various stresses for this 6-inch gun to one place of decimals. A similar procedure will apply for a gun built up of four or more parts.

With a 6-inch gun built up with carbon steel and of the dimensions just given, suppose a working chamber pressure of 16 tons/in.^{2} is produced, and it is required to know the firing stresses.

The initial stresses already found hold good, the powder stresses are

$$P_0 = 16,$$

$$P_1 = 11 \cdot 1 \frac{16}{24 \cdot 74} = 7 \cdot 178,$$

$$P_2 = 2 \cdot 7 \frac{16}{24 \cdot 74} = 1 \cdot 746,$$

$$P_3 = 0,$$

$$T_0 = 31 \cdot 2 \frac{16}{24 \cdot 74} = 20 \cdot 18,$$

$$T_1 = 17 \cdot 56 \frac{16}{24 \cdot 74} = 11 \cdot 36,$$

$$T_2 = 9 \cdot 16 \frac{16}{24 \cdot 74} = 5 \cdot 924,$$

$$T_3 = 6 \cdot 46 \frac{16}{24 \cdot 74} = 4 \cdot 178.$$

Combining these powder stresses with the initial stresses already found, gives the required firing stresses when the chamber pressure is that produced by the service change.

$$p_0 = 16,$$
$$p_1 = 11 \cdot 078,$$
$$p_2 = 4 \cdot 366,$$
$$p_3 = 0,$$
$$t_0 = 3 \cdot 98,$$
$$t'_1 = - 0 \cdot 94,$$
$$t_1 = 11 \cdot 80,$$
$$t'_2 = 5 \cdot 084,$$
$$t_2 = 14 \cdot 764,$$
$$t'_3 = 10 \cdot 398.$$

Case of a Cracked Tube.

If the A tube is cracked through longitudinally on both sides, its tensile strength is nil and the gun will not stand the same strain as before.

Thus in the 6-inch gun worked out, from the firing radial pressures,

$$p_1 = 15; \quad p_0 = 24 \cdot 74;$$

but with the A tube cracked p_0 will not be so large: the two outer tubes being undamaged we get as before

$$p_1 = 15.$$

Although the tensile strength of the A tube is nil, yet it serves to diminish the surface over which the powder pressure acts, and therefore

$$p_0 = p_1 \frac{r_1}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

This may be seen also from considering the equilibrium of the A tube; the forces acting on this tube are radial pressures,

$p_1 = 15$ tons/in.^{2} over a surface $2\pi r_1$, acting inwards,

p_0 over a surface $2\pi r_0$, acting outwards,

therefore

$$(2\pi r_1)\, p_1 = (2\pi r_0)\, p_0,$$

$$p_0 = p_1 \frac{r_1}{r_0},$$

$$= 15 \,.\, \frac{5 \cdot 6}{4} = 21 \cdot 0.$$

So that our maximum allowable pressure has been reduced from 24·74 to 21·0 tons/in.^{2}, due to the A tube being cracked.

Liners.

In the case of liners, no strength is accredited; for, being put in without shrinkage, they are taken as so much packing, and their effect as regards calculation of strength might be ignored but for the fact that they distribute the strain to a larger area. Of course, here, as in the case of shrinkage friction, with reference to longitudinal strength, any circumferential strength derived from the liner will be in addition to that calculated for.

Supposing the gun to have been designed for, and constructed originally with, a liner, then if p_0 represents the internal pressure on the liner, and p that transmitted to the interior of the A tube, and r_0 and r_1 the respective radii, the formula is simply

$$p_0 = p \frac{r}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

the liner acting as if cracked or segmental.

The Longitudinal Tension in the Gun.

Practically it is usual to take the longitudinal tension as uniform across a cross-section and as due to the powder pressure in the bore, treated as a closed vessel, closed at one end by the breech-screw, and at the other by the shot.

Thus supposing a breech-screw to gear into an A tube of which the internal radius is r_0 and the external radius of the gun is r_2, taking P_0 as the powder pressure, the average value R of the longitudinal tension will be found as follows:—The pressure multiplied by the sectional area of the chamber is resisted by a cross-section of the metal subjected to longitudinal tension (according to the design of the gun); for equilibrium, these must be equal, therefore in this case

$$P_0 \pi r_0^2 = R\pi (r_2^2 - r_0^2)$$

$$R = P_0 \frac{r_0^2}{r_2^2 - r_0^2} \text{ tons/in.}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

In all steel guns of modern construction the breech-screw gears into the layer of metal above the A tube; in smaller guns direct; in heavier pieces of the most recent construction

by means of a steel bush; the inner tube is thus relieved of longitudinal stress at the breech. For a gun consisting of a tube and jacket only, the formula would then become

$$R = P_0 \frac{r_0^2}{r_2^2 - r_1^2}.$$

Considering the longitudinal strength of the 6-inch B.L. gun,

$$P_0 = \frac{r_3^2 - r_1^2}{r_0^2} R;$$

with the given numerical values and putting $R = R_1 = 18$ gives $P_0 = 121$, and dividing this by a normal pressure of 17 tons, we get a longitudinal factor of safety $= 7 \cdot 1$.

In a heavy modern wire-built gun the longitudinal stress is taken up by the breech-piece and the jacket, but the jacket may not be taking its full share of the work, owing to the necessary connections between the two; hence a large factor of safety is obtained by considering the breech-piece only; let r and r' denote its internal and external radii, then

$$R = P_0 \frac{r_0^2}{r'^2 - r^2}.$$

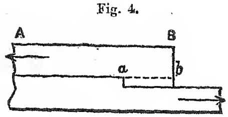
Practically the longitudinal strength is considered separately from the circumferential, and is specially provided for by shoulder, of which the *resistance to shearing* constitutes the longitudinal strength as calculated. No account is taken of frictional grip due to shrinkage, for it is considered as extremely probable that at the critical moment this becomes loosened by the elasticity of the different layers asserting itself more rapidly towards the interior as soon as the highest pressure has passed, while there is still a considerable longitudinal stress.

It is considered inadvisable to rely upon shrinkage in any way for longitudinal strength, and, consequently, any strength in this direction derived from the frictional grip will be in addition to the calculated strength.

The strain sustained by a shoulder is taken as a purely shearing one, and the strength of a shoulder is consequently dependent on its length; shearing strength, like resistance to tension, being directly proportional to the extent of surface where separation would take place. The strength is also here taken to be the same (in tons per square inch), which, if not strictly true, is rather in favour of the shearing strength.

The calculation, therefore, of length of shoulder for a given hoop is simple. For if AB is the supporting hoop of internal and external radii r_1, r_2 and length of shoulder ab ($= l$ say), it

Fig. 4.



is only necessary to make the cylindrical area of $ab =$ the annular sectional area of the hoop, or

$$\pi (r_2^2 - r_1^2) = 2\pi r_1 l,$$

whence

$$l = \frac{r_2^2 - r_1^2}{2r_1}.$$

The actual longitudinal strength of this arrangement would appear to be

$$\pi (r_2^2 - r_1^2) \mathrm{T},$$

or

$$2\pi r_1 l \mathrm{T},$$

T being the resistance to rupture by tension or shearing in tons per square inch of material where separation would take place.

Wire Gun Construction.

In a built-up gun, as seen from fig. 8, p. 165, of Part I., the curves of circumferential firing tensions show that the resistance offered by any tube when the gun is fired is unevenly distributed in the metal, the inner taking an undue share of resistance.

Great economy of material would be effected if we could make all the circumferential fibres take up a full uniform tension on firing; but to secure this condition only approximately, the number of layers of metal would have to be largely increased, and the cost, complication, and time of manufacture of a gun would be enormous.

But by adopting Mr. J. A. Longridge's plan of strengthening the tube by steel wire, wound round with appropriately varying tension, we are able to make the curve of circumferential firing tension a straight line for a given powder pressure, and now all parts of the wire coil are equally strained under the interior pressure, and take an equal share in the resistance.

For full theoretical investigation of this subject, *see* Mr. Longridge's "Treatise on the Application of Wire to the Construction of Ordnance" (1884), and a paper of 1887, "Further Investigations regarding Wire Gun Construction," also a Work by Lieutenant G. Moch, "Les Canons à Fils d'Acier."

By winding on many layers of steel wire on an A tube, we can get every layer of the wire coil to do its full share of resistance when the gun is fired, and so greatly strengthen the gun, provided each layer of wire is wound on with a properly adjusted tension.

A steel hoop shrunk over an A tube must not be subjected to a firing tension of over 18 to 24 tons/in.^{2}, whereas steel wire can safely be subjected to a tension of 50 tons/in.^{2}, therefore we have not only a much strengthened gun by using steel wire, but there is a considerable saving in the weight of metal to be used.

In a gun which has steel hoops shrunk on, the curve of radial pressure for each hoop is given by

$$p = \frac{a}{r^2} - b,$$

and the circumferential tension by

$$t = \frac{a}{r^2} + b,$$

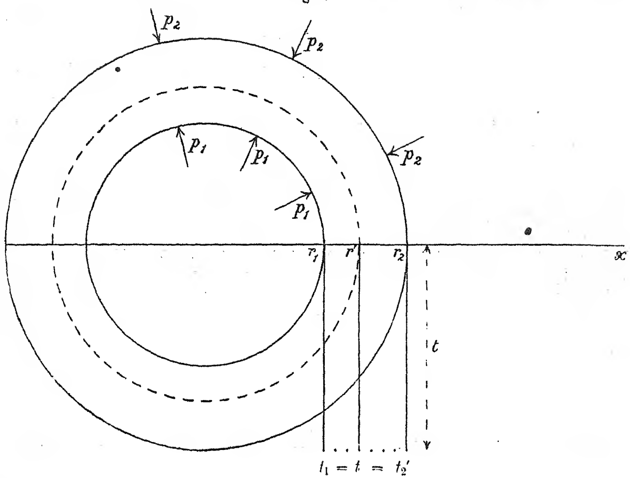
where a and b are constants to be determined for each hoop or tube.

These equations do not hold good for the firing stresses in a wire coil which has been wound round on an A tube; because the winding tension of each layer of wire has been adjusted so that the firing tension may be the same throughout the coil.

The jacket is required for the protection of the wire from damage and to provide the necessary longitudinal strength; it is fitted over the wire without any appreciable shrinkage.

When the gun is at rest the jacket will be free from stress, but when the gun is fired we may suppose the stress in it to be the powder stress only, on the assumption that the gun behaves as if homogeneous.

Fig. 5.



In fig. 5 let r_1 and r_2 be the internal and external radii of the wire coil, and let a powder pressure p_0 be applied so that the radial firing pressures at r_1, r_2 be p_1, p_2 respectively, and the tension throughout $= t$, so that

$$t_1 = t_2' = t.$$

To find the radial pressure p at any point r in the wire coil.

Consider the equilibrium of the portion of the wire between r_1 and r; we have

$$p_1 r_1 = pr + t(r - r_1) \quad \ldots\ldots\ldots\ldots \quad (19).$$

If the portion of wire coil between r and r_2 be considered, we have

$$pr = p_2 r_2 + t(r_2 - r),$$

and for the whole coil from r_1 to r_2,

$$p_1 r_1 = p_2 r_2 + t(r_2 - r_1).$$

From these last two equations we have, by subtraction,

$$p_1 r_1 - pr = t(r - r_1),$$

or

$$p_1 r_1 = pr + t(r - r_1),$$

which is equation (19) over again.

By writing x for r and y for p,

$$p_1 r_1 = xy + t(x - r_1),$$

or

$$x(y + t) = (p_1 + t)\, r_1 = \text{const.} = \text{A},$$

and moving the axis of x parallel to itself to a distance t, this reduces to

$$xy = \text{const., a rectangular hyperbola.}$$

From equation (19), $(p+t)r = p_1r_1+tr_1$ = a constant, say A, therefore

$$p+t = \frac{A}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

This gives a convenient equation to find the radial pressure at any point in the wire coil; the curve of these radial pressures is a rectangular hyperbola.

In the A tube and jacket only, the equations

$$p = \frac{a}{r^2} - b,$$

$$t = \frac{a}{r^2} + b,$$

hold good to find the stress at any point.

In wire guns, the limits which are usually accepted are as follows:—

MAXIMUM firing tensions to which the metal may be subjected, unless instructions to the contrary are given.

	Inner A tube.	Wire Coil.
Carbon steel	15 tons/in.^{2}	—
Nickel ,, ...	20 ,,	—
Wire	—	40 tons/in.^{2}

Maximum compression of any tube not to exceed 26 tons/in.^{2}.

The radius of bore to be measured from the axis of piece to the surface of the land.

In calculating the strength of wire guns, no allowance is to be made for any shrinkage that may be given to the jacket.

In wire guns, at present, the wire is wound on to the A tube after the inner A tube has been fitted into it, and so at first the A tube is not fully compressed, which gives an advantage in the first life of the gun, but on boring out for lining then the full compression takes place, and the advantage referred to is lost.

Similarly in the 6-inch Q.F., Mark II, where the 1-B tube is shrunk on to the A tube under the wire, the metal under the wire coil should be treated as homogeneous, because in boring out for lining the A tube will be compressed, and the advantage of the shrinkage lost.

The slight shrinkage between the inner A and A tubes being neglected, it follows that the limits of tension (18 and 24 tons/in.^{2}) do not enter into the calculations for various stresses in the built-up gun. Also, the limit of 40 tons/in.^{2} for the firing tension of the wire must be considered as a guide only, the 40 tons/in.^{2} may be exceeded in some cases up to about 50 tons/in.^{2}

The gunmaker fixes the diameter of the chamber, the thickness of the metal in the various layers and the length of travel of the shot, &c.; in this experience is the best guide. These points being settled on he can now find, from the limits assigned, what p_0, the maximum allowable powder-pressure in the bore, and also the corresponding circumferential tension in the wire coil in firing. Should the latter be excessive, or p_0 too small, then he can manipulate the radii until the desired result is attained.

The various radii having been fixed, the following examples show all the calculations of the stresses.

The dimensions across the 12″ B.L., XI., across middle of chamber are:—

$$r_0 = 8 \cdot 425 \text{ inches},$$
$$r_1 = 11 \cdot 74 \quad ,,$$
$$r_2 = 15 \cdot 31 \quad ,,$$
$$r_3 = 20 \cdot 11 \quad ,,$$
$$r_4 = 23 \cdot 9 \quad ,,$$

so that

$$r_1 - r_0 = 3 \cdot 315, \text{ the thickness of inner A tube},$$
$$r_2 - r_1 = 3 \cdot 57 \quad ,, \quad ,, \quad \text{A tube},$$
$$r_3 - r_2 = 4 \cdot 80 \quad ,, \quad ,, \quad \text{wire coil}.$$
$$r_4 - r_3 = 3 \cdot 79 \quad ,, \quad ,, \quad \text{jacket}.$$

The limits are

$$t_0 = 20,$$
$$(t_0) = -26,$$

hence, since firing tension = powder tension + initial tension,

$$t_0 = \mathrm{T}_0 + (t_0).$$

Therefore

$$\mathrm{T}_0 = 20 - (-26) = 46.$$

Treating the whole gun (inner A, A tube, wire coil and jacket) as one homogeneous mass for the powder stress,

$$\mathrm{P}_4 = 0 = \frac{a}{r_4^2} - b,$$

$$\mathrm{T}_0 = 46 = \frac{a}{r_0^2} + b,$$

and at any distance r, the radial pressure

$$\mathrm{P} = \frac{a}{r^2} - b.$$

From the first two equations

$$a = \frac{r_4^2 r_0^2}{r_4^2 + r_0^2} \cdot \mathrm{T}_0; \qquad b = \frac{r_0^2}{r_4^2 + r_0^2} \cdot \mathrm{T}_0,$$

so that

$$\mathrm{P} = \frac{r_0^2}{r^2} \cdot \frac{r_4^2 - r^2}{r_4^2 + r_0^2} \cdot \mathrm{T}_0,$$

from which

$$\mathrm{P}_0 = \frac{r_4^2 - r_0^2}{r_4^2 + r_0^2} \cdot 46 = 35 \cdot 84,$$

$$\mathrm{P}_1 = \frac{r_0^2}{r_1^2} \cdot \frac{r_4^2 - r_1^2}{r_4^2 + r_0^2} 46 = 15 \cdot 99,$$

so

$$\mathrm{P}_2 = 7 \cdot 32,$$
$$\mathrm{P}_3 = 2 \cdot 1,$$
$$\mathrm{P}_4 = 0.$$

Again

$$\mathrm{T}'_4 - \mathrm{P}_4 = 2b = \mathrm{T}_3 - \mathrm{P}_3 = \mathrm{T}_2 - \mathrm{P}_2 = \mathrm{T}_1 - \mathrm{P}_1 = \mathrm{T}_0 - \mathrm{P}_0.$$

Therefore

$$T'_4 = (T_0 - P_0) + P_4 = 10 \cdot 16 + 0 = 10 \cdot 16,$$
$$T_3 = 10 \cdot 16 + P_3 = 12 \cdot 26,$$
$$T_2 = 10 \cdot 16 + P_2 = 17 \cdot 48,$$
$$T_1 = 26 \cdot 15,$$
$$T_0 = 46.$$

This completes the powder stresses.

Firing stress, p, t.

In the A tube (which includes the inner A tube, both being regarded as homogeneous).

$$p_0 = \frac{r_2^2 - r_0^2}{r_2^2 + r_0^2}(t_0 + p_2) + p_2 \text{ (gunmaker's formula)},$$

from which, since $p_0 = 35 \cdot 84 = P_0$ and $t_0 = 20$, $p_2 = 16 \cdot 4$, so

$$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1, \text{ and } p_1 = 22 \cdot 3,$$

also

$$t'_1 = t_1 = (t_0 - p_0) + p_1 = -15 \cdot 8 + 22 \cdot 3 = 6 \cdot 5,$$
$$t'_2 = (t_0 - p_0) + p_2 = -15 \cdot 8 + 16 \cdot 4 = 0 \cdot 6.$$

In the wire coil each layer of wire is wound on, so that on firing the tension is the same this gives that $t_2 = t'_3 = t$ throughout.

Then, from considering the equilibrium of the whole coil,

$$p_2 r_2 = p_3 r_3 + t(r_3 - r_2),$$

$$t = \frac{p_2 r_2 - p_3 r_3}{r_3 - r_2} = 43 \cdot 47.$$

Therefore

$$t_2 = t'_3 = t = 43 \cdot 47.$$

In the jacket there is no initial stress, hence

$$t_3 = T_3 = 12 \cdot 26; \; p_3 = P_3 = 2 \cdot 1,$$
$$t'_4 = T'_4 = 10 \cdot 2; \; p_4 = P_4 = 0.$$

This completes the firing stresses.

Initial stresses (p), (t).

Inner A tube	$(p_0) = p_0 - P_0 = 0$	$: (t_0) = t_0 - T_0 = -26.$
	$(p_1) = p_1 - P_1 = 6 \cdot 31$	$: (t'_1) = t'_1 - T_1 = -19 \cdot 65.$
A tube	$(p_1) = 6 \cdot 31$	$: (t_1) = t_1 - T_1 = -19 \cdot 65.$
	$(p_2) = p_2 - P_2 = 9 \cdot 08$	$: (t'_2) = t'_2 - T_2 = -16 \cdot 88.$
Wire coil	$(p_2) = 9 \cdot 08$	$: (t_2) = 26.$
	$(p_3) = 0$	$: (t'_3) = 31 \cdot 21.$
Jacket	$(p_3) = 0$	$: (t_3) = 0.$
	$(p_4) = 0$	$: (t'_4) = 0.$

The powder and initial stresses above tabulated for the 12 inches across the chamber are shown on fig. 6. the firing stresses are on fig. 7.

In the 12-inch B.L. Mark XI, at a distance of 35 inches from the seat of the shot, the dimensions are :—

$$r_0 = 6 \text{ inches}$$
$$r_1 = 9{\cdot}29 \text{ ,,}$$
$$r_2 = 11{\cdot}83 \text{ ,,}$$
$$r_3 = 14{\cdot}95 \text{ ,,}$$
$$r_4 = 17{\cdot}95 \text{ ,,}$$

The various stresses areas follow :—

With limits $t_0 = 20$, $(t_0) = -26$,

$P_0 = 36{\cdot}76$	$p_0 = 36{\cdot}76$	$(p_0) = 0$
$P_1 = 12{\cdot}64$	$p_1 = 20{\cdot}2$	$(p_1) = 7{\cdot}56$
$P_2 = 6{\cdot}02$	$p_2 = 15{\cdot}66$	$(p_2) = 9{\cdot}64$
$P_3 = 2{\cdot}04$	$p_3 = 2{\cdot}04$	$(p_3) = 0$
$P_4 = 0$	$p_4 = 0$	$(p_4) = 0$
$T_0 = 46$	$t_0 = 20$	$(t_0) = -26$
$T_1 = 21{\cdot}9$	$t'_1 = 3{\cdot}44$	$(t'_1) = -18{\cdot}46$
	$t_1 = 3{\cdot}44$	$(t_1) = -18{\cdot}46$
$T_2 = 15{\cdot}26$	$t'_2 = -1{\cdot}10$	$(t'_2) = -16{\cdot}36$
$T_3 = 11{\cdot}28$	$t_2 = t'_3 = 49{\cdot}68$	$(t_2) = 34{\cdot}42$
	$t_3 = 11{\cdot}28$	$(t'_3) = 38{\cdot}40$
$T'_4 = 9{\cdot}24$	$t'_4 = 9{\cdot}24$	$(t_3) = 0$
		$(t'_4) = 0$

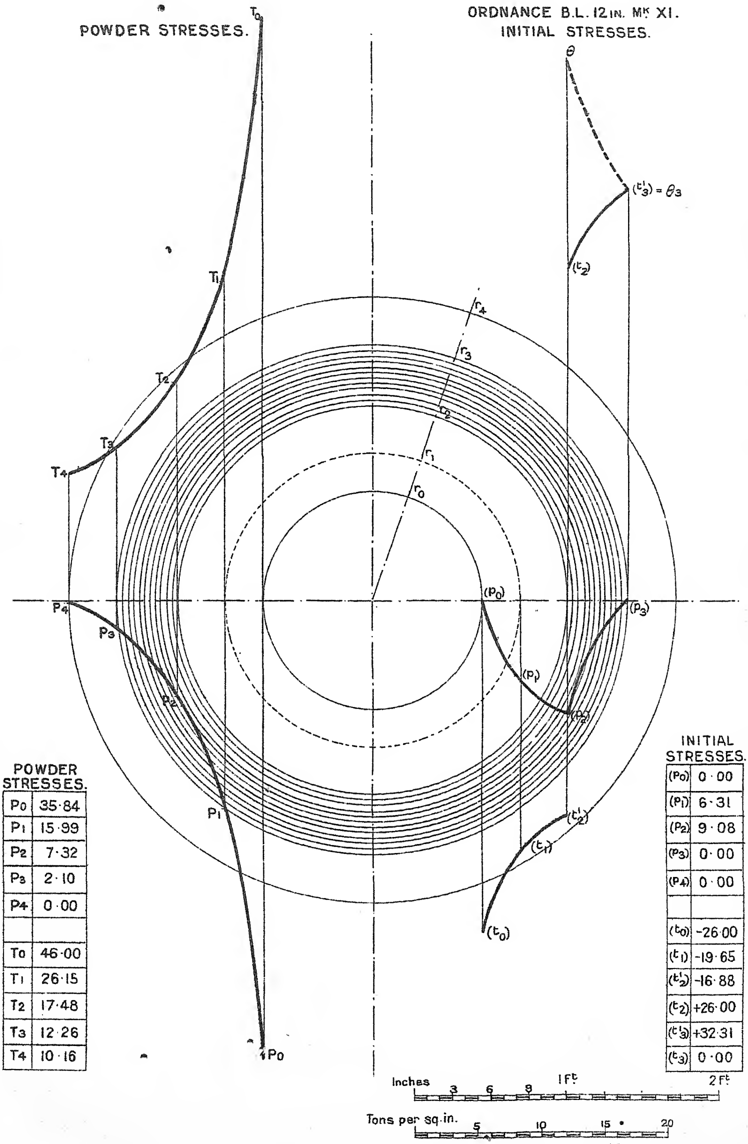
At 105·6 inches from the seat of the shot in the 12-inch B.L. XI,

$$r_0 = 6 \text{ inches}$$
$$r_1 = 8{\cdot}81 \text{ ,,}$$
$$r_2 = 11{\cdot}23 \text{ ,,}$$
$$r_3 = 14{\cdot}35 \text{ ,,}$$
$$r_4 = 16{\cdot}55 \text{ ,,}$$

and with limits $t_0 = 20$, $(t_0) = -26$,

$P_0 = 35{\cdot}2$	$p_0 = 35{\cdot}2$	$(p_0) = 0$
$P_1 = 13{\cdot}4$	$p_1 = 20{\cdot}4$	$(p_1) = 7{\cdot}0$
$P_2 = 6{\cdot}2$	$p_2 = 15{\cdot}4$	$(p_2) = 9{\cdot}2$
$P_3 = 1{\cdot}63$	$p_3 = 1{\cdot}63$	$(p_3) = 0$
$P_4 = 0$	$p_4 = 0$	$(p_4) = 0$
$T_0 = 46$	$t_0 = 20$	$(t_0) = -26$
$T_1 = 24{\cdot}2$	$t'_1 = t_1 = 5{\cdot}2$	$(t'_1) = t_1 = -19$
$T_2 = 17{\cdot}0$	$t'_2 = 0{\cdot}2$	$(t'_2) = -16{\cdot}8$
$T_3 = 12{\cdot}53$	$t_2 = t'_3 = 47{\cdot}7$	$(t_2) = 30{\cdot}7$
	$t_3 = 12{\cdot}53$	$(t'_3) = 35{\cdot}17$
$T'_4 = 10{\cdot}8$	$t'_4 = 10{\cdot}8$	$(t_3) = 0$
		$(t'_4) = 0$

FIGURE 6.



POWDER STRESSES.

P_0	35·84
P_1	15·99
P_2	7·32
P_3	2·10
P_4	0·00
T_0	46·00
T_1	26·15
T_2	17·48
T_3	12·26
T_4	10·16

INITIAL STRESSES.

(p_0)	0·00
(p_1)	6·31
(p_2)	9·08
(p_3)	0·00
(p_4)	0·00
(t_0)	-26·00
(t_1)	-19·65
(t'_2)	-16·88
(t_2)	+26·00
(t'_3)	+32·31
(t_3)	0·00

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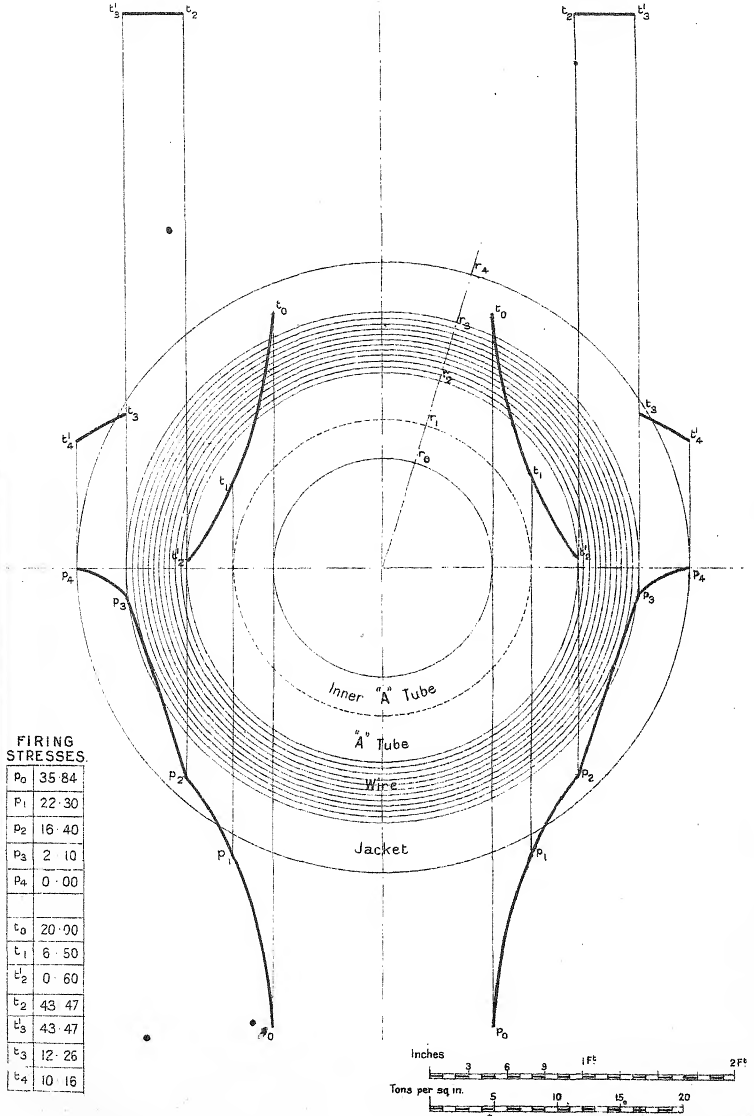
FIGURE 7.

FIRING STRESSES.

ORDNANCE B.L. 12 IN. Mk XI.



FIRING STRESSES.

p_0	35·84
p_1	22·30
p_2	16·40
p_3	2·10
p_4	0·00
t_0	20·00
t_1	6·50
t'_2	0·60
t_2	43·47
t'_3	43·47
t_3	12·26
t_4	10·16

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Winding Tension.

As shown in Part I, p. 177, the requisite winding on tension at a point r is given by

$$\theta = (t) + \frac{r^2 + r_0^2}{r^2 - r_0^2}(p),$$

where (t), (p) are the initial stresses at the point on the wire distant r from the centre of bore, and r_0, r_1, r_2, r_3 are the internal radii of the inner A tube, A tube, wire coil, and jacket respectively.

The expression for θ becomes

$$\theta = \frac{A}{r} + \frac{B}{r - r_0} + \frac{C}{r + r_0},$$

where

$$A = -(t + p_3) r_3,$$
$$B = (t + p_3) r_3 - (t + p_0) r_0,$$
$$C = (t + p_3) r_3 + (t + p_0) r_0.$$

This last formula for θ is convenient, because A, B, and C are the same for each layer of wire.

The winding tension of the last layer of wire must be the same as the tension in repose, so that at r_3,

$$\theta_3 = (t'_3),$$

and in the case of the 12-inch XI, across the middle of the powder chamber,

$$\theta_3 = (t'_3) = 31 \cdot 21.$$

The winding on tension of the inner or first layer can be obtained directly from

$$\theta_2 = (t_2) + \frac{r_2^2 + r_0^2}{r_2^2 - r_0^2}(p_2) = 42 \cdot 7 = (t_2) + (t'_2).$$

This value could be obtained equally well from

$$\theta_2 = \frac{A}{r_2} + \frac{B}{r_2 - r_0} + \frac{C}{r_2 + r_0}.$$

For practical purposes the $\theta_2\theta_3$ curve is replaced by the straight line $\theta_2\theta_3$, or else by a series of steps in which several layers are wound on at the same tension, followed by another series of layers at a tension reduced by the same amount from the previous series, and so on.

Shrinkage.

Shrinkage is the excess of the external diameter of the inner tube over the internal diameter of the outer tube, before they are put together and both are cold.

Fig. 8.

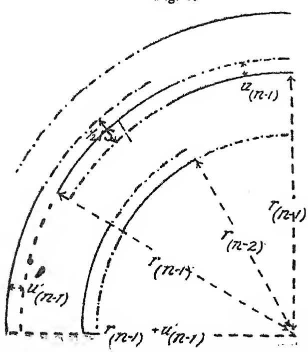
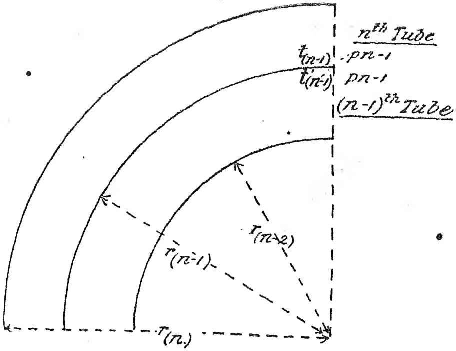


Fig. 9.



Hence, from the figure :—if S = shrinkage,

$$\frac{S}{2} = (r_{n-1} + u'_{n-1}) - (r_{n-1} - u_{n-1}) = u'_{n-1} + u_{n-1},$$

where u'_{n-1} and u_{n-1} are the radial displacements, from the unstrained position, of the outer and inner circumferential fibres of a tube and the superimposed hoop respectively, due to shrinkage.

Denoting the initial stresses at r_{n-1} in the completed gun by (t_{n-1}), (t'_{n-1}), and (p_{n-1}), then from equation (a) of (5),

$$E\frac{u_{n-1}}{r_{n-1}} = Ee_t = (t_{n-1}) + \sigma[(p_{n-1}) - R],$$

$$-E\frac{u'_{n-1}}{r_{n-1}} = -Ee'_t = (t'_{n-1}) + \sigma[(p_{n-1}) - R].$$

The negative sign is given against u'_{n-1} in the second equation because it denotes a compression, whereas u_{n-1} denotes an extension.

Hence by subtraction

$$\frac{E}{r_{n-1}}(u_{n-1} + u'_{n-1}) = (t_{n-1}) - (t'_{n-1}),$$

also

$$u_{n-1} + u'_{n-1} = \frac{S}{2},$$

therefore

$$_{n-1}S_n = \frac{2r_{n-1}}{E}[(t_{n-1}) - (t'_{n-1})].$$

This equation denotes the shrinkage between the $(n-1)^{th}$ and the n^{th} hoop; (t_{n-1}) is in tons/in.^{2}, E = 12,500; other dimensions are in inches.

The last equation may be written

$$E = \frac{(t_{n-1}) - (t'_{n-1})}{_{n-1}S_n/2r_{n-1}},$$

so that the shrinkage is the elongation in a bar of the metal of unit section, and equal in length to the diameter $2r_{n-1}$ under a tension equal to the difference of the initial circumferential tensions at the common surface of the two hoops.

The values (t_{n-1}), (t'_{n-1}) are the initial stresses, and as the powder pressure P_{n-1} at r_{n-1} increases them by equal amounts T_{n-1} to t_{n-1}, t'_{n-1} the firing stresses, their difference is unaltered, so that

$$(t_{n-1}) - (t'_{n-1}) = t_{n-1} - t'_{n-1}.$$

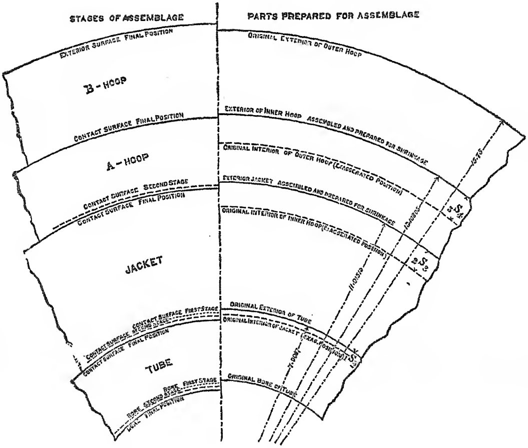
Hence, so long as the shrinkage *between* hoops is considered, it can be calculated either from the firing or from the initial stresses; and it is independent of the shrinkage imparted at other surfaces of contact of the coils, provided it is calculated as the shrinkage of the parts before assemblage.

If, however, the shrinkage is estimated for the difference between the internal diameter of a coil and the external diameter of the finished portion of the gun, then the initial stresses already set up in the gun must be taken into account and deducted (*see* 1902 Edition, p. 258).

With several layers of metal the addition of each part that is shrunk on modifies the initial stresses previously existing.

This is illustrated in diagrams in the American "Notes on the Construction of Ordnance," Nos. 31, 33, 35, by Lieutenant Rogers Birnie, showing the shrinkage (enlarged 50 times) of the different finished parts, and the intermediate states during assemblage, and the final state, when a jacket and two hoops are shrunk over the A tube of an 8-inch gun, shown in longitudinal section in the annexed fig. 10.

Fig. 10.



In a state of repose the tension of the outer fibres of the outside hoop is $8 \cdot 1$ tons/in.², and the circumferential pressure in the interior of the bore is $19 \cdot 9$ tons/in.²; so that, with $r_0 = 5$, $r_5 = 16$,

$${}_0S_1 \doteq 19 \cdot 9 \times 10 \div 12{,}500 = 0 \cdot 016,$$

or the contraction of the calibre is 16 thousandths of an inch, in consequence of the shrinkage while

$${}_4S_5 = 8 \cdot 1 \times 32 \div 12{,}500 = 0 \cdot 021,$$

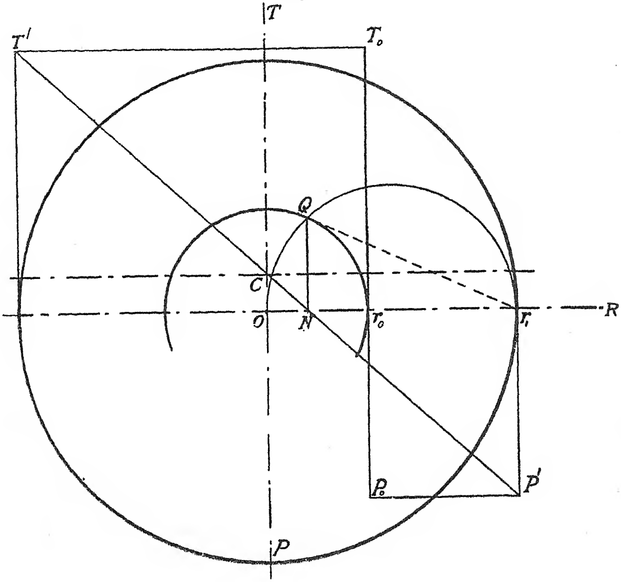
or the elongation of the external diameter due to the shrinkage is 21 thousandths of an inch.

The coefficient of expansion of steel per 1° F. is about $1 \div 150{,}000$: so that if ${}_n S_{n+1}$ denotes the shrinkage during manufacture, the temperature must be raised

$$150{,}000 \frac{{}_n S_{n+1}}{2r_n}$$

degrees Fahrenheit for the $(n+1)$th coil to be expanded sufficiently so as to slip over the nth coil; and this rise of temperature can also be represented geometrically in a similar manner.

Fig. 11.



It is found (Treatise on Service Ordnance, 1908, p. 28) that a satisfactory practical plan to follow is, "shrinkage is given by making the internal diameter of a steel tube $\frac{1}{500}$th smaller than the external diameter of the tube it goes over."

Graphical Construction of Gun Stresses.

The stresses in a gun can be graphically investigated by a rapid and simple method.

The following constructions are necessary:—

(a) Knowing that T and P are given by the equations $T = \frac{a}{r^2} + b$, $P = \frac{a}{r^2} - b$, and given at r_0 either $T = T_0$ or $P = P_0$, while at r_1, $P_1 = 0$, to find the centre of the curves, see fig. 11.

Describe the semicircle on Or_1 cutting the r_0 circle in Q, then join r_1Q, this is a tangent from r_1 to the r_0 circle, since the angle OQr_1 is a right angle: drop the ordinate QN and join NT′ or P′N cutting OT in C; this is the centre of the curves.

Proof.—With axes OR, OT the point T′ is $(-r_1, T_0)$, P′ is (r_1, p_0), also the co-ordinates of N are $\left(\frac{r_0^2}{r_1}, 0\right)$.

Since

$$T_0 = \frac{a}{r_0^2} + b,$$

and

$$P_1 = \frac{a}{r_1^2} - b = 0,$$

therefore

$$T_0 = b\,\frac{r_0^2 + r_1^2}{r_0^2} \quad \text{or} \quad b = T_0 \cdot \frac{r_0^2}{r_0^2 + r_1^2}.$$

The equation to line NT′ is

$$\frac{T - T_0}{-T_0} = \frac{r + r_1}{\frac{r_0^2}{r_1} + r_1} = \frac{r_1(r + r_1)}{r_0^2 + r_1^2}.$$

This cuts OT where $r = 0$.

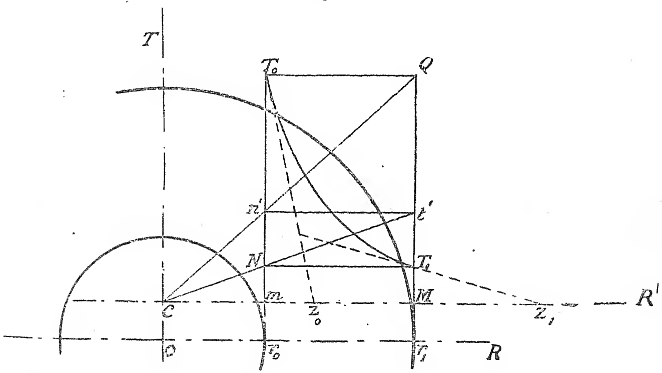
Therefore $OC = T_0 \frac{r_0^2}{r_0^2 + r_1^2}$, and this is equal to b.

Therefore C is the centre of the curves.

If P_0 is given instead of T_0, a similar proof holds good (see fig. 11). If P_0 and T_0 only be known, then, by bisecting the length T_0P_0, the centre is found.

(*b*) Given the axes of $T = \frac{a}{r^2} + b$ and one point (r_0, T_0) on the curve, to draw the curve (see fig. 12).

Fig. 12.



Make $OC = b$, referred to CT and a line CR′ at right angles to CT as axes, the equation for T becomes $T = \frac{a}{r^2}$.

The construction to find the value T_1 at a point r_1 is as follows:—

Join CQ, Q being the point (r_1, T_0), cutting r_0T_0 in n': Draw $n't'$ parallel to OR, join Ct' cutting r_0T_0 in N, draw NT_1. Then T_1 is a point on the curve, also if $Cz_0 = \frac{2}{3} r_0$ and

$Cz_1 = \frac{3}{2} r_1$, then z_0T_0 and z_1T_1 are tangents to the curve. *Proof.*—The equation to CQ is $\frac{r}{r_1} = \frac{T}{T_0}$, this cuts r_0T_0 in n', therefore

$$Mt' = mn' = \frac{r_0}{r_1} T_0.$$

The equation to Ct' is

$$\frac{r}{r_1} = \frac{T}{\frac{r_0}{r_1} T_0} = \frac{r_1}{r_0} \cdot \frac{T}{T_0}.$$

This cuts r_0T_0 where

$$Nm = T_1M = \frac{r_0^2}{r_1^2} T_0 = \frac{a}{r_1^2} \quad \text{since} \quad T_0r_0^2 = a$$

That is, T_1 lies on the curve $T = \frac{a}{r^2}$.

Again, the equation to the tangent at (r_1, T_1) is

$$T - T_1 = \frac{dT}{dr}(r - r_1)$$

$$= -\frac{2a}{r_1^3}(r - r_1) \quad \text{since} \quad T_1 = \frac{a}{r_1^2}.$$

This cuts CR' where $T = 0$, therefore

$$-\frac{a}{r_1^2} = -\frac{2a}{r_1^3}(Cz_1 - r_1) \qquad \text{or} \qquad Cz_1 = \frac{3r_1}{2}.$$

(*c*) Given two points on a rectangular hyperbola and one asymptote to find the other asymptote, *see* fig. 13.

In the firing stresses of a wire coil, *see* (20),

$$(p + t)\, r = \text{a constant, a rectangular hyperbola};$$

if the axes be changed to SX, SOr_2r_3 where $Os = t$, the uniform firing tension in the wire coil the equation becomes

$$pr = \text{a constant},$$

so that SX, SO are the asymptotes, the wire coil lying within r_2r_3.

Let $P_2\ (r_2, p_2)$ and $P_3\ (r_3, p_3)$ be the two given points and OP the given asymptote.

Complete the rectangle $p_2p_2'p_3p_3'$.

Draw the diagonal $p_2'p_3'$ to cut PO produced in S and draw SX parallel to OR. This is the required asymptote. The proof is obvious on completing the rectangles Sp_2 and Sp_3, for then these rectangles are seen to be equal; and SX and SP must be the asymptotes of the rectangular hyperbola pr = constant.

If both asymptotes SX and SP are known, but only one point on the rectangular hyperbola, say (p_3, r_3) is known, then another point is at once found, *see* fig. 13.

Thus, to find p_2 at r_2, complete the rectangle r_2p_3; join Sp'_3 and produce it to meet r_3p_3 in p_2'; then, drawing $p_2'p_2$ parallel to SX or OR to cut r_2p_3' in p_2 gives p_2 the required value of the radial pressure at r_2.

The proof is obvious, since the rectangle Sp_2 is seen to be equal to the rectangle Sp_3.

By means of the three constructions first proved, all the powder and firing stresses in a wire gun, or in a gun built up with tubes shrunk on, can be graphically drawn.

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FIG. 13.

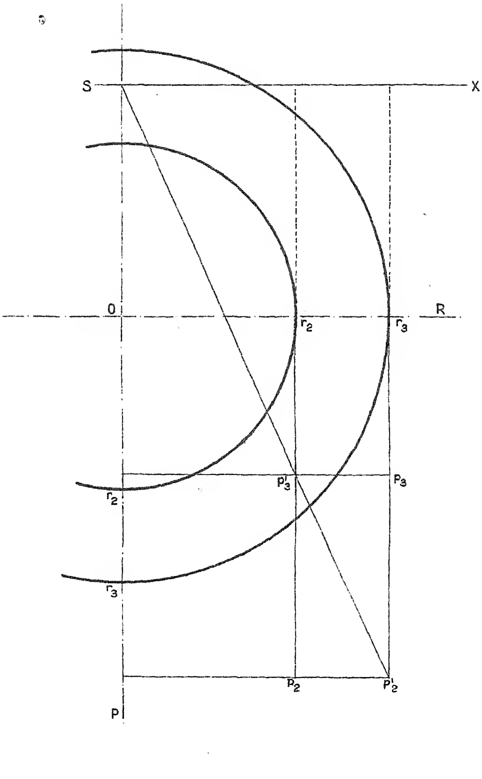

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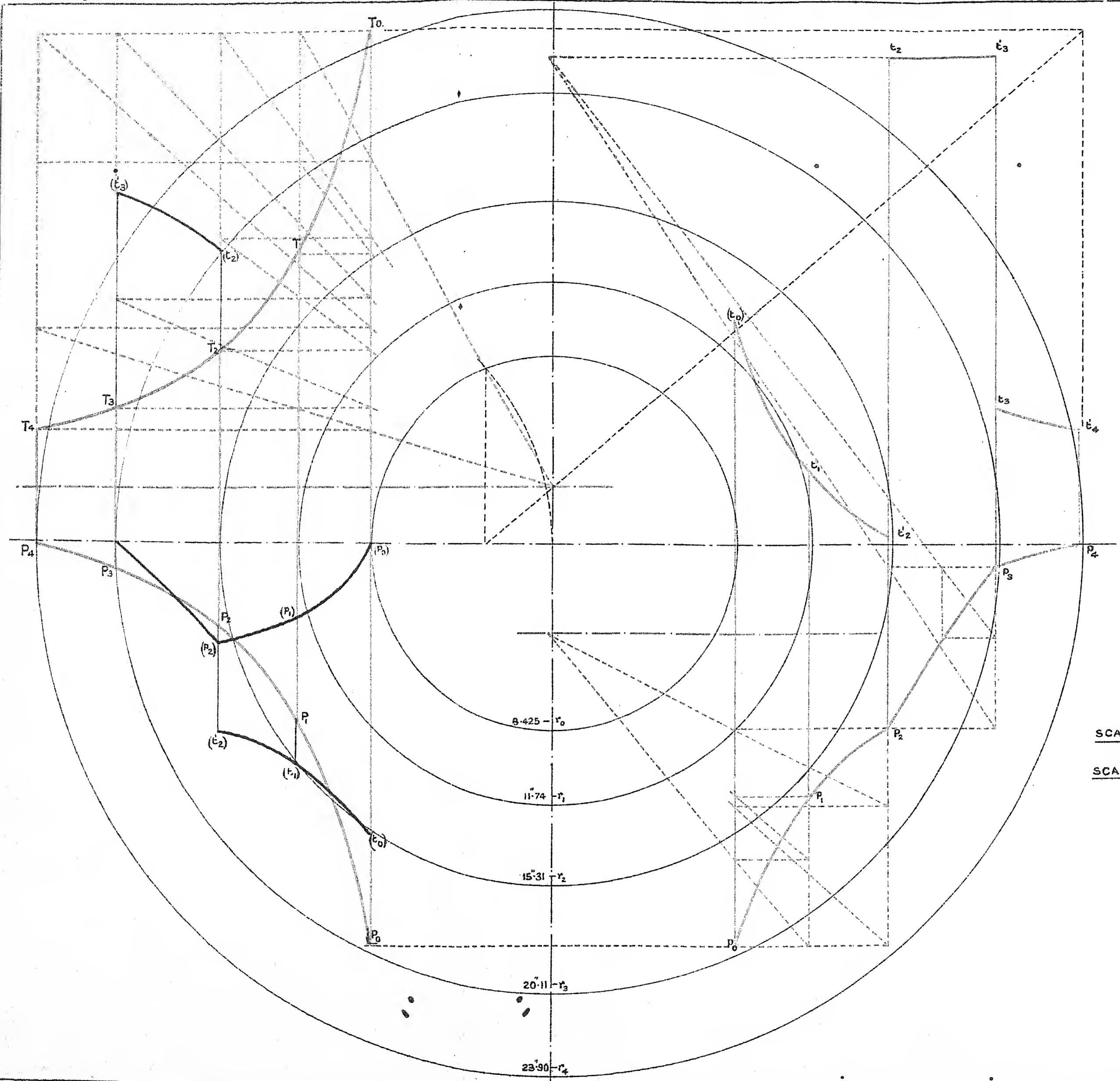
FIGURE 14.



STRESSES FOUND FROM FIGURE.					
P_0	35·9	p_0	35·9	(p_0)	0·0
P_1	15·9	p_1	22·4	(p_1)	6·5
P_2	7·4	p_2	16·4	(p_2)	9·0
P_3	2·1	p_3	2·1	(p_3)	0·0
P_4	0·0	p_4	0·0	(p_4)	0·0
T_0	46·0	t_0	20·0	(t_0)	−26·0
T_1	26·2	t_1	6·5	(t_1)	−19·7
T_2	17·4	t'_2	0·5	(t'_2)	−16·9
T_3	12·2	t_2	43·6	(t_2)	−26·2
T_4	10·2	t'_3	43·6	(t'_3)	31·4
		t_3	12·2	(t_3)	0·0
		t'_4	10·2	(t'_4)	0·0

SCALE OF SIZES, 1/5

SCALE OF STRESSES, 1" = 10 TONS PER SQ IN.

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The example taken is for the stresses across the centre of the chamber of the 12-inch B.L. XI gun, in which the data are (*see* fig. 14):—

$$r_0 = 8\cdot425 \text{ inches},$$
$$r_1 = 11\cdot74 \quad ,,$$
$$r_2 = 15\cdot31 \quad ,,$$
$$r_3 = 20\cdot11 \quad ,,$$
$$r_4 = 23\cdot9 \quad ,,$$

Also $t_0 = 20$, $(t_0) = -26$, so that

$$T_0 = 46,$$

then by following the procedure employed in the stress calculations already made, the powder stress curves for T and P are found graphically; knowing $T_0 = 46$ and $P_4 = 0$, the centre of the curves is first found, and then other points on the curves are obtained as before.

Next, in the firing stresses, $p_0 (= P_0)$ is now found, $t_0 = 20$ is known, thence the centre of the p, t curves is obtained at once, and the firing stress curves in the inner A and A tube are graphically found.

In the firing stresses for the wire coil p_2 is found, $p_3 = P_3$ is already found, and so two points (p_2, r_2) (p_3, r_3) are known, the asymptote giving the value of $t_2 = t = t_3'$ is graphically found, and any other points on the p_2p_3 curve can be found (as in fig. 13).

In the jacket the firing stresses are the same as the powder stresses.

The initial stresses are got by subtracting the powder stresses from the firing stresses.

Fig. 14 shows the complete stresses; the agreement of the various stresses is seen to be very close indeed to the calculated stresses (see p. 115).

Suppose, however, that in a wire gun the working maximum chamber pressure is fixed, say 18 tons/in.^{2}, and thence that P_0 is fixed at 36, also that the uniform maximum firing stress tension in the wire is also fixed, say $t_2' = t_3' = 45$; then by the constructions already given, the complete stresses can be found; the data in this case are

$$P_0 = 36,$$
$$t_2 = t_3' = 45.$$

The value of t_0 and of (t_0) can be deduced by this rapid graphical method; the procedure is as before when T_0 and (t_0) were given.

Captain Noble, I.O.M., A.O.D., who worked out the graphical method for obtaining fig. 14, also suggests the following for finding the radii of the A tube, wire coil, and jacket for given stresses in the metal (see figure).

Suppose the following data are given:—

$$P_0 = 36, \qquad T_0 = 46, \qquad r_0 = 6'', \qquad t_0 = 20.$$

The centre of the powder P_1T curves is known at once, it is 5 tons/in.^{2} above the centre line of bore. Knowing r_0, P_0, T_0, the P and T curves can be drawn, where the P curve cuts the line OR gives r_4.

On the right side of the figure the firing stress curves p and t can now be drawn, since t_0 and p_0 are known.

To find the wire layer, the firing tension of the outer ring of fibres of the A tube must be known; it is in most cases a small quantity. Assume it to be, say zero, so that $t_2' = 0$, and also that the fixed firing wire tension $t_2 = t_3' = 45$, then where the t curve just drawn cuts the centre line OR gives r_2.

To find r_3, the curve of firing pressure p in the jacket is the same as that for the powder pressure, therefore draw backwards from r_4 a copy of the P_4P_6 curve, where this is cut by the rectangular hyperbola $(p+t)r$ = constant of the wire coil radial pressure will give p_3 and r_3.

And now, to draw this rectangular hyperbola. The two asymptotes are known, since $t_2 = t_3' = t = 45$, also it cuts OR where $p = 0$, and the point (p_2, r_2) on it is also known, hence as many points as are required can be drawn, where the hyperbola cuts the curve p_4p_3 gives p_3, and drawing the ordinate gives r_3.

It is a matter of choice as to the value of r_1; values can be selected for it. In this graphical method the values of r_2, r_3, r_4 are obtained and the complete stress diagram is drawn at the same time.

In the example worked out on fig. 15, r_1 is taken as equal to 8 inches.

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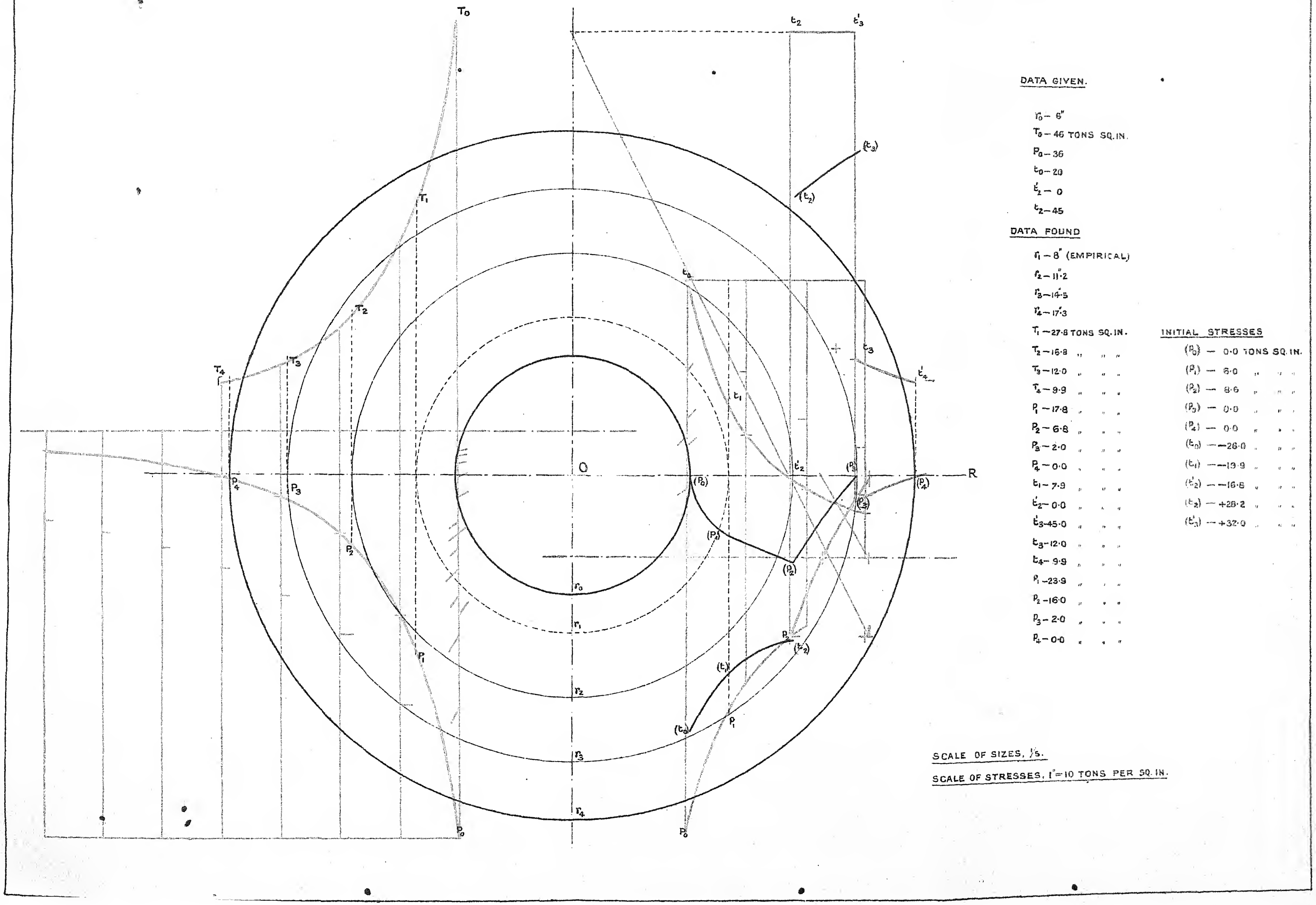
FIGURE 15.



Rifling.

A groove in a rifled gun advances a certain number of calibres for each complete turn it makes; this is called its "twist." Thus, if the twist is one turn in n calibres, the "pitch" of the helix, if uniform, is n calibres or nd inches, when the calibre is d inches. The angle which the groove at any particular point makes with the direction of axis of the gun is called the angle of rifling at that point.

Uniform Twist.

In fig. 16, B is the top groove at the commencement of rifling at the breech end, M is at the muzzle; α the angle of rifling $=$ MBP$_1$. On reaching A the groove has completed a complete turn. Then

$$AC = \pi d,$$

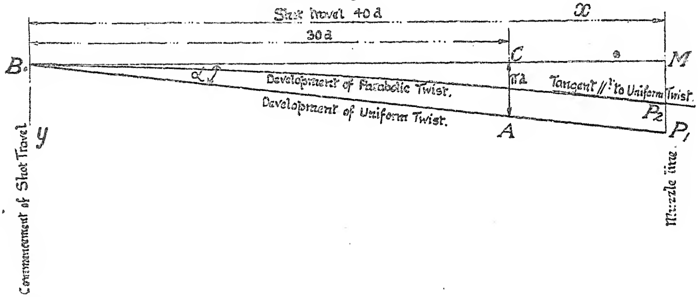
$$BC = nd,$$

and

$$\tan\alpha = \frac{AC}{BC} = \frac{P_1M}{MB} = \frac{\pi d}{nd} = \frac{\pi}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

In modern guns an average value of n is near 30, but the length of travel BM is greater than 35, so that the point A comes between B and M, as shown in fig. 16. In the R.M.L. and

Fig. 16.



early B.L. guns the reverse was the case—a complete turn took place beyond the muzzle of the gun.

Suppose

$$n = 30, \qquad \tan\alpha = \frac{\pi}{30}, \qquad \alpha = 5^\circ\ 59',$$

also

$$P_1M = BM \tan\alpha = l\frac{\pi}{n},$$

where l is the length of the rifling, generally called the total length of travel.

Increasing Twist.

Suppose the twist start from zero at B to 1 in n calibres at the muzzle, the angle of rifling increasing uniformly all the way to the muzzle at M, where the tangent to the rifling curve makes an angle $\tan^{-1}\frac{\pi}{n} = \alpha$ with the line BM (see fig. 16) parallel to the axis of the bore.

Let BP_2 be the line or groove: at M,

$$\tan\alpha = \frac{\pi}{30}, \; n = 30.$$

Therefore

$$\frac{d^2y}{dx^2} = \text{constant} = c,$$

$$\tan\theta = \frac{dy}{dx} = cx + c_1, \text{ but when } x = 0 \; \frac{dy}{dx} = 0, \text{ therefore } c_1 = 0,$$

$$y = \frac{c}{2}x^2 + c_2, \text{ but when } x = 0 \; \frac{dy}{dx} = 0, \text{ therefore } c_2 = 0,$$

and when $x = l$ inches, where l represents the length of rifling, generally called the "length of travel" of the shot,

$$\tan\theta = \tan\alpha = \frac{\pi}{n},$$

therefore

$$cl = \tan\alpha = \frac{\pi}{n}, \; c = \frac{\pi}{nl},$$

so that

$$y = \frac{\pi}{2nl}x^2,$$

or

$$x^2 = \frac{2nl}{\pi}y \quad \ldots \ldots \ldots \ldots \quad (2),$$

which is a parabola, whose vertex is at B and latus rectum is

$$\frac{2nl}{\pi}$$

Putting

$$x = l, \text{ gives } y = P_2M = l\frac{\pi}{2n};$$

$$P_1M = l\frac{\pi}{n}.$$

This shows that for a uniformly increasing twist from 0 to 1 in n, the angle turned through with uniformly increasing twist is half that turned with uniform twist; the velocity of rotation of the shot at the muzzle is the same.

The angle of rifling β at any point distant k inches from B is given by

$$\tan\beta = \frac{dy}{dx} = \frac{\pi}{nl}k = \frac{\pi}{q} \quad \ldots \ldots \ldots \ldots \quad (3),$$

where q is the twist of rifling.

At the start, if the twist is 1 in m and increase uniformly to 1 in n at the muzzle M, then the required curve is obtained thus:—

$$\frac{d^2y}{dx^2} = \text{constant } c,$$

$$\frac{dy}{dx} = cx + c, \text{ but when } x = 0, \frac{dy}{dx} = \frac{\pi}{m},$$

$$\text{and when } x = l, \frac{dy}{dx} = \frac{\pi}{n}.$$

Therefore

$$\frac{dy}{dx} = \frac{1}{l}\left(\frac{\pi}{n} - \frac{\pi}{m}\right)x + \frac{\pi}{m},$$

$$y = \frac{1}{2l}\left(\frac{\pi}{n} - \frac{\pi}{m}\right)x^2 + \frac{\pi}{m}x, \text{ since } x = 0 \text{ when } y = 0.$$

At any point distant K inches from B the twist of rifling q is given by

$$\frac{\pi}{q} = \frac{1}{l}\left(\frac{\pi}{n} - \frac{\pi}{m}\right)\mathrm{K} + \frac{\pi}{m} = \tan\beta,$$

where β is the angle of rifling, and

$$q = \frac{\pi}{\frac{\pi}{l}\left(\frac{1}{n} - \frac{1}{m}\right)\mathrm{K} + \frac{\pi}{m}} = \frac{m}{1 - \frac{\mathrm{K}}{l}\left(1 - \frac{m}{n}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Velocity of Rotation.

The velocity of rotation of the projectile at any point depends solely on the forward velocity and the twist of rifling at that point; so that at the muzzle it depends upon the muzzle velocity and the final twist of rifling at the muzzle.

If V denotes the forward axial velocity with which the shot leaves the muzzle, then the spin imparts to the points on the outside cylindrical surface a component velocity at right angles to the axis of magnitude—

$$\mathrm{V}\tan\beta = \frac{\pi \mathrm{V}}{n} \text{ f/s};$$

this is called the *linear velocity of rotation*.

The *angular velocity of rotation*, in radians/second, is obtained by dividing this by the radius of the shot in feet, a or $d \div 24$; it is, therefore,

$$\frac{\pi \mathrm{V}}{na} \quad \text{or} \quad \frac{24\pi \mathrm{V}}{nd};$$

and this again is converted into revolutions per second by dividing by 2π, since one revolution equals 2π radians; the shot, therefore, makes—

$$\frac{\mathrm{V}}{2na} \quad \text{or} \quad \frac{12\mathrm{V}}{nd} \text{ revs/sec.}$$

Thus, comparing the 6-inch gun and magazine rifle, in each of which $n = 30$; then for the same muzzle velocity, say 2000 f/s, the linear velocities of rotation will be the same, namely, 209 f/s, but the rifle bullet will make 2640 revs/second, against 133 revs/second of the 6-inch projectile.

Formerly it was considered requisite for a projectile to possess a given linear velocity of rotation to ensure its stability in flight, and for this reason the twist of rifling in howitzers, firing with low velocities, was made very quick, even up to one in 12 calibres.

But it is now found that the linear velocity of rotation should be a given fraction of the initial velocity, so that the same twist of rifling is suitable for high or low velocities, with a given projectile; but the determination of the appropriate twist from theoretical considerations is not a simple matter, and the twist must be settled by experiment to a great extent.

The investigation of the stability of an elongated projectile moving through the air in the direction of its axis with given angular velocity is very similar to that required for the

Table of Rotation for Stability of Projectiles.

(Calculated from Sir George Greenhill's formula by Major Cundill, R.A., and extended by Mr. A. G. Hadcock, late R.A., *vide Proc. R.A.I.*, vol. xi, No. 2, and vol. xiv, No. 3.)

Length of projectile in calibres.	Minimum twist at muzzle of gun requisite to give stability = 1 turn in *n* calibres.			
	Cast-iron common shell; cavity $=\frac{5}{27}$ths vol. of shell. (Density of cast iron 7·207.)	Palliser shell; cavity $=\frac{1}{8}$th vol. of shell. (Density of chilled iron 8·000.)	Solid steel bullet. (Density of steel 8·000.)	Solid lead and tin bullets of similar composition to M.-H. bullets. (Density of alloy 10·9.)
	n	*n*	*n*	*n*
2·0	63·87	71·08	72·21	84·29
2·1	59·84	66·59	67·66	78·98
2·2	56·31	62·67	63·67	74·32
2·3	53·19	59·19	60·14	70·20
2·4	50·41	56·10	57·00	66·53
2·5	47·91	53·32	54·17	63·24
2·6	45·65	50·81	51·62	60·26
2·7	43·61	48·53	49·30	57·55
2·8	41·74	46·45	47·19	55·09
2·9	40·02	44·54	45·25	52·72
3·0	38·45	42·79	43·47	50·74
3·1	36·99	41·16	41·82	48·82
3·2	35·64	39·66	40·30	47·04
3·3	34·39	38·27	38·84	45·38
3·4	33·22	36·97	37·56	43·84
3·5	32·13	35·75	36·33	42·40
3·6	31·11	34·62	35·17	41·05
3·7	30·15	33·55	34·09	39·79
3·8	29·25	32·55	33·07	38·61
3·9	28·40	31·61	32·11	37·48
4·0	27·60	30·72	31·21	36·43
4·1	26·85	29·88	30·36	35·43
4·2	26·13	29·08	29·55	34·49
4·3	25·45	28·33	28·78	33·59
4·4	24·81	27·61	28·05	32·74
4·5	24·20	26·93	27·36	31·94
4·6	23·65	26·32	26·74	31·21
4·7	23·06	25·66	26·08	30·44
4·8	22·53	25·08	25·48	29·74
4·9	22·03	24·51	24·91	29·07
5·0	21·56	23·98	24·36	28·44
5·1	21·08	23·46	23·84	27·83
5·2	20·64	22·97	23·34	27·24
5·3	20·22	22·50	22·86	26·68
5·4	19·81	22·05	22·40	26·14
5·5	19·42	21·61	21·96	25·63
5·6	19·04	21·19	21·53	25·13
5·7	18·68	20·79	21·12	24·66
5·8	18·33	20·40	20·73	24·20
5·9	18·00	20·03	20·35	23·75
6·0	17·67	19·67	19·98	23·33
7·0	14·99	16·68	16·95	19·78
8·0	13·02	14·48	14·72	17·18
9·0	11·50	12·80	13·00	15·18
10·0	10·31	11·47	11·85	13·60

stability of a top or gyrostat, spinning with its axis vertical, and the behaviour of the bodies have a close analogy. The annexed table on p. 128 shows the result of such calculations.

When a top is spun, the motion of the axis is at first unsteady, but this unsteadiness soon disappears, and the top then spins upright, when it is said to go to sleep; after a time the friction of the point reduces the spin to such an extent that the vertical position becomes unstable, and the axis again begins to wobble; the axis inclines more and more from the upright position, until finally the top falls over on its side (*see* Sir G. Greenhill's Dynamics, p. 194 to end).

So, too, an elongated projectile fired from a rifled gun is at first rather unsteady from the first portion of its flight, but the friction of the air soon destroys the irregular gyrations, and the shot, if provided with sufficient spin, proceeds steadily in the direction of the axis.

If the spin of the projectile died away more rapidly than the forward velocity, the projectile, like the top, would again become unsteady.

But the forward retardation of the shot is much greater than the angular retardation, so that the shot moves as if on an increasing screw; and practically, if once steady, the shot will continue so throughout its trajectory, in consequence of this *overscrew.*

In high-angle fire, however, the motion tends to become unsteady in the descending branch, in consequence of the great curvature of the trajectory.

The Total Rotating Thrust on the Driving Flanks of a Driving Band.

Take a cross-section of a gun, let P be the top groove PZ looking from the breech end towards the muzzle, PZ being parallel to the axis of the bore. PX is vertical and at right angles to PZ, PY is perpendicular to the plane PZX (see fig. 17).

Draw the line PT so that ZPT $= \theta$, the angle of rifling at P; similarly draw PN_0 in the plane $PZYN_0$, the quadrantal surface XZY is thus shifted round through the angle θ to the position XTN_0.

If the driving flank of the groove is radial to the bore along PX, then PN_0 in the plane XN_0 is the direction of the normal to the driving surface.

When the driving flank is not radial to the bore, but such that the normal to the driving flank, as measured in the cross-section of the bore, makes an angle δ with the radius of the bore, then the plane through the driving surface is now changed (from XPT) to QPT, and the angle QPX is $90° - \delta$. The normal to this new driving surface is PN in the plane XPN_0, and PN_0 becomes displaced to PN in the plane XPN_0 through an angle $N_0N = \psi$.

Since T is the pole of the plane XN_0N, $XQ' = N_0N = \psi$, and the angle $XTQ = \psi$, so that in the right-angled triangle QYT

$$\left.\begin{aligned} YTQ &= 90° - \psi \\ TY &= 90° - \theta \\ QY &= \delta \\ \text{and} \qquad \tan\psi \tan\delta &= \cos\theta \end{aligned}\right\} \qquad (5).$$

The forces acting upon the shot in the bore are :—

I. The thrust of the powder gas, G tons, in the direction of PZ.
II. The normal reaction of the surfaces in contact, R tons, in the direction PN.
III. The friction, μR tons, acting tangentially *backwards* in the direction TP.

The components of these impressed forces resolved parallel to the axis PZ are thus composed of

$$(G + R \cos ZN - \mu R \cos ZT) \text{ tons},$$

hence $\left(\text{from the formula } W\dfrac{f}{g} = P\right)$

$$\frac{W}{2240 \times g} \cdot \frac{d^2z}{dt^2} = G + R \cos ZN - \mu R \cos ZT \qquad (6),$$

where W is the weight of the shot in pounds, and Z the distance it has advanced in feet, while turning through an angle ω radians. Now, taking moments about the axis of the shot, the impressed couple on the shot along the axis PY at right angles to PZ (which is parallel to the axis of the shot), is

$$(rR \cos YN - r\mu R \cos YT) \text{ tons/feet},$$

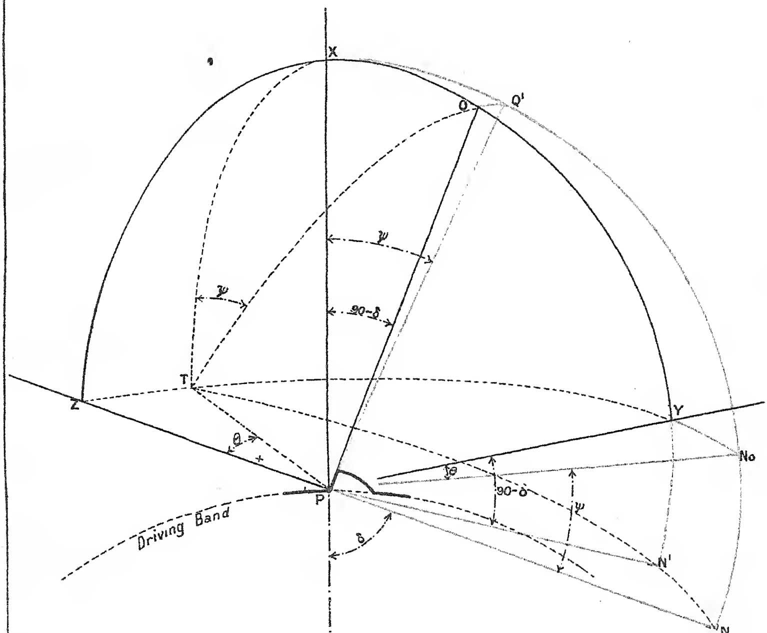
where r is the radius of the shot in *feet*, hence

$$\frac{W\rho^2}{2240g} \cdot \frac{d^2\omega}{dt^2} = rR \cos YN - r\mu R \cos YT \qquad (7).$$

Let the curve of rifling be $y = f(z)$

$$r\frac{d\omega}{dt} = \frac{dy}{dt} = \frac{dy}{dz} \cdot \frac{dz}{dt}.$$

FIGURE 17.



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Therefore

$$r\frac{d^2\omega}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2y}{dz^2}\left(\frac{dz}{dt}\right)^2 + \frac{dy}{dz}\cdot\frac{d^2z}{dt^2} \quad . \quad . \quad . \quad . \quad (8).$$

Also

$$\frac{dz}{dt} = v, \quad \frac{dy}{dz} = \tan\theta \quad . \quad . \quad . \quad . \quad (9),$$

therefore

$$r\frac{d^2\omega}{dt^2} = \frac{d^2y}{dz^2}\cdot v^2 + \tan\theta\frac{d^2z}{dt^2} \quad . \quad . \quad . \quad . \quad (10).$$

Also from (6)

$$\frac{d^2z}{dt^2} = \frac{G + R(\cos ZN - \mu\cos ZT)}{\dfrac{W}{2240\times g}} \quad . \quad . \quad . \quad . \quad (11),$$

and from (7)

$$r\frac{d^2\omega}{dt^2} = \frac{r^2 R(\cos YN - \mu\cos YT)}{\rho^2\dfrac{W}{2240\times g}} \quad . \quad . \quad . \quad . \quad (12).$$

Substituting these values in equation (10) gives

$$\frac{r^2}{\rho^2}R\frac{\cos YN - \mu\cos YT}{\dfrac{W}{2240\times g}} = \frac{d^2y}{dz^2}v^2 + \frac{G\tan\theta + R(\cos ZN - \mu\cos ZT)\tan\theta}{\dfrac{W}{2240\times g}},$$

or

$$R = \frac{G\tan\theta + \dfrac{Wv^2}{2240\times g}\cdot\dfrac{d^2y}{dz^2}}{\dfrac{r^2}{\rho^2}(\cos YN - \mu\cos YT) - (\cos ZN - \mu\cos ZT)\tan\theta} \quad . \quad . \quad . \quad . \quad (13),$$

but

$$\cos YT = \sin\theta,$$

$$\cos ZT = \cos\theta,$$

$$\cos YN = \cos YN_0\cos N_0N = \cos\theta\cos\psi,$$

$$\cos ZN = \cos ZN_0\cos N_0N = \sin\theta\cos\psi,$$

and from (5)

$$\tan\psi = \cos\theta\cot\delta,$$

therefore

$$\cos\psi = \frac{\sin\delta}{\sqrt{\sin^2\delta + \cos^2\theta\cos^2\delta}},$$

and

$$\cos YN = \frac{\sin\delta}{\sqrt{1 + \sin^2\delta\tan^2\theta}},$$

$$\cos ZN = -\frac{\tan\theta\sin\delta}{\sqrt{1+\sin^2\delta\tan^2\theta}}$$

so that

$$R = \frac{G\tan\theta + \dfrac{Wv^2}{2240\times g}\cdot\dfrac{d^2y}{dz^2}}{\dfrac{\dfrac{r^2}{\rho^2} + \tan^2\theta}{\sqrt{1+\sin^2\delta\tan^2\theta}}\sin\delta - \left(\dfrac{r^2}{\rho^2} - 1\right)\mu\sin\theta} \quad . \quad . \quad . \quad . \quad (14),$$

which gives the general expression for any curve of rifling $y = f(z)$ expressed in terms of θ and δ. Formula (14) and the above method of obtaining it is due to Sir G. Greenhill.

For a parabolic twist, starting at zero and increasing uniformly up to $\tan\theta = \frac{\pi}{n}$ at the muzzle

$$z^2 = \frac{2nl}{\pi} y = cy, \text{ say.}$$

where l is the length of rifling,

$$\tan\theta = \frac{dy}{dz} = \frac{2z}{c} = \frac{\pi z}{nl},$$

where z is the distance down the bore,

$$\frac{d^2y}{dz^2} = \frac{2}{c} = \frac{2\pi}{2nl} = \frac{\pi}{nl}.$$

Substituting these values of $\tan\theta$ and $\frac{d^2y}{dz^2}$ in (14) gives

$$\mathrm{R} = \frac{2\rho^2\left(\mathrm{G}z + \frac{\mathrm{W}v^2}{2240 \times g}\right)}{\frac{c^2r^2 + 4\rho^2z^2}{\sqrt{4z^2 \sin^2\delta + c^2}} \sin\delta - \frac{2\mu cz\,(r^2 - \rho^2)}{\sqrt{4z^2 + c^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

which is the formula given by Sir A. Noble in 'Phil. Magazine,' 1863 and 1864.

With a *uniform twist*, $\frac{d^2y}{dz^2} = 0$, $\tan\theta = \frac{\pi}{n}$, and if (as is now usual) the normal to the driving surface, that is, the line of action R is perpendicular to the radius, then the grooves have a radial flank and $\delta = 90^\circ$. The formula for R then becomes

$$\frac{\mathrm{R}}{\mathrm{G}} = \frac{\tan\theta}{\cos\theta\left(\frac{r^2}{\rho^2} + \tan^2\theta\right) - \left(\frac{r^2}{\rho^2} - 1\right)\mu\sin\theta},$$

since

$$\tan\theta = \frac{\pi}{n}, \ \cos\theta = \frac{n}{\sqrt{n^2 + \pi^2}}, \text{ and } \sin\theta = \frac{\pi}{\sqrt{n^2 + \pi^2}}.$$

(*a*) For a solid cylinder rotating about the longitudinal axis, $\rho = \frac{r}{\sqrt{2}}$.

(*b*) For a solid pointed shot, ρ is less than $\frac{r}{\sqrt{2}}$ because the mean radius of the head is less than r.

(*c*) For a hollow cylinder ρ will be greater than $\frac{r}{\sqrt{2}}$.

(*d*) For a pointed shell, ρ will be intermediate between (*b*) and (*c*) and approach (*a*) in value, that is $\rho = \frac{r}{\sqrt{2}}$.

The mathematical calculation of ρ for a pointed projectile is somewhat laborious especially when the shape of the cavity of a shell has to be considered: it is best to find ρ by graphical methods.

With $\frac{r^2}{\rho^2} = 2$.

$$\frac{\mathrm{R}}{\mathrm{G}} = \frac{\pi\sqrt{n^2 + \pi^2}}{2n^2 + \pi^2 - \mu\pi n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

Take a 12-inch gun, for which $n = 30$, $\mu = 0 \cdot 2$, $d = 12$, and a maximum chamber pressure $p_1 = 18$ tons/in.².

$$G = \pi \frac{d^2}{4} p \text{ tons,}$$

$R = \pi \,.\, d \,.\, t$ where t represents the thrust in tons per inch of circumference of bore.

$$\frac{R}{G} = \frac{\pi d t}{\pi \frac{d^2}{4} p} = \frac{\pi \sqrt{n^2+\pi^2}}{2n^2+\pi^2-\mu\pi n} = \frac{95}{1800+9 \cdot 9 - 18 \cdot 8} = \cdot 053,$$

which gives

$$R = \cdot 053\ G = \cdot 053 \pi \frac{d^2}{4} p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17).$$

At the point of maximum pressure $p = p_1 = 18$.

$R = \cdot 053 \times \pi \times 36 \times 18 = 108$ tons.

$t = \dfrac{\cdot 053}{4} p_1 d = 2 \cdot 86$ tons per inch of circumference at the time of maximum pressure.

Lastly, take the case of uniform rifling, $\delta = 90^\circ$, and *neglecting friction*, then

$$\tan \theta = \frac{\pi}{n} = \frac{\pi}{30} \quad \text{for} \quad n = 30,$$

$$\frac{d^2 y}{dz^2} = 0, \qquad \delta = 90^\circ, \qquad \mu = 0\,;$$

hence from (14) and (16)

$$\frac{R}{G} = \frac{\pi (n^2+\pi^2)^{1/2}}{2n^2+\pi^2} = \cdot 051,$$

$$\left.\begin{aligned} R &= \cdot 051\ G = \cdot 051\ \pi \frac{d^2}{4} p = \cdot 04\, d^2 p \\ t &= \frac{\cdot 051}{4}\, p d = \cdot 013\, p d \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18),$$

From (16), (17), and (18) for uniform rifling and $\delta = 90^\circ$ the circumferential thrust of R tons at any point is obtained at once when the pressure-space curve is known.

CHAPTER VII.

INTERIOR BALLISTICS.

WHEN a charge of cordite or other explosive is exploded in the chamber of the gun, the investigation of the relations connecting the pressure of the gases and the velocity of the projectile throughout the bore of a gun of known dimensions, with the various elements such as the capacity of the chamber, travel of the projectile, time, rate of burning of the explosive, &c., &c., is a part of the science called Interior Ballistics.

The subject of Interior Ballistics cannot be treated by rigid mathematical methods only, there exist such a number of factors which affect results that it is necessary to employ some empirical formulas. These formulas are deduced from the results of experiments made with closed-vessels and other instruments, but more especially from the results of firing guns of various calibre with various charges of the explosive which is used in the Service.

The following definitions are required in the practical treatment of Interior Ballistics :—

Let d represent the calibre of a gun in inches.

s the number of inches from the base of the projectile to any point along the axis of the bore : this is called the "travel" of the projectile to the selected point ; the "total length of travel" is the distance from the base of the projectile to the muzzle $= s_2$.

C the volume of the powder chamber in cubic inches (in.^{3}).

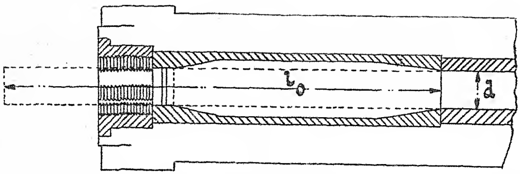
ϖ the weight of the charge in lbs.

Equivalent length of the powder chamber : l_0.

If we take a cylinder whose cross-section is the same as that of the bore and whose volume is the same as the capacity of the chamber, the length (l_0) of this cylinder is called the equivalent length of the powder chamber ; so that (fig. 1)

$$l_0 = \frac{C}{\pi \frac{d^2}{4}} \text{ inches} = 1 \cdot 27 \frac{C}{d^2} \text{ inches} = 0 \cdot 106 \frac{C}{d^2} \text{ feet ; and } C = \pi \frac{d^2}{4} l_0.$$

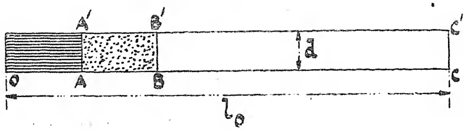
Fig. 1.



When a weight ϖ lbs. of explosive is in the powder chamber, the volume occupied by the charge includes a number of air interstices. In fig. 2, the chamber is represented as reduced to a cylinder whose cross-section is the same as that of the bore, its length OC $= l_0$ inches.

Suppose the air interstices of the charge removed, so that the volume occupied by the solid explosive itself is OA′ and the volume occupied by the charge is OB′.

Fig. 2.



Specific gravity or density, δ, of the explosive.

$$\delta = \frac{\text{weight of "solid" explosive}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{weight of explosive in OA}'}{\text{weight of water filling OA}'} = \frac{\varpi}{v_a/27\cdot73}$$

where v_a is the cubic inches in OA′ (fig. 2), and 27·73 cubic inches of water weigh 1 lb. at the standard temperature of 62° F.; therefore

$$\delta = \frac{27\cdot73\varpi}{v_a}.$$

With cordite M.D, the density δ is 1·58, hence

$$v_a = \text{OA}' = \frac{27\cdot73\varpi}{\delta} = \frac{27\cdot73\varpi}{1\cdot58}.$$

The gravimetric density of the charge: γ.

Allowing for the air interstices of the charge, γ is the mean density of the contents of the volume actually occupied by the charge. In fig. 2, this is represented as OB′ = V.

$$\gamma = \frac{\text{weight of explosive in charge}}{\text{weight of water filling the volume occupied by the charge}}$$

$$= \frac{\varpi}{v_b/27\cdot73} = \frac{27\cdot73\varpi}{v_b} = \frac{27\cdot73\varpi}{\text{volume occupied by charge}}.$$

Density of loading: Δ.

This is the mean density of the contents of the whole powder chamber, represented in fig. 2 as OC′ = C,

$$\Delta = \frac{\text{weight of charge in chamber}}{\text{weight of water filling chamber}}.$$

This allows for air interstices of the charge as well as for "air-space," therefore

$$\Delta = \frac{\varpi}{\text{C}/27\cdot73} = \frac{27\cdot73\varpi}{\text{C}}.$$

Thus, for a 12-inch B.L. IX, with a powder capacity 17930 cubic inches and a charge of 245 lbs. of cordite M.D,

$$\Delta = \frac{27\cdot73 \times 245}{17930} = 0\cdot39.$$

Also the number of cubic inches allotted to each pound of cordite in the chamber is equal to $\frac{17930}{245} = 70\cdot5$, hence $70\cdot5$ is the ratio of $27\cdot73$ to Δ, and $\frac{27\cdot73}{0\cdot39} = 70\cdot5$.

When the number of cubic inches allotted to each pound of the charge in the chamber is $27\cdot73$, then the loading density is $\Delta = 1$.

$$\textit{Expansion volume}: \ v_0 = \frac{1}{\Delta}.$$

When the contents of a whole charge are supposed evenly distributed throughout the space in the gun *behind the projectile*, then

$$v = \frac{\text{volume occupied per lb. of charge}}{27\cdot73}.$$

When the shot is rammed home, then before it moves,

$$v_0 = \frac{C}{27\cdot73\varpi} = \frac{1}{\Delta}.$$

When $\Delta = 1$, then $C = 27\cdot73\varpi$, $v_0 = 1$, and the unit expansion volume is $27\cdot73$ cubic inches per lb. of charge.

In the case of the 12-inch IX, $v_0 = \frac{1}{0\cdot39} = 2\cdot56$ expansion volumes in the chamber.

When the projectile has moved down the bore a distance s inches from the seat of the projectile, the space behind it is $\left(C + \frac{\pi}{4}d^2s\right)$ in.^{3}, and then the number of expansion volumes is (*see* fig. 3)

$$= \frac{C + \frac{\pi}{4}d^2s}{27\cdot73\varpi} = \frac{\frac{\pi}{4}d^2l_0 + \frac{\pi}{4}d^2s}{27\cdot73\varpi} = \frac{\frac{\pi}{4}d^2(l_0+s)}{27\cdot73\varpi} = \frac{\frac{\pi}{4}d^2l}{27\cdot73\varpi} = 0\cdot0283\frac{d^2l}{\varpi};$$

so that, if the total volume of the chamber and bore to the muzzle be B in.^{3}, then

$$B = C + \frac{\pi}{4}d^2s_2$$

where s_2 is the total travel, and

$$v_2 = \frac{B}{27\cdot73\varpi}.$$

In the case of the 12-inch IX, this gives

$$v_2 = \frac{63{,}100}{27\cdot73 \times 245} = 9\cdot3,$$

total expansion volumes in the bore.

v_1 is usually taken to represent the number of expansions to the point of maximum pressure: if this point be reached after a travel of s_1 inches, then

$$v_1 = \frac{C + \frac{\pi}{4}d^2s_1}{27\cdot73\varpi} = \frac{\frac{\pi}{4}d^2l_1}{27\cdot73\varpi} = 0\cdot0283\frac{d^2l_1}{\varpi},$$

where

$$l_1 = l_0 + s_1.$$

Equivalent length of initial air space : z_0.

In fig. 2 OA′ represents the actual volume occupied by the charge as a solid ; AC′ represents the total air space in the chamber ; the length AC = z_0 inches, so that

$$z_0 = OC - OA = \frac{C}{\frac{\pi}{4}d^2} - \frac{27\cdot 73\varpi}{\frac{\pi}{4}d^2\delta}.$$

But

$$C = \frac{27\cdot 73\varpi}{\Delta},$$

therefore

$$z_0 = 4 \,.\, \frac{27\cdot 73\varpi}{\pi d^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$$

$$= 35\cdot 3\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)\frac{\varpi}{d^2} \text{ inches}$$

$$= \ 2\cdot 94\,\frac{a^2\varpi}{d^2} \text{ feet},$$

where

$$a^2 = \frac{1}{\Delta} - \frac{1}{\delta}.$$

This formula is made great use of in the treatment of Interior Ballistics by Colonel Ingalls, U.S. Artillery (see *Journal of the R.A.*, December, 1909, vol. XXXVI), "*Interior Ballistics after Lissak upon Ingalls,*" by Captain J. H. Hardcastle, *p.a.c.*, late R.A. ; also in the confidential "*Internal Ballistics,*" by Major N. B. Heffernan, *p.a.c.*, R.A., published in 1907.

From closed-vessel experiments the pressure corresponding to different densities of loading is obtained. Plotting these pressures as ordinates and the corresponding values of $\frac{1}{\Delta} = v$, that is expansion volumes as abscissæ, a curve of p and v can be drawn, and then, by a quadrature of the curve, the theoretical work E in foot/tons realised in a gun by the expansion of 1 lb. of an explosive from one value v_1 to another v_2 can be obtained, unit expansion volume per lb. of explosive being $27\cdot 73$ cubic inches.

At any point of the curve let the pressure be p tons/in.², and suppose this constant while $v - \frac{dv}{2}$ changes to $v + \frac{dv}{2}$, that is, during a change of dv corresponding to $27\cdot 73\,dv$ cubic inches, the total pressure on the base of the shot in this interval is $p \times \frac{\pi}{4}d^2$, and the distance travelled by the shot is

$$\frac{27\cdot 73\,dv}{\frac{\pi}{4}d^2} \text{ inches},$$

hence the work done is

$$27\cdot 73p\,dv \text{ inch tons} = \frac{27\cdot 73}{12}p\,dv \text{ foot tons};$$

therefore

$$\Delta E = 2\cdot 31p\,dv \text{ foot tons},$$

and the total work done from $v = v_1$ to $v = v_2$ is

$$E = 2\cdot 31\int_{v_1}^{v_2} p\,dv \text{ foot tons}.$$

By this method the theoretical work capable of being done by 1 lb. of gunpowder or of cordite, Mark I or M.D., in expanding from volume unity to another can be found.

For ϖ lbs. of explosive

$$E = 2\cdot31\varpi\int_{v_1}^{v_2} p\,dv \text{ foot tons.}$$

If the curve connecting p and v be of an adiabatic nature, pv^{γ} = const. = C, then

$$\int_{v_1}^{v_2} p\,dv = C\int\frac{dv}{v^{\gamma}} = \frac{C}{\gamma-1}\left[\frac{1}{v_1^{\gamma-1}} - \frac{1}{v_2^{\gamma-1}}\right]$$

$$= \frac{p_1v_1 - p_2v_2}{\gamma-1} \quad \text{or} \quad \frac{p_1v_1}{\gamma-1}\left[1-\left(\frac{v_1}{v_2}\right)^{\gamma-1}\right],$$

and then

$$E = 2\cdot31\varpi\frac{p_1v_1 - p_2v_2}{\gamma-1} \text{ foot tons}$$

or

$$2\cdot31\varpi\frac{p_1v_1}{\gamma-1}\left[1-\left(\frac{v_1}{v_2}\right)^{\gamma-1}\right],$$

so that E can be tabulated for all possible values of $\frac{v_1}{v_2}$.

When a gun is fired, the rate of ignition and the rate of combustion have a great effect on the pressures produced; of these two, the rate of combustion, however, has by far the greater effect. The greater the total surface of the explosive, the quicker the rate of evolution of the gases. The rate of combustion itself is a function of the pressure, so that the pressure is a function of the shape, size, chemical components, &c., of the explosive.

When the charge in a gun is ignited, a certain amount of pressure must be reached before the projectile's driving-band is "engraved." The rate of combustion is increased by the pressure, and before the projectile has moved far a high pressure has been reached. The projectile is meanwhile rapidly increasing its velocity and the volume behind it, so that very soon the pressure will begin to fall on account of this increasing volume and because of the loss of temperature of the gases from the conversion of heat into energy, also from the decreased surface of the explosive.

From the point of maximum pressure to the muzzle there should be a continuous fall of pressure, the decreasing pressure influencing the rate of combustion. The pressure-space curve along the whole length of travel is somewhat of the form in fig. 3.

The following method of treating interior ballistics involves no rigid theory; it is merely an attempt to simplify a very difficult problem and to arrive at a practical solution, namely, to find the weight and size of cordite M.D. which will produce in a given gun a required muzzle velocity and a given maximum pressure.

With cordite Mark I, or M.D., a charge is not, as a rule, wholly consumed when the point of maximum pressure is reached; it is however found that the pressure-space curve from this point to the muzzle is of the nature of an adiabatic.

(*a*) From the results of Sir A. Noble's experiments with closed vessel, &c., Mr. A. G. Hadcock finds that from the point of maximum pressure to the muzzle, the curve connecting the pressure with the expansion volume, v, approximates very closely (for cordite M.D.) to the formula

$$(p-1)\,v^{1\cdot28} = 107\cdot14.$$

(*b*) And that up to the point of maximum pressure the curve which connects p and v may be taken as a quadrant of an ellipse.

(*c*) Also that when employing (*a*) and (*b*) for finding the energy imparted to the projectile, a factor must be employed, which is given by

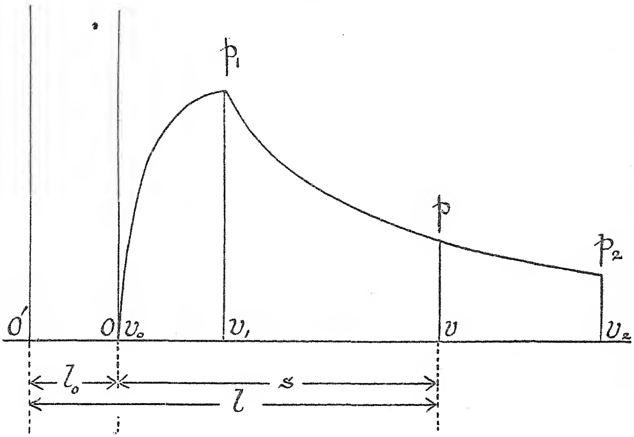
$$\text{factor} = 0{\cdot}9 + 0{\cdot}008d$$

where d is the calibre in inches.

For friction, &c., a deduction of 1 ton/in.^{2} per foot of travel is made in foot tons, this is equal to

$$\text{area of bore} \times \text{shot travel in feet} = \frac{\pi d^2}{4} \cdot \frac{s}{12} \text{ foot tons.}$$

Fig. 3.



Accepting (*a*), (*b*), and (*c*), the following method is given for finding the weight of charge of cordite M.D., also the size which will just be burned at the muzzle to produce a given muzzle velocity and a given maximum chamber pressure.

The following data are required:—

W lbs., weight of projectile.
s_2 inches, total length of travel.
d inches, calibre of gun.
C cubic inches, capacity of chamber.
V f/s, the required muzzle velocity.
p_1 tons/in^2, the maximum pressure allowed.

The following are then to be found:—

ϖ lbs, weight of charge, cordite M.D.
D inches, the actual diameter of the cord of cordite M.D.

In finding ϖ and D, the following curves are also obtained:—the pressure-space, velocity-space, time-space, film of cordite-space; these give the pressure, velocity, time, and thickness of cordite burned, at any point along the bore of the gun.

The volume v in (a) and (b) is

$$v = \frac{\text{capacity}}{27\cdot73\varpi} = \frac{\frac{\pi}{4}d^2l}{27\cdot73\varpi} = \frac{d^2l}{35\cdot3\varpi},$$

where l inches is the distance to any point in the bore, measured from the rear end of equivalent length of the chamber.

Substituting this value in the equation

$$(p-1)\,v^{1\cdot28} = 107\cdot14,$$

then

$$(p-1)\left(\frac{\pi}{4}d^2l\right)^{1\cdot28} = 107\cdot14\,(27\cdot73\varpi)^{1\cdot28},$$

$$\frac{1}{1\cdot28} = 0\cdot78, \text{ therefore } (p-1)^{0\cdot78}\frac{d^2l}{\varpi} = (107\cdot14)^{0\cdot78}\frac{27\cdot73\times4}{\pi},$$

$$(p-1)^{0\cdot78}d^2l = 1361\varpi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A).$$

In fig. 3, p_1 and v_1 are the pressure and volumes of expansion at the point of maximum pressure; p_2, v_2 represent corresponding values at the muzzle; v_0 represents the volumes of expansion in the powder chamber itself. The area of the curve from v_0 to v_1 is $\frac{\pi}{4}(v_1 - v_0)\,p_1$, and from v_1 to v_2 the area is

$$\int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2}\left(\frac{107\cdot14}{v^{1\cdot28}} + 1\right)dv$$

$$= \frac{107\cdot14}{0\cdot28}\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1)$$

$$= 382\cdot5\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1).$$

The total area is therefore

$$(v_1 - v_0)\,p_1\frac{\pi}{4} + 382\cdot5\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1).$$

The work in foot tons per lb. of charge is

$$e = 2\cdot31\left[(v_1 - v_0)\,p_1\frac{\pi}{4} + 382\cdot5\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1)\right] \quad . \quad . \quad . \quad . \quad (B).$$

For ϖ lbs. of charge then, the work done on the projectile throughout the bore is equal to the muzzle energy in foot tons, and with the introduction of the factor and deduction for friction, &c.,

$$E = \frac{WV^2}{2g\times2240}$$

$$= \text{factor}\left\{2\cdot31\varpi\left[(v_1 - v_0)\,p_1\frac{\pi}{4} + 382\cdot5\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1)\right]\right.$$

$$\left. - \text{area of bore} \times \text{shot travel in feet}\right\},$$

or,

$$E = f\left(\varpi e - \frac{\pi}{4}d^2\times\frac{s_2}{12}\right) \text{ foot tons};$$

this gives

$$\varpi e = \frac{E}{f} + \frac{\pi}{4}d^2\frac{s_2}{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C).$$

From the equation

$$(p-1)\,v^{1\cdot28} = 107\cdot14 \text{ (Hadcock's empirical formula for M.D. cordite),}$$

at point of maximum pressure,

$$v_1 = \left(\frac{107\cdot14}{p_1-1}\right)^{\frac{1}{1\cdot28}} = \left(\frac{107\cdot14}{p_1-1}\right)^{0\cdot78},$$

so that when p_1 is fixed, v_1 is also known, and in the equation (B), when p_1 is known, and hence also v_1, then for any chosen value of v_2 this last equation becomes of the form

$$e = \lambda v_0 + \mu, \quad \tan\theta = \frac{e-\mu}{v_0} = \lambda,$$

which is a straight line, where λ and μ are constants, got from the known values of p_1, v_1, and the chosen value of v_2, where

$$\lambda = -\ 2\cdot31\, p_1 \frac{\pi}{4} = \tan\theta,$$

$$\mu = 2\cdot31\left[v_1 p_1 \frac{\pi}{4} + 382\cdot5\left(\frac{1}{v_1^{0\cdot28}} - \frac{1}{v_2^{0\cdot28}}\right) + (v_2 - v_1)\right] \text{ in foot tons per lb. of cordite.}$$

Since λ does not contain v_2, it follows that for a fixed value of p_1, v_1 is known, and for various values of v_2, the relation between e and v_0 can be represented by a series of parallel straight lines; v_0 being represented as abscissæ and e as ordinates.

Example in drawing parallel lines for the case of $p_1 = 16$ tons/in.^{2} (See Diagram A, by Captain R. K. Hezlet, R.A.)

Select any value of v_2, say 10, then from (B) evaluate e for $v_0 = 2$, $v_0 = 3$, etc., as an example. The straight line got from joining the points (e_2, $v_0 = 2$) and (e_3, $v_0 = 3$) is drawn; e can now be read off this line for all values of v_0 and for $v_2 = 10$.

To draw the other lines it is necessary to find the co-ordinate values of e and v_0 for one point only and for another selected value of v_2, say $v_2 = 11$; a line through this point parallel to the first gives e in terms of all values of v_0 and for $v_2 = 11$.

From the diagram of the series of parallel lines, e can be read off for all values of v_2 and all values of v_0, so that the diagram gives a complete record of e for all values of v_0 and v_1 got from (B), when

$$p_1 = 16, \quad v_1 = 4\cdot64, \quad \lambda = \tan\theta = -29\cdot05$$

For any maximum pressure p_1 a similar diagram of parallel straight lines can be drawn.

On any one diagram for p_1 use is made of the equation $v_0 = \dfrac{C}{27\cdot73\varpi}$, and that in any gun under discussion $\dfrac{v_2}{v_0} = \dfrac{B}{C} = \text{constant}$.

From the intersection of the plotting on the diagram of $v_0 = \dfrac{C}{27\cdot73\varpi}$ and $\dfrac{v_2}{v_0} = \dfrac{B}{C}$, the weight of charge ϖ is read off.

An example will very readily show the method of procedure, which is exactly the same for any gun.

Example for finding ϖ. (Diagram A.)

60-pr. B.L. :—

$$W = 60 \text{ lbs.}$$

$$d = 5 \text{ inches.}$$

$$s_2 = 11 \cdot 5 \text{ feet.}$$

$$C = 618 \cdot 75 \text{ ins.}^3.$$

$$B \text{ (total bore)} = 3361 \text{ ins.}^3.$$

It is required to find the charge of cordite M.D. which will give M.V = 2030 f/s and

$$p_1 = 16$$

$$\text{Muzzle energy } E = \frac{WV^2}{2g \times 2240} = 1818 \text{ foot tons}$$

$$\text{Factor, } f = 0 \cdot 9 + (0 \cdot 008 \times 5) = 0 \cdot 94.$$

From

$$\varpi e = \frac{E}{f} + \frac{\pi}{4} d^2 s$$

$$= \frac{1818}{0 \cdot 94} + \frac{\pi}{4} \times 25 \times 11 \cdot 5 = 2160,$$

$$\varpi = \frac{2160}{e}.$$

Now

$$\frac{1}{\Delta} = v_0 = \frac{C}{27 \cdot 73 \varpi} = \frac{618 \cdot 75}{27 \cdot 73 \varpi},$$

therefore

$$\frac{v_0}{e} = \frac{618 \cdot 75}{27 \cdot 73 \varpi e} = \frac{618 \cdot 75}{27 \cdot 73 \times 2160} = \frac{618 \cdot 75}{59{,}900}$$

$$= \frac{2 \cdot 07}{200} = \frac{3 \cdot 1}{300} \text{ (a straight line).}$$

From Diagram A :—

$$v_0 = 2 \cdot 07 \quad \text{when} \quad e = 200,$$

and

$$v_0 = 3 \cdot 1 \quad \text{when} \quad e = 300.$$

On the Diagram A, for $p_1 = 16$, plot any two selected points such as $(2 \cdot 07, 200)$, $(3 \cdot 1, 300)$, and draw the straight line through them.

Again,

$$\frac{v_2}{v_0} = \frac{B}{C} = \frac{3361}{618 \cdot 75} = \frac{12}{2 \cdot 21} = \frac{13}{2 \cdot 395};$$

since

$$v_0 = 2 \cdot 21 \quad \text{when} \quad v_2 = 12 \quad \text{and} \quad v_0 = 2 \cdot 395 \quad \text{when} \quad v_2 = 13.$$

Draw the straight line joining the points selected, $(12, 2 \cdot 21)$, $(13, 2 \cdot 395)$.

The intersection (see Diagram) of the two last drawn curves or straight lines gives a value of $e = 225$, from which

$$\varpi = \frac{2160}{225} = 9 \cdot 6 \text{ lbs.}$$

DIAGRAM A.

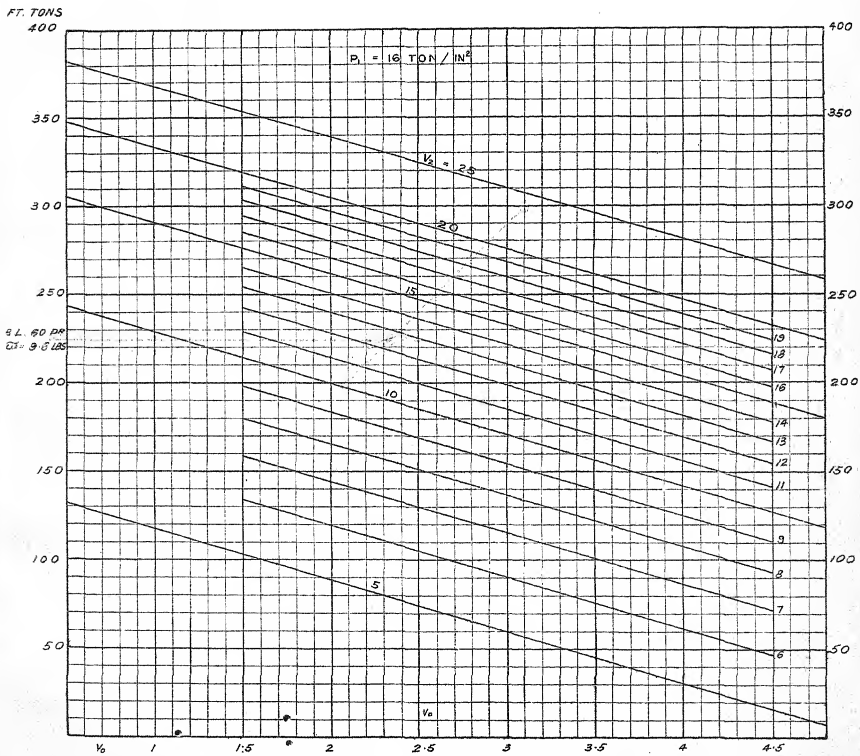


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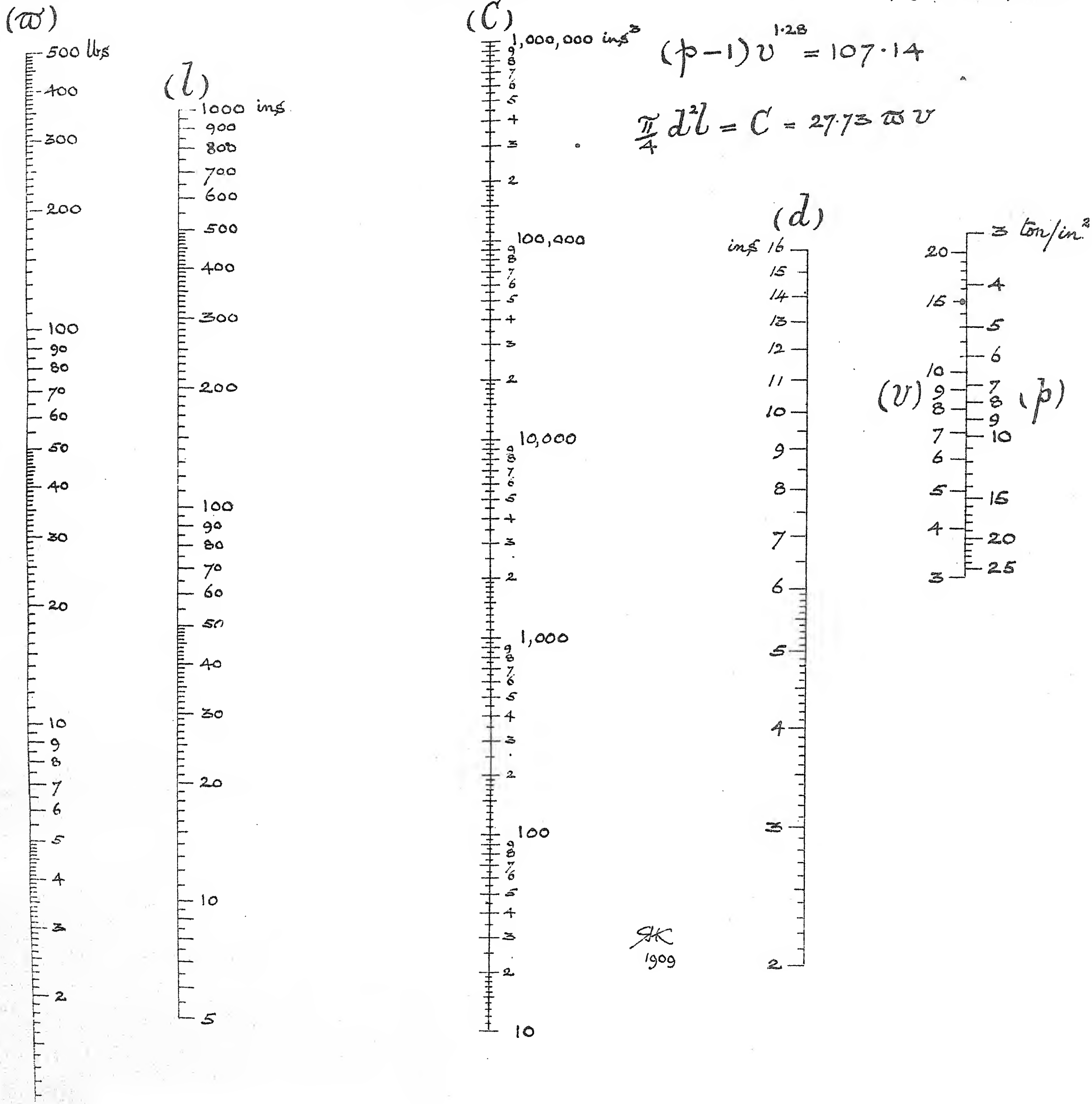
DIAGRAM B.


(ϖ)
500 lbs
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
(l)
1000 ins
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
5
(C)
1,000,000 ins³
100,000
10,000
1,000
100
10
$(p-1)v^{1\cdot 28} = 107\cdot 14$
$\frac{\pi}{4} d^2 l = C = 27\cdot 73\, \varpi v$
(d)
ins 16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
(v)
20
15
10
9
8
7
6
5
4
3
(p)
3 ton/in²
4
5
6
7
8
9
10
15
20
25
SK
1909

The Pressure-space Curve between Point of Maximum Pressure and the Muzzle.

From page 140, formula (A),

$$(p-1)^{0\cdot78}\, d^2 l = 1361\varpi.$$

A graphic representation of this formula is most useful (*see Diagram B*).

Writing

$$d^2 l = 1361\varpi\,(p-1)^{-0\cdot78}$$

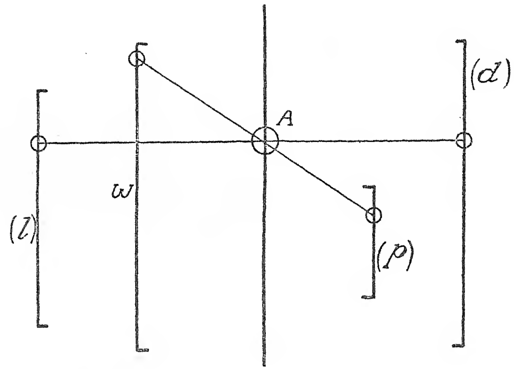
$$2\log d + \log l = \log\varpi - 0\cdot78\log(p-1) + \log 1361,$$

which is of the form

$$\phi(d) + \psi(l) = f(\varpi) - \mathrm{F}(p).$$

An equation of this form can, as shown by M. d'Ocagne in "Calcul-graphique et Nomographie," Paris, 1908, be represented by a "nomogram," in which the scales of d, l, ϖ, p lie on four parallel straight lines, while a fifth central undivided "line of reference" serves to connect the alignment of corresponding values of d (calibre) and l (travel plus E.L.C.), ϖ (weight of charge) and p (pressure) (fig. 4).

Fig. 4.



The nomogram is read in the following way :—

Supposing l, d, ϖ are known, and it is required to find the value of p to satisfy

$$(p-1)^{0\cdot78}\, d^2 l = 1361\varpi.$$

Join the given values of l and d on their respective scales by a straight line cutting the reference line in a point A (fig. 4).

Join the given value of ϖ on its scale to A and produce to cut the scale of p, the point of intersection will give the required value of p.

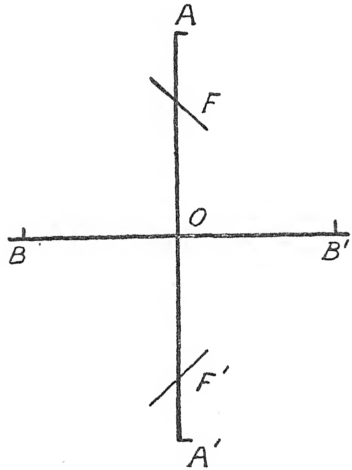
On Diagram B, worked out and drawn by Captain R. K. Hezlet, R.A., a complete representation is given which is applicable for variations of 1 to 500 lbs. for ϖ, 5 to 1000 inches for l, 10 to 1,000,000 cubic inches for capacity of chamber C, 2 to 16 inches for d, 3 to 25 tons/in.^{2} for p; l is measured from the rear end of the equivalent length of the chamber.

From Diagram B (the central scale has in this case been divided to show the actual capacities in cubic inches) a number of points can be at once plotted in the pressure-space curve between the point of maximum pressure and the muzzle; this part of the curve can be drawn (see Diagram C).

The Pressure-space Curve up to the Point of Maximum Pressure.

This is a quadrant of an ellipse (see p. 138), in which the semi-major axis is the length representing the maximum pressure p_1 tons/in.^{2}, the semi-minor axis is the length representing the travel s inches up to the point of maximum pressure, this "s" inches is found by means of Diagram B; p_1, d, C, ϖ being known, l is found, hence $s = (l - l_0)$ inches is found, where l_0 is the equivalent length of the chamber.

Suppose OA $= a$, OB $= b$ represent the semi-major and semi-minor axes respectively. With centre B and radius equal to a describe a circle cutting AOA′ in F and F′, then F and F′ are the foci.



Take a string of length $2a$ equal to the major axis, fasten the ends at F and F′, then a pencil in the loop will describe a true ellipse; the required quadrant of the pressure-space curve up to the maximum pressure is thus quickly drawn with accuracy (see Diagram C).

Velocity along the Bore: Velocity-space Curve. (Diagram C.)

This is obtained quickly with the help of a planimeter, which gives the area, A, of the pressure-space curve up to any point along the bore; an example from the 60-pr. B.L. is given:—

60-pr. B.L.—

$$d = 5, \quad \varpi = 9 \cdot 44 \text{ lbs.}, \quad p_1 = 16, \quad C = 618 \cdot 75,$$

$$\text{E.L.C.} = l_0 = 31 \cdot 5 \text{ inches}, \quad W = 60 \text{ lbs.}, \quad \text{total travel, } 138 \text{ inches.}$$

Distance from end of equivalent length of chamber to point of maximum pressure is $s_1 = 62$ inches.

From the pressure-space diagram:—

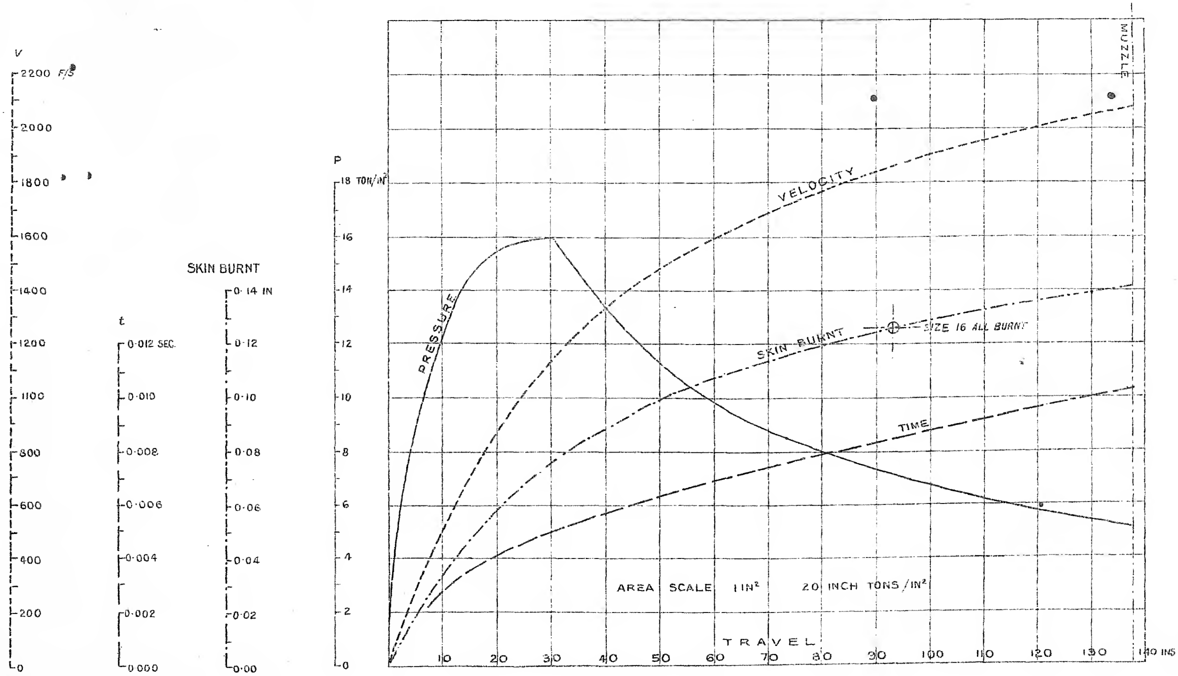
1 square inch represents 20 inch tons/in.^{2}.

Empirical factor $= 0 \cdot 9 [0 \cdot 9 + 0 \cdot 008d] = 0 \cdot 9 \times 0 \cdot 94 = 0 \cdot 846$.

To face page 144 Chapter VII.

DIAGRAM C.

B. L. 60 PR. ∂ = 5 IN. ω = 60 LB. ϖ = 9·44 LB. SIZE 16 MAXM PRESS. 16 TON/IN² TRAVEL 138 IN. CHAMBER = 618·75 IN³ EQUIV LENGTH = 31·5 IN TRAVEL TO MAXM PRESS 30·5 IN



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The velocity at any point is given by

$$\frac{Wv^2}{2g \times 2240} \text{ foot/tons} = A \times \tfrac{20}{12} \times 0 \cdot 846 \times \pi \frac{d^2}{4},$$

$$v \text{ f/s} = \sqrt{A \times 20 \times 0 \cdot 846 \times 62} = 255 \sqrt{A}.$$

Travel s inches.	l inches.	p tons/in.^{2}.	Area, sq. inches.	$v = 255\sqrt{A}$, feet per second.
0	31·5	—	—	0
20	—	—	11·60	868
30·5	62	16	—	—
40	—	—	27·82	1345
60	—	9·8	39·37	1598
80	—	8·1	48·20	1770
100	—	6·1	55·61	1900
120	—	5·8	61·86	2005
138	—	5·1	66·70	2080

See Diagram C, which is worked out and drawn by Captain R. K. Hezlet, R.A.

Time-space, Film Burnt-space Curves, and Size of Cordite to be Used. (Diagram C.)

Use is made of Mansell's table for film of cordite burnt in 0·001 second under various pressures. Temperature of charge 80° F. For the 60-pr. B.L., the following table shows the tabulated figures which provide the data, and a similar procedure applies to any gun :—

Travel, s inches.	$\bar{v}$ f/s, mean velocity over the interval.	Δt secs., time over the interval.	t secs., time to any point along bore.	$\bar{p}$ tons/in^2, mean pressure over the interval.	Mansell's factor × 1000. Temperature of charge 80° F.	ΔF inches, film-burnt (reduction of diameter) during the interval.	F inches thickness burnt up to any point along bore.
0							
	270	0·00309		8·6	11·78	0·0363	
10			0·00309				0·0363
	700	0·00119		14·7	19·92	0·0236	
20			0·00428				0·0599
	1020	0·00082		15·8	21·40	0·0175	
30			0·00510				0·0774
	1250	0·00067		14·7	19·92	0·0133	
40			0·00577				0·0907
	1490	0·00112		11·3	15·38	0·0172	
60			0·00689				0·1079
	1700	0·00098		8·8	12·04	0·0118	
80			0·00787				0·1197
	1840	0·00091		7·4	10·17	0·0092	
100			0·00878				0·1289
	1960	0·00085		6·3	8·70	0·0074	
120			0·00963				0·1363
	2050	0·00073		5·6	7·77	0·0057	
138			0·01036				0·1420*

* This is close to the actual size (0·1265) of size 16 M.D. Hence size 16 M.D. should be used. (See Diagram C.) This size 16 would be all burnt at a point about 40 inches from the muzzle.

GUNNERY TABLES.

CONTENTS.

INTRODUCTION.

These ballistic tables are based upon experiments carried out at Shoeburyness in 1904–6 with projectiles of two-calibre head (Ordnance Committee Reports Nos. 1179, 1185 and 1192).

The laws of resistance of the air deduced from these experiments are expressed by the following equations in which C is the ballistic coefficient and r the retardation (in foot-seconds), or resistance per unit of projectile's mass :—

$$4000 \text{ f/s} > v > 2600 \text{ f/s},$$

$$Cr = [7 \cdot 1865702 - 10]\, v^{1 \cdot 67}.$$

$$2600 \text{ f/s} > v > 2000 \text{ f/s},$$

$$Cr = [7 \cdot 7671157 - 10]\, v^{1 \cdot 5}.$$

$$2000 \text{ f/s} > v > 1460 \text{ f/s},$$

$$Cr = [6 \cdot 7768067 - 10]\, v^{1 \cdot 8}$$

$$1460 \text{ f/s} > v > 1190 \text{ f/s},$$

$$Cr = [2 \cdot 9795830 - 10]\, v^{3}.$$

$$1190 \text{ f/s} > v > 1040 \text{ f/s},$$

$$Cr = [2 \cdot 3689459 - 20]\, v^{6 \cdot 45}.$$

$$1040 \text{ f/s} > v > 840 \text{ f/s},$$

$$Cr = [2 \cdot 7777107 - 10]\, v^{3}.$$

$$840 \text{ f/s} > v > 0 \text{ f/s},$$

$$Cr = [6 \cdot 8717017 - 10]\, v^{1 \cdot 6}.$$

N.B.—The figures in the brackets are the logarithms of the coefficients of v.

TABLE I. gives values of K, Cr, and p for velocities from 100 f/s to 4000 f/s.

TABLES II., III., IV., V., VI., and VIII. are in appearance similar to those in Text Book of Gunnery, Part I., 1907, which they now supersede.

TABLE VII. is compiled from Glaisher's *Hygrometrical Tables*, and is useful for obtaining values of the coefficient of tenuity, τ, for all readings of the wet and dry bulb.

From this table, δ, the density in grains/ft.^{3} for the reading of the barometer in inches, and of the wet and dry bulb thermometer in degrees Fahrenheit of the meteorological record on the day of an experiment can be obtained, and thence the tenuity factor $\tau = \frac{\delta}{\Delta}$, where $\Delta = 534 \cdot 22$ is the standard density in grains/ft.^{3} for dry air at 62° F. and a 30-inch barometric height.

To use the table, look out the number corresponding to the readings given by the wet and dry bulb; this gives the grains/ft.^{3} at a barometric height of 29 inches; and then the table of proportional parts (used in conjunction with the column "Difference for 1 inch in barometer") shows the addition to be made for the extra height of the barometer above 29 inches. The standard barometric height in the table is taken very low, at 29 inches, in order to avoid negative proportional parts.

Examples:—

1. On March 7, 1879, the meteorological record was

$$\text{Thermometer} \begin{Bmatrix} \text{Wet } 37^\circ \\ \text{Dry } 39^\circ \end{Bmatrix} \text{Barometer } 29 \cdot 95 \left\{ \begin{array}{r} 538 \cdot 9 \\ 16 \cdot 7 \\ 0 \cdot 9 \\ \hline \end{array} \right.$$

$$\delta = 556 \cdot 5$$

$$\tau = \frac{\delta}{\Delta} = \frac{556 \cdot 5}{534 \cdot 22} = 1 \cdot 042.$$

2. On March 11, 1879, the readings were

$$\text{Thermometer} \begin{Bmatrix} \text{Wet } 42^\circ \\ \text{Dry } 45^\circ \end{Bmatrix} \text{Barometer } 30 \cdot 25 \left\{ \begin{array}{r} 532 \cdot 3 \\ 18 \cdot 3 \\ 3 \cdot 7 \\ 0 \cdot 9 \\ \hline \end{array} \right.$$

$$\delta = 555 \cdot 2$$

$$\tau = \frac{\delta}{\Delta} = \frac{555 \cdot 2}{534 \cdot 22} = 1 \cdot 039$$

TABLE I.

Values of K, Cr, and p.

v	K	$K\left(\frac{v}{1000}\right)^3 = Cr = pg$	p	v	K	$K\left(\frac{v}{1000}\right)^3 = Cr = pg$	p
f/s.			lbs.	f/s.			lbs.
100	1179·51	1·18	0·037	560	105·74	18·6	0·577
110	1032·18	1·37	0·043	580	100·67	19·6	0·610
120	913·80	1·58	0·049	600	96·01	20·7	0·644
130	816·93	1·79	0·056	620	91·70	21·9	0·679
140	736·41	2·02	0·063	640	87·71	23·0	0·714
150	668·61	2·26	0·070	660	84·01	24·2	0·750
160	610·85	2·50	0·078	680	80·57	25·3	0·787
170	561·14	2·76	0·086	700	77·37	26·5	0·824
180	517·99	3·02	0·094	720	74·38	27·8	0·862
190	480·23	3·29	0·102	740	71·58	29·0	0·901
200	446·95	3·58	0·111	760	68·96	30·3	0·940
210	417·44	3·87	0·120	780	66·49	31·6	0·980
220	391·12	4·16	0·129	800	64·18	32·9	1·02
230	367·52	4·47	0·139	820	62·00	34·2	1·06
240	346·26	4·79	0·149	840	59·94	35·5	1·10
250	327·04	5·11	0·159	860	59·94	38·1	1·18
260	309·56	5·44	0·169	880	59·94	40·8	1·27
270	293·63	5·78	0·180	900	59·94	43·7	1·36
280	279·04	6·13	0·190	920	59·94	46·7	1·45
290	265·67	6·48	0·201	940	59·94	49·8	1·55
300	253·36	6·84	0·212	960	59·94	53·0	1·65
310	241·99	7·21	0·224	980	59·94	56·4	1·75
320	231·47	7·58	0·236	1000	59·94	59·9	1·86
330	221·71	7·97	0·248	1020	59·94	63·6	1·98
340	212·63	8·36	0·260	1040	59·94	67·4	2·09
350	204·17	8·75	0·272	1060	64·01	76·2	2·37
360	196·28	9·16	0·284	1080	68·27	86·0	2·67
380	181·97	10·0	0·310	1100	72·73	96·8	3·01
400	169·36	10·8	0·337	1120	77·40	108·7	3·38
420	158·18	11·7	0·364	1140	82·27	121·9	3·79
440	148·21	12·6	0·392	1160	87·36	136·4	4·24
460	139·26	13·6	0·421	1180	92·67	152·3	4·73
480	131·21	14·5	0·451	1190	95·41	160·8	4·99
500	123·92	15·5	0·481	1200	95·41	164·9	5·12
520	117·30	16·5	0·512	1220	95·41	173·3	5·38
540	111·26	17·5	0·544	1240	95·41	181·9	5·65

Table I—*continued.*

v	K	$K\left(\frac{v}{1000}\right)^3 = Cr = pg$	p	v	K	$K\left(\frac{v}{1000}\right)^3 = Cr = pg$	p
f/s.			lbs.	f/s.			lbs.
1260	95·41	190·9	5·93	2280	53·73	636·8	19·78
1280	95·41	200·1	6·22	2300	53·03	645·2	20·04
1300	95·41	209·6	6·51	2320	52·35	653·7	20·30
1320	95·41	219·5	6·82	2340	51·68	662·1	20·57
1340	95·41	229·6	7·13	2360	51·02	670·6	20·83
1360	95·41	240·0	7·45	2380	50·38	679·2	21·10
1380	95·41	250·7	7·79	2400	49·75	687·8	21·36
1400	95·41	261·8	8·13	2420	49·13	696·4	21·63
1420	95·41	273·2	8·49	2440	48·53	705·0	21·90
1440	95·41	284·9	8·85	2460	47·94	713·7	22·17
1460	95·41	296·9	9·22	2480	47·36	722·4	22·44
1480	93·86	304·3	9·45	2500	46·80	731·2	22·71
1500	92·36	311·7	9·68	2520	46·24	740·0	22·99
1520	90·91	319·3	9·92	2540	45·70	748·8	23·26
1540	89·49	326·9	10·15	2560	45·16	757·7	23·54
1560	88·12	334·5	10·39	2580	44·64	766·6	23·81
1580	86·78	342·3	10·63	2600	44·12	775·5	24·09
1600	85·48	350·1	10·88	2620	43·67	785·5	24·40
1620	84·21	358·0	11·12	2640	43·24	795·5	24·71
1640	82·99	366·0	11·37	2660	42·80	805·6	25·03
1660	81·79	374·1	11·62	2680	42·38	815·8	25·34
1680	80·62	382·3	11·87	2700	41·96	826·0	25·66
1700	79·48	390·5	12·13	2720	41·55	836·2	25·98
1720	78·37	398·8	12·39	2740	41·15	846·5	26·30
1740	77·29	407·2	12·65	2760	40·75	856·8	26·62
1760	76·24	415·7	12·91	2780	40·36	867·2	26·94
1780	75·22	424·2	13·18	2800	39·98	877·7	27·26
1800	74·21	432·8	13·44	2820	39·60	888·1	27·59
1820	73·24	441·5	13·72	2840	39·23	898·7	27·92
1840	72·28	450·3	13·99	2860	38·87	909·3	28·25
1860	71·35	459·1	14·26	2880	38·51	919·9	28·58
1880	70·44	468·0	14·54	2900	38·17	930·6	28·91
1900	69·55	477·1	14·82	2920	37·81	941·4	29·24
1920	68·68	486·1	15·10	2940	37·47	952·2	29·58
1940	67·83	495·3	15·39	2960	37·13	963·0	29·92
1960	67·00	504·5	15·67	2980	36·80	973·9	30·25
1980	66·19	513·8	15·96	3000	36·48	984·8	30·59
2000	65·40	523·2	16·25	3020	36·15	995·8	30·93
2020	64·43	531·1	16·50	3040	35·84	1007	31·28
2040	63·49	539·0	16·74	3060	35·53	1018	31·62
2060	62·56	546·9	16·99	3080	35·22	1029	31·97
2080	61·66	554·9	17·24	3100	34·92	1040	32·32
2100	60·78	562·9	17·49	3120	34·62	1052	32·66
2120	59·93	571·0	17·74	3140	34·33	1063	33·01
2140	59·09	579·1	17·99	3160	34·04	1074	33·37
2160	58·27	587·2	18·24	3180	33·76	1086	33·72
2180	57·47	595·4	18·49	3200	33·48	1097	34·07
2200	56·69	603·6	18·75	3220	33·20	1108	34·43
2220	55·92	611·9	19·01	3240	32·93	1120	34·79
2240	55·18	620·1	19·26	3260	32·66	1132	35·15
2260	54·44	628·5	19·52	3280	32·39	1143	35·51

Table I—*continued.*

v	K	$K\left(\frac{v}{1000}\right)^3 = Cv = pg$	p
f/s.			lbs.
3300	32 ·13	1155	35 ·87
3320	31 ·88	1166	36 ·24
3340	31 ·62	1178	36 ·60
3360	31 ·37	1190	36 ·97
3380	31 ·13	1202	37 ·34
3400	30 ·88	1214	37 ·71
3420	30 ·64	1226	38 ·08
3440	30 ·40	1238	38 ·45
3460	30 ·17	1250	38 ·82
3480	29 ·94	1262	39 ·20
3500	29 ·71	1274	39 ·58
3520	29 ·49	1286	39 ·95
3540	29 ·27	1298	40 ·33
3560	29 ·05	1311	40 ·72
3580	28 ·83	1323	41 ·10
3600	28 ·62	1335	41 ·48
3620	28 ·41	1348	41 ·87
3640	28 ·20	1360	42 ·25
3660	28 ·00	1373	42 ·64
3680	27 ·80	1385	43 ·03
3700	27 ·60	1398	43 ·43
3720	27 ·40	1411	43 ·82
3740	27 ·21	1423	44 ·21
3760	27 ·01	1436	44 ·61
3780	26 ·82	1449	45 ·00
3800	26 ·64	1462	45 ·40
3820	26 ·45	1474	45 ·80
3840	26 ·27	1487	46 ·20
3860	26 ·09	1500	46 ·61
3880	25 ·91	1513	47 ·01
3900	25 ·73	1526	47 ·42
3920	25 ·56	1539	47 ·82
3940	25 ·38	1553	48 ·23
3960	25 ·21	1566	48 ·64
3980	25 ·05	1579	49 ·05
4000	24 ·88	1592	49 ·46

TABLE II.

Time t in seconds, between velocity V and v f/s. $t = \mathrm{C}\,[\mathrm{T}(\mathrm{V}) - \mathrm{T}(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ.
f/s.											+
10	0 ·186	1 ·027	1 ·855	2 ·670	3 ·472	4 ·262	5 ·040	5 ·807	6 ·562	7 ·306	0 ·790
11	8 ·040	8 ·762	9 ·474	10 ·177	10 ·869	11 ·552	12 ·225	12 ·889	13 ·544	14 ·190	·683
12	14 ·828	15 ·457	16 ·077	16 ·690	17 ·295	17 ·892	18 ·482	19 ·064	19 ·639	20 ·206	·597
13	20 ·767	21 ·321	21 ·868	22 ·408	22 ·942	23 ·470	23 ·991	24 ·507	25 ·016	25 ·519	·528
14	26 ·017	26 ·509	26 ·996	27 ·477	27 ·952	28 ·423	28 ·888	29 ·348	29 ·804	30 ·254	·471
15	30 ·699	31 ·140	31 ·576	32 ·008	32 ·435	32 ·857	33 ·276	33 ·690	34 ·100	34 ·506	·422
16	34 ·908	35 ·305	35 ·699	36 ·089	36 ·475	36 ·857	37 ·236	37 ·611	37 ·982	38 ·350	·382
17	38 ·715	39 ·076	39 ·433	39 ·788	40 ·139	40 ·487	40 ·831	41 ·173	41 ·512	41 ·847	·348
18	42 ·179	42 ·509	42 ·836	43 ·160	43 ·481	43 ·799	44 ·114	44 ·427	44 ·737	45 ·044	·318
19	45 ·349	45 ·652	45 ·952	46 ·249	46 ·544	46 ·836	47 ·126	47 ·414	47 ·699	47 ·982	·292
20	48 ·263	48 ·542	48 ·818	49 ·092	49 ·364	49 ·634	49 ·902	50 ·168	50 ·431	50 ·693	·270
21	50 ·953	51 ·210	51 ·466	51 ·720	51 ·972	52 ·222	52 ·470	52 ·716	52 ·961	53 ·204	·250
22	53 ·445	53 ·684	53 ·921	54 ·157	54 ·391	54 ·624	54 ·855	55 ·084	55 ·311	55 ·537	·233
23	55 ·762	55 ·985	56 ·206	56 ·426	56 ·644	56 ·861	57 ·076	57 ·290	57 ·503	57 ·714	·217
24	57 ·923	58 ·131	58 ·338	58 ·544	58 ·748	58 ·951	59 ·152	59 ·352	59 ·551	59 ·749	·203
25	59 ·945	60 ·140	60 ·334	66 ·527	60 ·718	60 ·908	61 ·097	61 ·285	61 ·472	61 ·657	·190
26	61 ·842	62 ·025	62 ·207	62 ·388	62 ·568	62 ·747	62 ·924	63 ·101	63 ·277	63 ·451	·179
27	63 ·625	63 ·797	63 ·969	64 ·139	64 ·309	64 ·477	64 ·644	64 ·811	64 ·977	65 ·142	·168
28	65 ·305	65 ·468	65 ·630	65 ·791	65 ·951	66 ·110	66 ·268	66 ·425	66 ·582	66 ·738	·159
29	66 ·893	67 ·047	67 ·200	67 ·352	67 ·503	67 ·654	67 ·804	67 ·953	68 ·101	68 ·248	·151
30	68 ·394	68 ·540	68 ·685	68 ·829	68 ·973	69 ·116	69 ·258	69 ·399	69 ·540	69 ·680	·143
31	69 ·819	69 ·957	70 ·095	70 ·232	70 ·368	70 ·503	70 ·638	70 ·772	70 ·906	71 ·039	·135
32	71 ·171	71 ·302	71 ·433	71 ·563	71 ·693	71 ·822	71 ·950	72 ·078	72 ·205	72 ·331	·129
33	72 ·457	72 ·582	72 ·707	72 ·831	72 ·954	73 ·077	73 ·199	73 ·321	73 ·442	73 ·563	·123

34	7 3·683	3·802	3·921	4·039	4·157	4·274	4·391	4·507	4·622	4·737	0·117
35	4·852	4·966	5·079	5·192	5·304	5·416	5·528	5·639	5·749	5·859	·112
36	5·969	6·078	6·186	6·294	6·402	6·509	6·615	6·721	6·827	6·932	·107
37	7 7·037	7·141	7·244	7·348	7·451	7·554	7·656	7·758	7·859	7·960	·103
38	8·060	8·160	8·260	8·359	8·457	8·555	8·653	8·751	8·848	8·945	·098
39	9·041	9·137	9·232	9·327	9·422	9·516	9·610	9·704	9·797	9·890	·094
40	7 9·982	*0·074	*0·166	*0·257	*0·348	*0·439	*0·529	*0·619	*0·709	*0·798	·091
41	8 0·887	0·975	1·063	1·151	1·239	1·326	1·413	1·499	1·585	1·671	·087
42	1·757	1·842	1·927	2·011	2·095	2·179	2·263	2·346	2·429	2·512	·084
43	8 2·594	2·676	2·758	2·839	2·920	3·001	3·082	3·162	3·242	3·321	·081
44	3·401	3·480	3·559	3·637	3·715	3·793	3·871	3·948	4·025	4·102	·078
45	4·179	4·255	4·331	4·407	4·482	4·557	4·632	4·707	4·782	4·856	·075
46	8 4·930	5·003	5·077	5·150	5·223	5·295	5·368	5·440	5·512	5·583	·072
47	5·655	5·726	5·797	5·867	5·938	6·008	6·078	6·148	6·217	6·287	·070
48	6·356	6·424	6·493	6·561	6·629	6·697	6·765	6·832	6·900	6·967	·068
49	8 7·033	7·100	7·166	7·232	7·298	7·364	7·430	7·495	7·560	7·625	·066
50	7·689	7·754	7·818	7·882	7·946	8·010	8·073	8·136	8·199	8·262	·064
51	8·325	8·387	8·450	8·512	8·574	8·635	8·697	8·758	8·819	8·880	·061
52	8 8·941	9·001	9·062	9·122	9·182	9·242	9·301	9·361	9·420	9·479	·060
53	9·538	9·597	9·655	9·713	9·772	9·830	9·888	9·945	*0·003	*0·060	·058
54	9 0·117	0·174	0·231	0·288	0·344	0·401	0·457	0·513	0·568	0·624	·057
55	9 0·680	0·735	0·790	0·845	0·900	0·955	1·009	1·064	1·118	1·172	·055
56	1·226	1·280	1·333	1·387	1·440	1·493	1·546	1·599	1·652	1·705	·053
57	1·757	1·809	1·861	1·913	1·965	2·017	2·068	2·120	2·171	2·222	·052
58	9 2·273	2·324	2·375	2·425	2·476	2·526	2·576	2·626	2·676	2·726	·050
59	2·775	2·825	2·874	2·923	2·972	3·021	3·070	3·119	3·167	3·216	·049
60	3·264	3·312	3·360	3·408	3·456	3·504	3·551	3·599	3·646	3·693	·048
61	9 3·740	3·787	3·834	3·881	3·927	3·974	4·020	4·066	4·112	4·158	·047
62	4·204	4·250	4·295	4·341	4·386	4·431	4·476	4·521	4·566	4·611	·045
63	4·655	4·700	4·744	4·789	4·833	4·877	4·921	4·965	5·009	5·053	·044
64	9 5·096	5·140	5·183	5·226	5·269	5·312	5·355	5·398	5·440	5·483	·043
65	5·525	5·568	5·610	5·652	5·694	5·736	5·778	5·820	5·861	5·903	·042
66	5·944	5·986	6·027	6·068	6·109	6·150	6·191	6·232	6·272	6·313	·040
67	9 6·354	6·394	6·434	6·474	6·514	6·554	6·594	6·634	6·674	6·714	·040
68	6·753	6·793	6·832	6·871	6·910	6·949	6·988	7·027	7·066	7·105	·039
69	7·143	7·182	7·220	7·259	7·297	7·335	7·373	7·411	7·449	7·487	·037

Table II—*continued.*

$t = C\,[T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
70	9 7 ·524	7 ·562	7 ·599	7 ·637	7 ·674	7 ·712	7 ·749	7 ·786	7 ·823	7 ·860	0 ·037
71	7 ·897	7 ·934	7 ·970	8 ·007	8 ·044	8 ·080	8 ·116	8 ·153	8 ·189	8 ·225	·036
72	8 ·261	8 ·297	8 ·333	8 ·369	8 ·405	8 ·441	8 ·476	8 ·512	8 ·547	8 ·582	·036
73	9 8 ·617	8 ·653	8 ·688	8 ·723	8 ·758	8 ·793	8 ·828	8 ·863	8 ·897	8 ·932	·035
74	8 ·966	9 ·001	9 ·035	9 ·069	9 ·103	9 ·137	9 ·171	9 ·205	9 ·239	9 ·273	·034
75	9 ·307	9 ·341	9 ·374	9 ·408	9 ·441	9 ·475	9 ·508	9 ·542	9 ·575	9 ·608	·034
76	9 9 ·641	9 ·674	9 ·707	9 ·740	9 ·772	9 ·805	9 ·838	9 ·871	9 ·903	9 ·936	·033
77	9 ·968	*0 ·000	*0 ·032	*0 ·065	*0 ·097	*0 ·129	*0 ·161	*0 ·193	*0 ·225	*0 ·257	·032
78	10 0 ·288	0 ·320	0 ·351	0 ·383	0 ·414	0 ·446	0 ·477	0 ·508	0 ·539	0 ·571	·032
79	10 0 ·602	0 ·633	0 ·664	0 ·695	0 ·725	0 ·756	0 ·787	0 ·818	0 ·848	0 ·879	·031
80	0 ·909	0 ·940	0 ·970	1 ·000	1 ·030	1 ·061	1 ·091	1 ·121	1 ·151	1 ·181	·031
81	1 ·210	1 ·240	1 ·270	1 ·300	1 ·329	1 ·359	1 ·388	1 ·418	1 ·447	1 ·477	·030
82	10 1 ·506	1 ·535	1 ·564	1 ·593	1 ·622	1 ·651	1 ·680	1 ·709	1 ·738	1 ·767	·029
83	1 ·796	1 ·825	1 ·853	1 ·882	1 ·910	1 ·939	1 ·967	1 ·995	2 ·023	2 ·052	·029
84	20 ·80	2 ·108	2 ·136	2 ·164	2 ·192	2 ·220	2 ·247	2 ·275	2 ·302	2 ·329	·028
85	10 2 ·356	2 ·384	2 ·411	2 ·438	2 ·464	2 ·491	2 ·518	2 ·545	2 ·571	2 ·597	·027
86	2 ·623	2 ·650	2 ·676	2 ·702	2 ·728	2 ·754	2 ·779	2 ·805	2 ·830	2 ·856	·026
87	2 ·881	2 ·907	2 ·932	2 ·957	2 ·982	3 ·007	3 ·032	3 ·056	3 ·081	3 ·105	·025
88	10 3 ·130	3 ·155	3 ·179	3 ·203	3 ·227	3 ·252	3 ·276	3 ·300	3 ·323	3 ·347	·025
89	3 ·371	3 ·394	3 ·418	3 ·442	3 ·465	3 ·488	3 ·511	3 ·535	3 ·558	3 ·581	·023
90	3 ·604	3 ·627	3 ·649	3 ·672	3 ·695	3 ·718	3 ·740	3 ·762	3 ·784	3 ·807	·023
91	10 3 ·829	3 ·851	3 ·873	3 ·895	3 ·917	3 ·939	3 ·960	3 ·982	4 ·004	4 ·026	·022
92	4 ·047	4 ·068	4 ·089	4 ·111	4 ·132	4 ·153	4 ·174	4 ·195	4 ·216	4 ·237	·021
93	4 ·257	4 ·278	4 ·299	4 ·320	4 ·340	4 ·361	4 ·381	4 ·401	4 ·421	4 ·441	·021
94	10 4 ·461	4 ·482	4 ·502	4 ·522	4 ·541	4 ·561	4 ·581	4 ·601	4 ·620	4 ·640	·020
95	4 ·659	4 ·679	4 ·698	4 ·718	4 ·737	4 ·756	4 ·775	4 ·794	4 ·813	4 ·832	·019
96	4 ·851	4 ·870	4 ·888	4 ·907	4 ·926	4 ·945	4 ·963	4 ·982	5 ·000	5 ·018	·019
97	10 5 ·036	5 ·055	5 ·073	5 ·091	5 ·109	5 ·127	5 ·145	5 ·163	5 ·181	5 ·199	·018
98	5 ·216	5 ·234	5 ·252	5 ·270	5 ·287	5 ·305	5 ·322	5 ·339	5 ·356	5 ·374	·018
99	5 ·391	5 ·408	5 ·425	5 ·442	5 ·459	5 ·476	5 ·493	5 ·510	5 ·527	5 ·544	·017

100	10 5·560	5·577	5·594	5·611	5·627	5·644	5·660	5·676	5·692	5·709	0·017
101	5·725	5·741	5·757	5·773	5·789	5·805	5·821	5·837	5·853	5·869	·016
102	5·884	5·900	5·916	5·932	5·947	5·963	5·978	5·994	6·009	6·024	·016
103	10 6·039	6·055	6·070	6·085	6·100	6·115	6·130	6·145	6·160	6·175	·015
104	6·190	6·205	6·219	6·234	6·248	6·263	6·277	6·291	6·305	6·320	·015
105	6·334	6·348	6·361	6·375	6·389	6·403	6·416	6·429	6·442	6·456	·014
106	10 6·469	6·482	6·495	6·508	6·521	6·534	6·546	6·559	6·571	6·584	·013
107	6·596	6·609	6·621	6·633	6·645	6·657	6·669	6·681	6·692	6·704	·012
108	6·716	6·728	6·739	6·751	6·762	6·773	6·784	6·796	6·807	6·818	·011
109	10 6·829	6·840	6·851	6·862	6·872	6·883	6·893	6·904	6·914	6·925	·011
110	6·935	6·946	6·956	6·966	6·976	6·986	6·996	7·006	7·016	7·026	·010
111	7·035	7·045	7·055	7·065	7·074	7·084	7·093	7·103	7·112	7·121	·010
112	10 7·130	7·139	7·148	7·157	7·166	7·175	7·184	7·193	7·202	7·211	·009
113	7·220	7·229	7·237	7·246	7·254	7·263	7·271	7·279	7·287	7·296	·009
114	7·304	7·312	7·320	7·328	7·336	7·344	7·352	7·360	7·368	7·376	·008
115	10 7·384	7·392	7·399	7·407	7·414	7·422	7·429	7·437	7·444	7·452	·008
116	7·459	7·467	7·474	7·481	7·488	7·495	7·502	7·509	7·516	7·523	·007
117	7·530	7·537	7·544	7·551	7·558	7·565	7·571	7·578	7·585	7·592	·007
118	10 7·598	7·605	7·611	7·618	7·624	7·631	7·637	7·643	7·649	7·656	·007
119	7·662	7·668	7·674	7·681	7·687	7·693	7·699	7·705	7·711	7·717	·006
120	7·723	7·729	7·735	7·741	7·747	7·753	7·759	7·765	7·771	7·777	·006
121	10 7·783	7·789	7·795	7·801	7·807	7·813	7·818	7·824	7·830	7·836	·006
122	7·842	7·848	7·853	7·859	7·865	7·871	7·876	7·882	7·887	7·893	·006
123	7·899	7·905	7·910	7·916	7·921	7·927	7·932	7·938	7·943	7·949	·006
124	10 7·954	7·960	7·965	7·971	7·976	7·982	7·987	7·993	7·998	8·004	·006
125	8·009	8·014	8·019	8·025	8·030	8·036	8·041	8·046	8·051	8·057	·006
126	8·062	8·067	8·072	8·078	8·083	8·088	8·093	8·098	8·103	8·108	·005
127	10 8·113	8·119	8·124	8·129	8·134	8·139	8·144	8·149	8·154	8·159	·005
128	8·164	8·169	8·174	8·179	8·184	8·189	8·194	8·199	8·204	8·209	·005
129	8·213	8 218	8·223	8·228	8·233	8·238	8·242	8·247	8·252	8·257	·005
130	10 8·262	8·267	8·272	8·276	8·281	8·285	8·290	8·295	8·300	8·304	·004
131	8·309	8·314	8·319	8·323	8·328	8·332	8·337	8·342	8·346	8·351	·004
132	8·355	8·360	8·365	8·369	8·374	8·378	8·383	8·387	8·392	8·396	·004
133	10 8·400	8·405	8·409	8·414	8·418	8·422	8·427	8·431	8·436	8·440	·004
134	8·444	8·449	8·453	8·458	8·462	8·466	8·470	8·475	8·479	8·483	·004
135	8·487	8·492	8·496	8·500	8·504	8·508	8·512	8·517	8·521	8·525	·004

Table II—*continued*.

$t = C[T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
136	10 8·529	8·533	8·537	8·542	8·546	8·550	8·554	8·558	8·562	8·566	0·004
137	8·570	8·574	8·579	8·583	8·587	8·591	8·595	8·599	8·603	8·607	·004
138	8·611	8·615	8·619	8·623	8·627	8·631	8·635	8·639	8·643	8·647	·004
139	10 8·650	8·654	8·658	8·662	8·666	8·670	8·674	8·678	8·682	8·685	·004
140	8·689	8·693	8·697	8·700	8·704	8·708	8·712	8·715	8·719	8·723	·004
141	8·727	8·730	8·734	8·738	8·742	8·745	8·749	8·753	8·757	8·760	·004
142	10 8·764	8·768	8·772	8·775	8·779	8·782	8·786	8·790	8·793	8·797	·004
143	8·800	8·804	8·807	8·811	8·814	8·818	8·821	8·825	8·828	8·832	·004
144	8·835	8·839	8·843	8·846	8·850	8·853	8·857	8·860	8·864	8·867	·003
145	10 8·870	8·874	8·877	8·881	8·884	8·887	8·891	8·894	8·897	8·901	·003
146	8·904	8·907	8·911	8·914	8·918	8·921	8·925	8·928	8·931	8·935	·003
147	8·938	8·941	8·945	8·948	8·951	8·954	8·958	8·961	8·964	8·967	·003
148	10 8·971	8·974	8·977	8·980	8·984	8·987	8·990	8·993	8·997	9·000	·003
149	9·003	9·006	9·010	9·013	9·016	9·019	9·023	9·026	9·029	9·033	·003
150	9·036	9·039	9·042	9·045	9·049	9·052	9·055	9·058	9·061	9·064	·003
151	10 9·067	9·071	9·074	9·077	9·080	9·083	9·087	9·090	9·093	9·096	·003
152	9·099	9·102	9·105	9·109	9·112	9·115	9·118	9·121	9·124	9·127	·003
153	9·130	9·133	9 136	9·139	9·142	9·145	9·148	9·152	9·155	9·158	·003
154	10 9·161	9·164	9·167	9·170	9·173	9·176	9·179	9·182	9·185	9·188	·003
155	9·191	9·194	9·197	9·200	9·203	9·206	9·209	9·212	9·215	9·218	·003
156	9·221	9·224	9·227	9·230	9·233	9·236	9·239	9·242	9·245	9·248	·003
157	10 9·251	9·254	9·257	9·260	9·263	9·266	9·269	9·272	9·275	9·278	·003
158	9·280	9·283	9·286	9·289	9·292	9·295	9·298	9·301	9·304	9·306	·003
159	9·309	9·312	9·315	9·318	9·321	9·324	9·327	9·330	9·333	9·335	·003
160	10 9·338	9·341	9·344	9·347	9·350	9·353	9·356	9·359	9·361	9·364	·003
161	9·367	9·370	9·373	9·376	9·379	9·381	9·384	9·387	9·390	9·392	·003
162	9·395	9·398	9·401	9·403	9·406	9·409	9·412	9·414	9·417	9·420	·003
163	10 9·423	9·426	9·428	9·431	9·434	9·437	9·439	9·442	9·445	9·448	·003
164	9·450	9·453	9·456	9·459	9·461	9·464	9·467	9·469	9·472	9·475	·003
165	9·477	9·480	9·483	9·486	9·488	9·491	9·494	9·496	9·499	9·502	·003

166	10 9·504	9·507	9·510	9·513	9·515	9·518	9·521	9·523	9·526	9·529	0·003
167	9·531	9·534	9·537	9·539	9·542	9·545	9·547	9·550	9·552	9·554	·003
168	9·557	9·559	9·562	9·565	9·567	9·570	9·573	9·575	9·578	9·581	·003
169	10 9·583	9·586	9·589	9·591	9·594	9·597	9·599	9·602	9·604	9·606	·003
170	9·609	9·611	9·614	9·616	9·619	9·621	9·624	9·626	9·629	9·631	·002
171	9·634	9·636	9·639	9·641	9·644	9·646	9·649	9·651	9·654	9·656	·002
172	10 9·659	9·661	9·664	9·666	9·669	9·671	9·674	9·676	9·679	9·681	·002
173	9·684	9·686	9·689	9·691	9·694	9·696	9·699	9·701	9·704	9·706	·002
174	9·709	9·711	9·714	9·716	9·719	9·721	9·724	9·726	9·729	9·731	·002
175	10 9·733	9·736	9·738	9·741	9·743	9·746	9·748	9·751	9·753	9·756	·002
176	9·758	9·761	9·763	9·766	9·768	9·770	9·773	9·775	9·777	9·780	·002
177	9·782	9·784	9·787	9·789	9·791	9·794	9·796	9·798	9·801	9·803	·002
178	10 9·805	9·808	9·810	9·812	9·815	9·817	9·819	9·822	9·824	9·826	·002
179	9·829	9·831	9·833	9·836	9·838	9·840	9·843	9·845	9·847	9·850	·002
180	9·852	9·854	9·857	9·859	9·861	9·864	9·866	9·868	9·871	9·873	·002
181	10 9·875	9·878	9·880	9·882	9·885	9·887	9·889	9·891	9·894	9·896	·002
182	9·898	9·900	9·903	9·905	9·907	9·909	9·912	9·914	9·916	9·918	·002
183	9·920	9·923	9·925	9·927	9·929	9·931	9·934	9·936	9·938	9·940	·002
184	10 9·942	9·945	9·947	9·949	9·951	9·954	9·956	9·958	9·960	9·963	·002
185	9·965	9·967	9·969	9·972	9·974	9·976	9·978	9·980	9·983	9·985	·002
186	9·987	9·989	9·991	9·993	9·996	9·998	*0·000	*0·002	*0·004	*0·006	·002
187	11 0·008	0·011	0·013	0·015	0·017	0·019	0·021	0·023	0·025	0·028	·002
188	0·030	0·032	0·034	0·036	0·038	0·040	0·042	0·045	0·047	0·049	·002
189	0·051	0·053	0·055	0·057	0·059	0·062	0·064	0·066	0·068	0·070	·002
190	11 0·072	0·074	0·076	0·078	0·080	0·083	0·085	0·087	0·089	0·091	·002
191	0·093	0·095	0·097	0·099	0·101	0·103	0·105	0·107	0·109	0·111	·002
192	0·113	0·115	0·117	0·119	0·121	0·124	0·126	0·128	0·130	0·132	·002
193	11 0·134	0·136	0·138	0·140	0·142	0·144	0·146	0·148	0·150	0·152	·002
194	0·154	0·156	0·158	0·160	0·162	0·164	0·166	0·168	0·170	0·172	·002
195	0·174	0·176	0·178	0·180	0·182	0·184	0·186	0·188	0·190	0·192	·002
196	11 0·194	0·196	0·198	0·200	0·202	0·204	0·206	0·208	0·210	0·212	·002
197	0·214	0·216	0·218	0·220	0·222	0·224	0·226	0·228	0·230	0·232	·002
198	0·234	0·236	0·238	0·240	0·242	0·244	0·246	0·248	0·250	0·252	·002
199	11 0·253	0·255	0·257	0·259	0·261	0·263	0·265	0·267	0·269	0·271	·002
200	0·272	0·274	0·276	0·278	0·280	0·282	0·284	0·286	0·288	0·290	·002
201	0·291	0·293	0·295	0·297	0·299	0·301	0·303	0·305	0·307	0·308	·002

Table II—*continued.*

$t = C[T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
202	11 0·310	0·312	0·314	0·316	0·318	0·320	0·322	0·323	0·325	0·327	0·002
203	0·329	0·331	0·333	0·335	0·336	0·338	0·340	0·342	0·344	0·346	·002
204	0·347	0·349	0·351	0·353	0·355	0·357	0·359	0·361	0·362	0·364	·002
205	11 0·366	0·368	0·370	0·372	0·374	0·375	0·377	0·379	0·381	0·383	·002
206	0·384	0·386	0·388	0·390	0·392	0·393	0·395	0·397	0·399	0·401	·002
207	0·402	0·404	0·406	0·408	0·410	0·412	0·414	0·415	0·417	0·419	·002
208	11 0·421	0·423	0·425	0·427	0·429	0·430	0·432	0·434	0·436	0·438	·002
209	0·439	0·441	0·443	0·444	0. 446	0·448	0·449	0·451	0·453	0·455	·002
210	0·456	0·458	0·460	0·462	0·464	0·465	0·467	0·469	0·471	0·473	·002
211	11 0·474	0·476	0·478	0·480	0·482	0·483	0·485	0·487	0·489	0·491	·002
212	0·492	0·494	0·496	0·498	0·499	0·501	0·503	0·504	0·506	0·508	·002
213	0·509	0·511	0·513	0·515	0·516	0·518	0·520	0·521	0·523	0·525	·002
214	11 0·526	0·528	0·530	0 532	0·534	0·535	0·537	0·539	0·541	0·542	·002
215	0·544	0·546	0·548	0 549	0·551	0·553	0·554	0·556	0·558	0·559	·002
216	0·561	0·563	0·564	0·566	0·568	0·569	0·571	0·573	0·574	0·576	·002
217	11 0·578	0·580	0·581	0·583	0·585	0·587	0·588	0·590	0·592	0·594	·002
218	0·595	0·597	0·599	0·601	0·602	0·604	0·605	0·607	0·608	0·610	·002
219	0·611	0·613	0·614	0·616	0·618	0·620	0·621	0·623	0·625	0·627	·002
220	11 0·628	0·630	0·632	0·634	0·635	0·637	0·638	0·640	0·641	0·643	·002
221	0·644	0·646	0·648	0·650	0·651	0·653	0·654	0·656	0·658	0·660	·002
222	0·661	0·663	0·665	0·667	0·668	0·670	0·671	0·673	0·674	0·676	·002
223	11 0·677	0·679	0·681	0·683	0·684	0·686	0·687	0·689	0·690	0·692	·002
224	0·693	0·695	0·697	0·699	0·700	0·702	0·703	0·705	0·706	0·708	·002
225	0·709	0·711	0·713	0·715	0·716	0·718	0·719	0·721	0·722	0·724	·002
226	11 0·725	0·727	0·728	0·730	0·731	0·733	0·735	0·736	0·738	0·740	·002
227	0·741	0·743	0·745	0·746	0·748	0·749	0·751	0·752	0·754	0·755	·002
228	0·757	0·758	0·760	0·762	0·763	0·765	0·767	0·768	0·770	0·772	·002
229	11 0·773	0·775	0·776	0·778	0·779	0·781	0·782	0·784	0·785	0·787	·002
230	0·788	0·790	0·792	0·794	0·795	0·797	0·798	0·800	0·801	0·803	·002
231	0·804	0·806	0·807	0·809	0·810	0·812	0·813	0·815	0·816	0·818	·002

232	11 0·819	0·821	0·822	0·824	0·825	0·827	0·828	0·830	0·831	0·833	0·002
233	0·834	0·836	0·837	0·839	0·840	0·842	0·843	0·845	0·846	0·848	·002
234	0·849	0·851	0·852	0·854	0·855	0·857	0·858	0·860	0·861	0·863	·002
235	11 0·864	0·866	0·867	0·869	0·870	0·872	0·873	0·875	0·876	0·878	·002
236	0·879	0·881	0·882	0·884	0·885	0·887	0·888	0·890	0·891	0·893	·002
237	0·894	0·896	0·897	0·899	0·900	0·902	0·903	0·905	0·906	0·908	·002
238	11 0·909	0·911	0·912	0·914	0·915	0·917	0·918	0·920	0·921	0·923	·002
239	0·924	0·926	0·927	0·929	0·930	0·932	0·933	0·934	0·936	0·937	·002
240	0·938	0·940	0·941	0·943	0·944	0·946	0·947	0·949	0·950	0·952	·002
241	11 0·953	0·955	0·956	0·958	0·959	0·961	0·962	0·963	0·965	0·966	·001
242	0·967	0·969	0·970	0·972	0·973	0·974	0·976	0·977	0·978	0·980	·001
243	0·981	0·982	0·984	0·985	0·987	0·988	0·990	0·991	0·993	0·994	·001
244	11 0·996	0·997	0·999	1·000	1·002	1·003	1·005	1·006	1·007	1·009	·001
245	1·010	1·011	1·013	1·014	1·016	1·017	1·019	1·020	1·022	1·023	·001
246	1·024	1·026	1·027	1·029	1·030	1·032	1·033	1·034	1·036	1·037	·001
247	11 1·038	1·040	1·041	1·042	1·044	1·045	1·046	1·048	1·049	1·050	·001
248	1·052	1·053	1·054	1·056	1·057	1·058	1·060	1·061	1·062	1·064	·001
249	1·065	1·066	1·068	1·069	1·071	1·072	1·074	1·075	1·077	1·078	·001
250	11 1·079	1·081	1·082	1·084	1·085	1·086	1·088	1·089	1·090	1·092	·001
251	1·093	1·094	1·096	1·097	1·098	1·100	1·101	1·102	1·104	1·105	·001
252	1·106	1·108	1·109	1·111	1·112	1·113	1·115	1·116	1·117	1·119	·001
253	11 1·120	1·121	1·123	1·124	1·125	1·127	1·128	1·129	1·131	1·132	·001
254	1·133	1·135	1·136	1·137	1·139	1·140	1·141	1·143	1·144	1·145	·001
255	1·147	1·148	1·149	1·151	1·152	1·153	1·155	1·156	1·157	1·159	·001
256	11 1·160	1·161	1·162	1·164	1·165	1·166	1·168	1·169	1·170	1·172	·001
257	1·173	1·174	1·175	1·177	1·178	1·179	1·181	1·182	1·183	1·185	·001
258	1·186	1·187	1·188	1·190	1·191	1·192	1·194	1·195	1·196	1·198	·001
259	11 1·199	1·200	1·201	1·203	1·204	1·205	1·207	1·208	1·209	1·211	·001
260	1·212	1·213	1·215	1·216	1·217	1·219	1·220	1·221	1·223	1·224	·001
261	1·225	1·227	1·228	1·229	1·230	1·232	1·233	1·234	1·235	1·237	·001
262	11 1·238	1·239	1·240	1·242	1·243	1·244	1·245	1·247	1·248	1·249	·001
263	1·250	1·252	1·253	1·254	1·256	1·257	1·258	1·259	1·261	1·262	·001
264	1·263	1·264	1·266	1·267	1·268	1·269	1·271	1·272	1·273	1·274	·001
265	11 1·276	1·277	1·278	1·279	1·281	1·282	1·283	1·284	1·286	1·287	·001
266	1·288	1·289	1·291	1·292	1·293	1·294	1·295	1·297	1·298	1·299	·001
267	1·300	1·301	1·303	1·304	1·305	1·306	1·308	1·309	1·310	1·311	·001

Table II—*continued.*

$t = C\,[T(V) - T(v)]$.

f/s.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
268	11 1·313	1·314	1·315	1·316	1·317	1·319	1·320	1·321	1·322	1·323	+ 0·001
269	1·325	1·326	1·327	1·328	1·329	1·331	1·332	1·333	1·334	1·335	·001
270	1·337	1·338	1·339	1·340	1·341	1·343	1·344	1·345	1·346	1·347	·001
271	11 1·349	1·350	1·351	1·352	1·353	1·355	1·356	1·357	1·358	1·359	·001
272	1·361	1·362	1·363	1·364	1·365	1·367	1·368	1·369	1·370	1·371	·001
273	1·373	1·374	1·375	1·376	1·377	1·379	1·380	1·381	1·382	1·383	·001
274	11 1·385	1·386	1·387	1·388	1·389	1·391	1·392	1·393	1·394	1·395	·001
275	1·397	1·398	1·399	1·400	1·401	1·402	1·404	1·405	1·406	1·407	·001
276	1·408	1·410	1·411	1·412	1·413	1·414	1·415	1·417	1·418	1·419	·001
277	11 1·420	1·421	1·422	1·424	1·425	1·426	1·427	1·428	1·429	1·430	·001
278	1·432	1·433	1·434	1·435	1·436	1·437	1·438	1·439	1·441	1·442	·001
279	1·443	1·444	1·445	1·446	1·447	1·448	1·450	1·451	1·452	1·453	·001
280	11 1·454	1·455	1·456	1·458	1·459	1·460	1·461	1·462	1·463	1·464	·001
281	1·466	1·467	1·468	1·469	1·470	1·471	1·472	1·474	1·475	1·476	·001
282	1·477	1·478	1·479	1·480	1·482	1·483	1·484	1·485	1·486	1·487	·001
283	11 1·488	1·490	1·491	1·492	1·493	1·494	1·495	1·496	1·498	1·499	·001
284	1·500	1·501	1·502	1·503	1·504	1·505	1·507	1·508	1·509	1·510	·001
285	1·511	1·512	1·513	1·514	1·515	1·516	1·518	1·519	1·520	1·521	·001
286	11 1·522	1·523	1·524	1·525	1·526	1·527	1·529	1·530	1·531	1·532	·001
287	1·533	1·534	1·535	1·536	1·537	1·538	1·540	1·541	1·542	1·543	·001
288	1·544	1·545	1·546	1·547	1·548	1·549	1·550	1·551	1·552	1·553	·001
289	11 1·554	1·556	1·557	1·558	1·559	1·560	1·561	1·562	1·563	1·564	·001
290	1·565	1·566	1·567	1·568	1·569	1·570	1·572	1·573	1·574	1·575	·001
291	1·576	1·577	1·578	1·579	1·580	1·581	1·582	1·583	1·584	1·585	·001
292	11 1·586	1·588	1·589	1·590	1·591	1·592	1·593	1·594	1·595	1·596	·001
293	1·597	1·598	1·599	1·600	1·601	1·602	1·604	1·605	1·606	1·607	·001
294	1·608	1·609	1·610	1·611	1·612	1·613	1·614	1·615	1·616	1·617	·001
295	11 1·618	1·620	1·621	1·622	1·623	1·624	1·625	1·626	1·627	1·628	·001
296	1·629	1·630	1·631	1·632	1·633	1·634	1·635	1·636	1·637	1·638	·001
297	1·639	1·640	1·641	1·642	1·643	1·644	1·645	1·646	1·647	1·648	·001

298	11 1·649	1·650	1·651	1·652	1·653	1·654	1·655	1·656	1·657	1·658	·001
299	1·659	1·661	1·662	1·663	1·664	1·665	1·666	1·667	1·668	1·669	·001
300	1·670	1·671	1·672	1·673	1·674	1·675	1·676	1·677	1·678	1·679	·001
301	11 1·680	1·681	1·682	1·683	1·684	1·685	1·686	1·687	1·688	1·689	·001
302	1·690	1·691	1·692	1·693	1·694	1·695	1·696	1·697	1·698	1·699	·001
303	1·700	1·701	1·702	1·703	1·704	1·705	1·706	1·707	1·708	1·709	·001
304	11 1·710	1·711	1·712	1·713	1·714	1·715	1·716	1·717	1·718	1·719	·001
305	1·720	1·721	1·722	1·723	1·724	1·725	1·726	1·727	1·728	1·729	·001
306	1·730	1·731	1·732	1·733	1·734	1·735	1·736	1·737	1·738	1·739	·001
307	11 1·739	1·740	1·741	1·742	1·743	1·744	1·745	1·746	1·747	1·748	·001
308	1·749	1·750	1·751	1·752	1·753	1·754	1·755	1·756	1·757	1·758	·001
309	1·759	1·760	1·761	1·762	1·763	1·764	1·765	1·766	1·767	1·768	·001
310	11 1·768	1·769	1·770	1·771	1·772	1·773	1·774	1·775	1·776	1·777	·001
311	1·778	1·779	1·780	1·781	1·782	1·783	1·784	1·785	1·786	1·787	·001
312	1·787	1·788	1·789	1·790	1·791	1·792	1·793	1·794	1·795	1·796	·001
313	11 1·797	1·798	1·799	1·800	1·801	1·802	1·803	1·804	1·805	1·806	·001
314	1·806	1·807	1·808	1·809	1·810	1·811	1·812	1·813	1·814	1·815	·001
315	1·816	1·817	1·818	1·819	1·820	1·821	1·822	1·823	1·824	1·825	·001
316	11 1·825	1·826	1·827	1·828	1·829	1·830	1·831	1·832	1·833	1·834	·001
317	1·834	1·835	1·836	1·837	1·838	1·839	1·840	1·841	1·842	1·843	·001
318	1·844	1·845	1·846	1·847	1·848	1·849	1·850	1·851	1·852	1·853	·001
319	11 1·853	1·854	1·855	1·856	1·857	1·858	1·859	1·860	1·861	1·862	·001
320	1·862	1·863	1·864	1·865	1·866	1·867	1·868	1·869	1·870	1·871	·001
321	1·871	1·872	1·873	1·874	1·875	1·876	1·877	1·878	1·879	1·880	·001
322	11 1·880	1·881	1·882	1·883	1·884	1·885	1·886	1·887	1·888	1·888	·001
323	1·889	1·890	1·891	1·892	1·893	1·894	1·895	1·896	1·896	1·897	·001
324	1·898	1·899	1·900	1·901	1·902	1·903	1·904	1·904	1·905	1·906	·001
325	11 1·907	1·908	1·909	1·910	1·911	1·911	1·912	1·913	1·914	1·915	·001
326	1·916	1·917	1·918	1·918	1·919	1·920	1·921	1·922	1·923	1·924	·001
327	1·925	1·925	1·926	1·927	1·928	1·929	1·930	1·931	1·931	1·932	·001
328	11 1·933	1·934	1·935	1·936	1·936	1·937	1·938	1·939	1·940	1·941	·001
329	1·942	1·943	1·944	1·944	1·945	1·946	1·947	1·948	1·949	1·950	·001
330	1·951	1·952	1·952	1·953	1·954	1·955	1·956	1·957	1·958	1·959	·001
331	11 1·959	1·960	1·961	1·962	1·963	1·964	1·965	1·966	1·967	1·967	·001
332	1·968	1·969	1·970	1·971	1·972	1·973	1·974	1·975	1·975	1·976	·001
333	1·977	1·978	1·979	1·980	1·981	1·981	1·982	1·983	1·984	1·985	·001

Table II—*continued.*

$t = C\,[T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s. 334	11 1·985	1·986	1·987	1·988	1·989	1·990	1·991	1·992	1·992	1·993	+ 0·001
335	1·994	1·995	1·996	1·997	1·998	1·998	1·999	2·000	2·001	2·002	·001
336	2·002	2·003	2·004	2·005	2·006	2·006	2·007	2·008	2·009	2·010	·001
337	11 2·010	2·011	2·012	2·013	2·014	2·015	2·016	2·017	2·017	2·018	·001
338	2·019	2·020	2·021	2·022	2·023	2·023	2·024	2·025	2·026	2·027	·001
339	2·027	2·028	2·029	2·030	2·031	2·031	2·032	2·033	2·034	2·035	·001
340	11 2·035	2·036	2·037	2·038	2·039	2·040	2·040	2·041	2·042	2·043	·001
341	2·044	2·044	2·045	2·046	2·047	2·048	2·048	2·049	2·050	2·051	·001
342	2·052	2·052	2·053	2·054	2·055	2·056	2·056	2·057	2·058	2·059	·001
343	11 2·060	2·060	2·061	2·062	2·063	2·064	2·064	2·065	2·066	2·067	·001
344	2·068	2·068	2·069	2·070	2·071	2·072	2·072	2·073	2·074	2·075	·001
345	2·076	2·076	2·077	2·078	2·079	2·080	2·080	2·081	2·082	2·083	·001
346	11 2·084	2·084	2·085	2·086	2·087	2·088	2·088	2·089	2·090	2·091	·001
347	2·092	2·092	2·093	2·094	2·095	2·096	2·096	2·097	2·098	2·099	·001
348	2·100	2·100	2·101	2·102	2·103	2·104	2·104	2·105	2·106	2·107	·001
349	11 2·108	2·108	2·109	2·110	2·111	2·112	2·112	2·113	2·114	2·115	·001
350	2·116	2·116	2·117	2·118	2·119	2·120	2·120	2·121	2·122	2·123	·001
351	2·124	2·124	2·125	2·126	2·127	2·128	2·128	2·129	2·130	2·131	·001
352	11 2·131	2·132	2·133	2·134	2·135	2·135	2·136	2·137	2·138	2·139	·001
353	2·139	2·140	2·141	2·142	2·143	2·143	2·144	2·145	2·146	2·147	·001
354	2·147	2·148	2·149	2·150	2·150	2·151	2·152	2·153	2·153	2·154	·001
355	11 2·155	2·156	2·156	2·157	2·158	2·159	2·159	2·160	2·161	2·162	·001
356	2·162	2·163	2·164	2·165	2·165	2·166	2·167	2·168	2·168	2·169	·001
357	2·170	2·171	2·171	2·172	2·173	2·174	2·174	2·175	2·176	2·177	·001
358	11 2·177	2·178	2·179	2·180	2·180	2·181	2·182	2·183	2·183	2·184	·001
359	2·185	2·186	2·186	2·187	2·188	2·189	2·189	2·190	2·191	2·192	·001
360	2·192	2·193	2·194	2·195	2·196	2·196	2·197	2·198	2·199	2·200	·001

361	11 2·200	2·201	2·202	2·203	2·203	2·204	2·205	2·205	2·206	2·207	·001
362	2·207	2·208	2·209	2·210	2·211	2·211	2·212	2·213	2·214	2·215	·001
363	2·215	2·216	2·217	2·218	2·218	2·219	2·220	2·220	2·221	2·222	·001
364	11 2·222	2·223	2·224	2·225	2·225	2·226	2·227	2·227	2·228	2·229	·001
365	2·229	2·230	2·231	2·232	2·232	2·233	2·234	2·235	2·235	2·236	·001
366	2·237	2·238	2·239	2·240	2·240	2·241	2·242	2·242	2·243	2·244	·001
367	11 2·244	2·245	2·246	2·247	2·247	2·248	2·249	2·249	2·250	2·251	·001
368	2·251	2·252	2·253	2·254	2·254	2·255	2·256	2·256	2·257	2·258	·001
369	2·258	2·259	2·260	2·261	2·262	2·262	2·263	2·264	2·265	2·266	·001
370	11 2·266	2·267	2·268	2·269	2·269	2·270	2·271	2·271	2·272	2·273	·001
371	2·273	2·274	2·275	2·276	2·276	2·277	2·278	2·278	2·279	2·280	·001
372	2·280	2·281	2·282	2·283	2·283	2·284	2·285	2·285	2·286	2·287	·001
373	11 2·287	2·288	2·289	2·290	2·290	2·291	2·292	2·292	2·293	2·294	·001
374	2·294	2·295	2·296	2·296	2·297	2·298	2·299	2·299	2·300	2·301	·001
375	2·301	2·302	2·303	2·304	2·304	2·305	2·306	2·306	2·307	2·308	·001
376	11 2·308	2·309	2·310	2·311	2·311	2·312	2·313	2·313	2·314	2·315	·001
377	2·315	2·316	2·317	2·318	2·319	2·319	2·320	2·321	2·321	2·322	·001
378	2·322	2·323	2·324	2·325	2·325	2·326	2·327	2·327	2·328	2·329	·001
379	11 2·329	2·330	2·331	2·331	2·332	2·333	2·333	2·334	2·335	2·335	·001
380	2·336	2·336	2·337	2·338	2·338	2·339	2·340	2·340	2·341	2·342	·001
381	2·342	2·343	2·344	2·345	2·345	2·346	2·347	2·347	2·348	2·349	·001
382	11 2·349	2·350	2·351	2·351	2·352	2·353	2·353	2·354	2·355	2·355	·001
383	2·356	2·357	2·357	2·358	2·359	2·359	2·360	2·361	2·361	2·362	·001
384	2·363	2·363	2·364	2·365	2·365	2·366	2·367	2·367	2·368	2·369	·001
385	11 2·369	2·370	2·371	2·371	2·372	2·373	2·373	2·374	2·375	2·375	·001
386	2·376	2·377	2·377	2·378	2·379	2·379	2·380	2·381	2·381	2·382	·001
387	2·383	2·383	2·384	2·385	2·385	2·386	2·387	2·387	2·388	2·389	·001
388	11 2·389	2·390	2·391	2·391	2·392	2·393	2·393	2·394	2·395	2·395	·001
389	2·396	2·397	2·397	2·398	2·399	2·399	2·400	2·401	2·401	2·402	·001
390	2·402	2·403	2·404	2·404	2·405	2·406	2·406	2·407	2·408	2·408	·001
391	11 2·409	2·410	2·411	2·411	2·412	2·413	2·413	2·414	2·415	2·415	·001
392	2·416	2·417	2·417	2·418	2·419	2·419	2·420	2·421	2·421	2·422	·001
393	2·422	2·423	2·423	2·424	2·425	2·425	2·426	2·427	2·427	2·428	·001
394	11 2·428	2·429	2·430	2·430	2·431	2·432	2·432	2·433	2·434	2·434	·001
395	2·435	2·436	2·436	2·437	2·438	2·438	2·439	2·440	2·440	2·441	·001
396	2·441	2·442	2·443	2·443	2·444	2·445	2·445	2·446	2·447	2·447	·001

Table II—*continued.*

$t = C[T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
397	11 2·448	2·449	2·449	2·450	2·451	2·451	2·452	2·453	2·453	2·454	0·001
398	2·454	2·455	2·455	2·456	2·457	2·457	2·458	2·459	2·459	2·460	·001
399	2·460	2·461	2·462	2·462	2·463	2·464	2·464	2·465	2·466	2 466	·001
400	11 2·467										

TABLE III.

Distance s in feet, between velocity V and v f/s. $s = C\,[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
10	3571 ·5	3656 ·1	3740 ·1	3823 ·6	3906 ·7	3989 ·2	4071 ·3	4153 ·0	4234 ·2	4314 ·9	82 ·6
11	4395 ·2	4475 ·0	4554 ·4	4633 ·4	4712 ·0	4790 ·2	4867 ·9	4945 ·3	5022 ·3	5098 ·8	78 ·2
12	5175 ·0	5250 ·8	5326 ·3	5401 ·3	5476 ·0	5550 ·4	5624 ·3	5698 ·0	5771 ·2	5844 ·2	74 ·4
13	5916 ·8	5989 ·0	6061 ·0	6132 ·6	6203 ·9	6274 ·8	6345 ·5	6415 ·8	6485 ·9	6555 ·6	71 ·0
14	6625 ·0	6694 ·2	6763 ·0	6831 ·6	6899 ·8	6967 ·8	7035 ·5	7102 ·9	7170 ·1	7237 ·0	68 ·0
15	7303 ·6	7369 ·9	7436 ·0	7501 ·8	7567 ·4	7632 ·7	7697 ·7	7762 ·5	7827 ·1	7891 ·4	65 ·3
16	7955 ·4	8019 ·3	8082 ·9	8146 ·2	8209 ·3	8272 ·2	8334 ·9	8397 ·3	8459 ·6	8521 ·6	62 ·9
17	8583 ·3	8644 ·9	8706 ·2	8767 ·3	8828 ·3	8889 ·0	8949 ·5	9009 ·7	9069 ·8	9129 ·7	60 ·7
18	9189 ·4	9248 ·9	9308 ·2	9367 ·3	9426 ·2	9484 ·9	9543 ·4	9601 ·7	9659 ·8	9717 ·8	58 ·7
19	9775 ·6	9833 ·2	9890 ·6	9947 ·8	*0004 ·9	*0061 ·8	*0118 ·5	*0175 ·0	*0231 ·4	*0287 ·6	56 ·9
20	1 0343 ·6	0399 ·4	0455 ·1	0510 ·6	0566 ·0	0621 ·2	0676 ·2	0731 ·1	0785 ·8	0840 ·3	55 ·2
21	0894 ·7	0949 ·0	1003 ·1	1057 ·0	1110 ·8	1164 ·4	1217 ·9	1271 ·2	1324 ·4	1377 ·5	53 ·6
22	1 1430 ·4	1483 ·1	1535 ·7	1588 ·2	1640 ·5	1692 ·7	1744 ·8	1796 ·7	1848 ·5	1900 ·1	52 ·2
23	1951 ·6	2003 ·0	2054 ·2	2105 ·3	2156 ·3	2207 ·1	2257 ·8	2308 ·4	2358 ·9	2409 ·2	50 ·8
24	2459 ·4	2509 ·5	2559 ·4	2609 ·3	2659 ·0	2708 ·5	2758 ·0	2807 ·3	2856 ·6	2905 ·7	49 ·5
25	1 2954 ·6	3003 ·5	3052 ·3	3100 ·9	3149 ·4	3197 ·8	3246 ·1	3294 ·3	3342 ·4	3390 ·3	48 ·4
26	3438 ·2	3485 ·9	3533 ·5	3581 ·0	3628 ·4	3675 ·7	3722 ·9	3770 ·0	3817 ·0	3863 ·9	47 ·3
27	3910 ·6	3957 ·3	4003 ·9	4050 ·3	4096 ·7	4142 ·9	4189 ·1	4235 ·1	4281 ·1	4327 ·0	46 ·2
28	1 4372 ·7	4418 ·4	4463 ·9	4509 ·4	4554 ·8	4600 ·0	4645 ·2	4690 ·3	4735 ·3	4780 ·2	45 ·2
29	4825 ·0	4869 ·7	4914 ·3	4958 ·9	5003 ·3	5047 ·6	5091 ·8	5136 ·0	5180 ·1	5224 ·1	44 ·3
30	5268 ·0	5311 ·8	5355 ·5	5399 ·1	5442 ·7	5486 ·2	5529 ·6	5572 ·9	5616 ·1	5659 ·2	43 ·5
31	1 5702 ·2	5745 ·2	5788 ·1	5830 ·9	5873 ·6	5916 ·2	5958 ·8	6001 ·3	6043 ·7	6086 ·0	42 ·6
32	6128 ·2	6170 ·3	6212 ·4	6254 ·4	6296 ·3	6338 ·1	6379 ·9	6421 ·6	6463 ·2	6504 ·7	41 ·8
33	6546 ·2	6587 ·6	6628 ·9	6670 ·1	6711 ·3	6752 ·4	6793 ·4	6834 ·3	6875 ·2	6916 ·0	41 ·1
34	1 6956 ·7	6997 ·3	7037 ·9	7078 ·4	7118 ·8	7159 ·2	7199 ·5	7239 ·7	7279 ·9	7320 ·0	40 ·4
35	7360 ·0	7399 ·9	7439 ·8	7479 ·6	7519 ·3	7559 ·0	7598 ·6	7638 ·2	7677 ·7	7717 ·1	39 ·7
36	7756 ·4	7795 ·7	7834 ·9	7874 ·0	7913 ·1	7952 ·1	7991 ·1	8030 ·0	8068 ·8	8107 ·6	39 ·0

Table III—*continued.*

$s = C[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s. 37	1 8146·3	8184·9	8223·5	8262·0	8300·5	8338·9	8377·2	8415·5	8453·7	8491·8	+ 38·4
38	8529·9	8567·9	8605·9	8643·8	8681·6	8719·4	8757·1	8794·8	8832·4	8870·0	37·8
39	8907·5	8944·9	8982·3	9019·6	9056·9	9094·1	9131·3	9168·4	9205·4	9242·4	37·2
40	1 9279·3	9316·2	9353·0	9389·8	9426·5	9463·2	9499·8	9536·3	9572·8	9609·2	36·7
41	9645·6	9682·0	9718·3	9754·5	9790·7	9826·8	9862·9	9898·9	9934·8	9970·7	36·1
42	2 0006·6	0042·4	0078·2	0113·9	0149·6	0185·2	0220·7	0256·2	0291·7	0327·1	35·6
43	2 0362·5	0397·8	0433·0	0468·2	0503·4	0538·5	0573·6	0608·6	0643·6	0678·5	35·1
44	0713·4	0748·2	0783·0	0817·7	0852·4	0887·1	0921·7	0956·2	0990·7	1025·2	34·7
45	1059·6	1093·9	1128·2	1162·5	1196·7	1230·9	1265·0	1299·1	1333·2	1367·2	34·2
46	2 1401·2	1435·1	1468·9	1502·7	1536·5	1570·3	1604·0	1637·6	1671·2	1704·8	33·8
47	1738·3	1771·8	1805·2	1838·6	1872·0	1905·3	1938·5	1971·8	2005·0	2038·1	33·3
48	2071·2	2104·3	2137·3	2170·2	2203·2	2236·1	2268·9	2301·7	2334·5	2367·2	32·9
49	2 2399·9	2432·6	2465·2	2497·8	2530·3	2562·8	2595·3	2627·7	2660·0	2692·4	32·5
50	2724·7	2756·9	2789·1	2821·3	2853·5	2885·6	2917·7	2949·7	2981·7	3013·6	32·1
51	3045·5	3077·4	3109·3	3141·1	3172·8	3204·6	3236·3	3267·9	3299·5	3331·1	31·8
52	2 3362·7	3394·2	3425·7	3457·1	3488·5	3519·9	3551·2	3582·5	3613·7	3645·0	31·4
53	3676·2	3707·3	3738·4	3769·5	3800·6	3831·6	3862·5	3893·5	3924·4	3955·3	31·0
54	3986·1	4016·9	4047·7	4078·4	4109·1	4139·8	4170·4	4201·0	4231·6	4262·1	30·7
55	2 4292·6	4323·1	4353·5	4383·9	4414·3	4444·6	4474·9	4505·2	4535·5	4565·7	30·3
56	4595·8	4626·0	4656·1	4686·1	4716·2	4746·2	4776·2	4806·1	4836·0	4865·9	30·0
57	4895·8	4925·6	4955·4	4985·2	5014·9	5044·6	5074·3	5103·9	5133·5	5163·1	29·7
58	2 5192·6	5222·1	5251·6	5281·1	5310·5	5339·9	5369·2	5398·6	5427·9	5457·1	29·4
59	5486·4	5515·6	5544·8	5573·9	5603·1	5632·2	5661·2	5690·3	5719·3	5748·2	29·1
60	5777·2	5806·1	5835·0	5863·9	5892·7	5921·5	5950·3	5979·0	6007·7	6036·4	28·8
61	2 6065·1	6093·7	6122·3	6150·9	6179·5	6208·0	6236·5	6265·0	6293·4	6321·8	28·5
62	6350·2	6378·5	6406·9	6435·2	6463·4	6491·7	6519·9	6548·1	6576·3	6604·4	28·3
63	6632·5	6660·6	6688·7	6716·7	6744·7	6772·7	6800·6	6828·6	6856·5	6884·3	28·0
64	2 6912·2	6940·0	6967·8	6995·6	7023·3	7051·0	7078·7	7106·4	7134·0	7161·7	27·7
65	7189·3	7216·9	7244·4	7271·9	7299·3	7326·8	7354·3	7381·7	7409·1	7436·5	27·5
66	7463·8	7491·1	7518·4	7545·7	7572·9	7600·1	7627·3	7654·5	7681·6	7708·7	27·2

67	2 7735·8	7762·9	7789·9	7816·9	7843·9	7870·9	7897·8	7924·7	7951·6	7978·5	27·0
68	8005·4	8032·2	8059·0	8085·8	8112·6	8139·3	8166·0	8192·7	8219·4	8246·0	26·7
69	8272·6	8299·2	8325·8	8352·3	8378·8	8405·3	8431·8	8458·3	8484·7	8511·1	26·5
70	2 8537·5	8563·9	8590·2	8616·6	8642·9	8669·2	8695·4	8721·6	8747·8	8774·0	26·3
71	8800·2	8826·4	8852·5	8878·6	8904·6	8930·7	8956·7	8982·7	9008·7	9034·7	26·1
72	9060·6	9086·6	9112·5	9138·4	9164·2	9190·1	9215·9	9241·7	9267·4	9293·2	25·9
73	2 9318·9	9344·6	9370·3	9396·0	9421·6	9447·3	9472·9	9498·5	9524·0	9549·6	25·7
74	9575·1	9600·6	9626·1	9651·6	9677·0	9702·4	9727·8	9753·2	9778·5	9803·9	25·4
75	9829·2	9854·5	9879·8	9905·1	9930·3	9955·5	9980·7	*0005·9	*0031·0	*0056·2	25·2
76	3 0081·3	0106·4	0131·5	0156·5	0181·5	0206·6	0231·6	0256·5	0281·5	0306·4	25·0
77	0331·4	0356·3	0381·1	0406·0	0430·9	0455·7	0480·5	0505·3	0530·1	0554·8	24·8
78	0579·5	0604·2	0628·9	0653·6	0678·2	0702·9	0727·5	0752·1	0776·7	0801·2	24·7
79	3 0825·8	0850·3	0874·8	0899·3	0923·7	0948·2	0972·6	0997·0	1021·4	1045·8	24·5
80	1070·2	1094·5	1118·8	1143·1	1167·4	1191·7	1215·9	1240·1	1264·3	1288·5	24·3
81	1312·7	1336·9	1361·0	1385·1	1409·2	1433·3	1457·4	1481·4	1505·5	1529·5	24·1
82	3 1553·5	1577·5	1601·4	1625·4	1649·3	1673·2	1697·1	1721·0	1744·8	1768·7	23·9
83	1792·5	1816·3	1840·1	1863·9	1887·6	1911·4	1935·1	1958·8	1982·5	2006·2	23·7
84	2029·8	2053·4	2077·0	2100·5	2123·9	2147·3	2170·6	2193·9	2217·1	2240·3	23·4
85	3 2263·4	2286·5	2309·5	2332·5	2355·4	2378·2	2401·0	2423·8	2446·5	2469·1	22·9
86	2491·7	2514·2	2536·7	2559·1	2581·5	2603·8	2626·1	2648·3	2670·5	2692·6	22·3
87	2714·7	2736·7	2758·7	2780·6	2802·4	2824·2	2846·0	2867·7	2889·4	2911·0	21·8
88	3 2932·6	2954·1	2975·6	2997·0	3018·4	3039·7	3061·0	3082·2	3103·4	3124·5	21·3
89	3145·6	3166·6	3187·6	3208·6	3229·5	3250·3	3271·1	3291·9	3312·6	3333·3	20·8
90	3353·9	3374·5	3395·0	3415·5	3435·9	3456·3	3476·6	3496·9	3517·2	3537·4	20·4
91	3 3557·6	3577·7	3597·8	3617·8	3637·8	3657·8	3677·7	3697·5	3717·3	3737·1	20·0
92	3756·9	3776·6	3796·2	3815·8	3835·4	3854·9	3874·4	3893·8	3913·2	3932·6	19·5
93	3951·9	3971·1	3990·3	4009·5	4028·7	4047·8	4066·9	4085·9	4104·9	4123·8	19·1
94	3 4142·7	4161·6	4180·4	4199·2	4217·9	4236·6	4255·3	4273·9	4292·5	4311·0	18·7
95	4329·5	4348·0	4366·4	4384·8	4403·2	4421·5	4439·8	4458·0	4476·2	4494·3	18·3
96	4512·4	4530·5	4548·6	4566·6	4584·6	4602·5	4620·4	4638·3	4656·1	4673·9	17·9
97	3 4691·6	4709·3	4727·0	4744·7	4762·3	4779·8	4797·4	4814·9	4832·3	4849·7	17·5
98	4867·1	4884·5	4901·8	4919·1	4936·3	4953·5	4970·7	4987·9	5005·0	5022·1	17·2
99	5039·1	5056·1	5073·1	5090·0	5106·9	5123·8	5140·6	5157·4	5174·2	5190·9	16·9
100	3 5207·6	5224·3	5240·9	5257·5	5274·1	5290·6	5307·1	5323·6	5340·0	5356·4	16·5
101	5372·8	5389·1	5405·4	5421·7	5438·0	5454·2	5470·4	5486·5	5502·6	5518·7	16·2
102	5534·8	5550·8	5566·8	5582·7	5598·6	5614·5	5630·4	5646·2	5662·0	5677·8	15·9

Table III—*continued.*

$s = C\,[S\,(V) - S\,(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
103	3 5693 ·6	5709 ·3	5724 ·9	5740 ·6	5756 ·2	5771 ·8	5787 ·4	5802 ·9	5818 ·4	5833 ·9	15 ·6
104	5849 ·3	5864 ·7	5880 ·0	5895 ·2	5910 ·4	5925 ·4	5940 ·4	5955 ·3	5970 ·1	5984 ·9	15 ·0
105	5999 ·6	6014 ·2	6028 ·7	6043 ·1	6057 ·5	6071 ·8	6086 ·0	6100 ·2	6114 ·3	6128 ·3	14 ·3
106	3 6142 ·2	6156 ·1	6169 ·9	6183 ·6	6197 ·3	6210 ·9	6224 ·4	6237 ·9	6251 ·3	6264 ·6	13 ·6
107	6277 ·8	6291 ·0	6304 ·1	6317 ·1	6330 ·1	6343 ·0	6355 ·9	6368 ·7	6381 ·4	6394 ·0	12 ·9
108	6406 ·6	6419 ·1	6431 ·6	6444 ·0	6456 ·3	6468 ·6	6480 ·8	6493 ·0	6505 ·1	6517 ·1	12 ·3
109	3 6529 ·1	6541 ·0	6552 ·8	6564 ·6	6576 ·3	6588 ·0	6599 ·6	6611 ·2	6622 ·7	6634 ·2	11 ·7
110	6645 ·6	6656 ·9	6668 ·2	6679 ·4	6690 ·6	6701 ·7	6712 ·7	6723 ·7	6734 ·7	6745 ·6	11 ·1
111	6756 ·4	6767 ·2	6777 ·9	6788 ·6	6799 ·3	6809 ·9	6820 ·4	6830 ·9	6841 ·3	6851 ·7	10 ·6
112	3 6862 ·0	6872 ·3	6882 ·5	6892 ·7	6902 ·8	6912 ·9	6922 ·9	6932 ·9	6942 ·8	6952 ·7	10 ·1
113	6962 ·5	6972 ·3	6982 ·0	6991 ·7	7001 ·4	7011 ·0	7020 ·5	7030 ·0	7039 ·5	7048 ·9	9 ·6
114	7058 ·3	7067 ·6	7076 ·9	7086 ·2	7095 ·4	7104 ·5	7113 ·6	7122 ·7	7131 ·7	7140 ·7	9 ·1
115	3 7149 ·6	7158 ·5	7167 ·4	7176 ·2	7185 ·0	7193 ·7	7202 ·4	7211 ·1	7219 ·7	7228 ·2	8 ·7
116	7236 ·7	7245 ·2	7253 ·7	7262 ·1	7270 ·5	7278 ·8	7287 ·1	7295 ·3	7303 ·5	7311 ·7	8 ·3
117	7319 ·8	7327 ·9	7336 ·0	7344 ·0	7352 ·0	7360 ·0	7367 ·9	7375 ·8	7383 ·6	7391 ·4	8 ·0
118	3 7399 ·2	7406 ·9	7414 ·6	7422 ·3	7429 ·9	7437 ·5	7445 ·0	7452 ·5	7460 ·0	7467 ·5	7 ·6
119	7474 ·9	7482 ·3	7489 ·7	7497 ·1	7504 ·4	7511 ·8	7519 ·1	7526 ·4	7533 ·7	7541 ·0	7 ·4
120	7548 ·3	7555 ·6	7562 ·8	7570 ·1	7577 ·3	7584 ·6	7591 ·8	7599 ·0	7606 ·2	7613 ·4	7 ·3
121	3 7620 ·5	7627 ·7	7634 ·8	7641 ·9	7649 ·0	7656 ·1	7663 ·2	7670 ·3	7677 ·4	7684 ·5	7 ·1
122	7691 ·5	7698 ·6	7705 ·6	7712 ·6	7719 ·6	7726 ·6	7733 ·5	7740 ·5	7747 ·5	7754 ·4	7 ·0
123	7761 ·3	7768 ·3	7775 ·2	7782 ·1	7789 ·0	7795 ·9	7802 ·7	7809 ·6	7816 ·4	7823 ·3	6 ·9
124	3 7830 ·1	7836 ·9	7843 ·7	7850 ·5	7857 ·2	7864 ·0	7870 ·8	7877 ·6	7884 ·3	7891 ·0	6 ·8
125	7897 ·7	7904 ·4	7911 ·1	7917 ·8	7924 ·4	7931 ·1	7937 ·7	7944 ·4	7951 ·0	7957 ·6	6 ·7
126	7964 ·2	7970 ·8	7977 ·4	7984 ·0	7990 ·6	7997 ·2	8003 ·7	8010 ·2	8016 ·7	8023 ·2	6 ·6
127	3 8029 ·7	8036 ·2	8042 ·7	8049 ·2	8055 ·6	8062 ·1	8068 ·5	8075 ·0	8081 ·4	8087 ·8	6 ·5
128	8094 ·2	8100 ·6	8107 ·0	8113 ·4	8119 ·7	8126 ·1	8132 ·4	8138 ·8	8145 ·1	8151 ·4	6 ·4
129	8157 ·7	8164 ·0	8170 ·3	8176 ·6	8182 ·8	8189 ·1	8195 ·3	8201 ·6	8207 ·8	8214 ·0	6 ·3
130	3 8220 ·2	8226 ·4	8232 ·6	8238 ·8	8244 ·9	8251 ·1	8257 ·2	8263 ·3	8269 ·5	8275 ·6	6 ·2
131	8281 ·7	8287 ·8	8293 ·9	8300 ·0	8306 ·1	8312 ·2	8318 ·2	8324 ·3	8330 ·3	8336 ·4	6 ·1
132	8342 ·4	8348 ·4	8354 ·4	8360 ·4	8366 ·4	8372 ·3	8378 ·3	8384 ·2	8390 ·2	8396 ·1	6 ·0

133	3 8402·1	8408·0	8413·9	8419·8	8425·7	8431·6	8437·5	8443·3	8449·2	8455·0	5·9
134	8460·9	8466·7	8472·5	8478·3	8484·1	8489·9	8495·7	8501·5	8507·3	8513·1	5·8
135	8518·8	8524·6	8530·3	8536·0	8541·8	8547·5	8553·2	8558·9	8564·6	8570·3	5·7
136	3 8575·9	8581·6	8587·2	8592·8	8598·5	8604·1	8609·7	8615·3	8620·9	8626·5	5·6
137	8632·1	8637·7	8643·3	8648·9	8654·4	8660·0	8665·5	8671·0	8676·6	8682·1	5·6
138	8687·6	8693·1	8698·6	8704·1	8709·5	8715·0	8720·4	8725·9	8731·3	8736·8	5·5
139	3 8742·2	8747·6	8753·0	8758·5	6763·9	8769·3	8774·7	8780·0	8785·4	8790·7	5·4
140	8796·1	8801·4	8806·7	8812·1	8817·4	8822·7	8828·0	8833·3	8838·6	8843·9	5·3
141	8849·2	8854·5	8859·7	8865·0	8870·2	8875·5	8880·7	8885·9	8891·1	8896·3	5·2
142	3 8901·5	8906·7	8911·9	8917·1	8922·3	8927·4	8932·6	8937·7	8942·9	8948·0	5·1
143	8953·2	8958·3	8963·4	8968·5	8973·6	8978·7	8983·8	8988·9	8994·0	8999·1	5·1
144	9004·1	9009·1	9014·1	9019·2	9024·2	9029·2	9034·2	9039·2	9044·3	9049·3	5·0
145	3 9054·3	9059·3	9064·3	9069·2	9074·2	9079·1	9084·1	9089·0	9093·9	9098·9	4·9
146	9103·8	9108·7	9113·6	9118·5	9123·4	9128·3	9133·2	9138·1	9143·0	9147·9	4·9
147	9152·8	9157·7	9162·6	9167·5	9172·4	9177·2	9182·1	9187·0	9191·9	9196·8	4·8
148	3 9201·6	9206·5	9211·4	9216·2	9221·1	9225·9	9230·7	9235·6	9240·4	9245·2	4·8
149	9250·1	9254·9	9259·8	9264·6	9269·5	9274·3	9279·1	9283·9	9288·7	9293·5	4·8
150	9298·3	9303·1	9307·9	9312·8	9317·6	9322·4	9327·2	9332·0	9336·8	9341·6	4·8
151	3 9346·3	9351·1	9355·9	9360·7	9365·5	9370·2	9375·0	9379·8	9384·6	9389·3	4·7
152	9394·1	9398·8	9403·6	9408·3	9413·1	9417·8	9422·6	9427·3	9432·1	9436·8	4·7
153	9441·6	9446·3	9451·0	9455·8	9460·5	9465·2	9469·9	9474·6	9479·4	9484·1	4·7
154	3 9488·8	9493·5	9498·2	9502·9	9507·6	9512·3	9517·0	9521·7	9526·4	9531·1	4·7
155	9535·8	9540·5	9545·2	9549·9	9554·6	9559·2	9563·9	9568·6	9573·2	9577·9	4·6
156	9582·5	9587·2	9591·9	9596·5	9601·2	9605·8	9610·5	9615·2	9619·8	9624·5	4·6
157	3 9629·1	9633·7	9638·3	9643·0	9647·6	9652·2	9656·8	9661·4	9666·1	9670·7	4·6
158	9675·3	9679·9	9684·5	9689·2	9693·8	9698·4	9703·0	9707·6	9712·2	9716·8	4·6
159	9721·4	9726·0	9730·6	9735·2	9739·8	9744·3	9748·9	9753·5	9758·1	9762·7	4·5
160	3 9767·2	9771·8	9776·3	9780·9	9785·5	9790·0	9794·6	9799·2	9803·7	9808·3	4·5
161	9812·8	9817·3	9821·9	9826·4	9830·9	9835·5	9840·0	9844·5	9849·0	9853·6	4·5
162	9858·1	9862·6	9867·1	9871·7	9876·2	9880·7	9885·2	9889·8	9894·3	9898·8	4·5
163	3 9903·3	9907·8	9912·3	9916·8	9921·3	9925·8	9930·3	9934·8	9939·2	9943·7	4·5
164	9948·2	9952·7	9957·2	9961·7	9966·1	9970·6	9975·1	9979·5	9984·0	9988·5	4·5
165	9992·9	9997·4	*0001·8	*0006·3	*0010·7	*0015·2	*0019·6	*0024·1	*0028·5	*0033·0	4·5
166	4 0037·4	0041·8	0046·3	0050·7	0055·1	0059·5	0064·0	0068·4	0072·8	0077·2	4·4
167	0081·6	0086·0	0090·4	0094·8	0099·2	0103·7	0108·1	0112·5	0116·9	0121·3	4·4
168	0125·7	0130·1	0134·5	0138·9	0143·3	0147·7	0152·1	0156·4	0160·8	0165·2	4·4

Table III—*continued.*

$s = C[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
169	4 0169 ·5	0173 ·9	0178 ·3	0182 ·6	0187 ·0	0191 ·4	0195 ·8	0200 ·1	0204 ·5	0208 ·9	4 ·4
170	0213 ·2	0217 ·6	0221 ·9	0226 ·3	0230 ·6	0234 ·9	0239 ·3	0243 ·6	0247 ·9	0252 ·3	4 ·3
171	0256 ·6	0260 ·9	0265 ·2	0269 ·5	0273 ·9	0278 ·2	0282 ·5	0286 ·9	0291 ·2	0295 ·5	4 ·3
172	4 0299 ·8	0304 ·1	0308 ·4	0312 ·7	0317 ·0	0321 ·4	0325 ·7	0330 ·0	0334 ·3	0338 ·6	4 ·3
173	0342 ·9	0347 ·2	0351 ·5	0355 ·8	0360 ·1	0364 ·4	0368 ·6	0372 ·9	0377 ·2	0381 ·4	4 ·3
174	0385 ·7	0390 ·0	0394 ·2	0398 ·5	0402 ·8	0407 ·0	0411 ·3	0415 ·6	0419 ·8	0424 ·1	4 ·3
175	4 0428 ·3	0432 ·6	0436 ·8	0441 ·1	0445 ·3	0449 ·6	0453 ·8	0458 ·1	0462 ·3	0466 ·6	4 ·3
176	0470 ·8	0475 ·0	0479 ·2	0483 ·4	0487 ·6	0491 ·9	0496 ·1	0500 ·3	0504 ·6	0508 ·8	4 ·2
177	0513 ·0	0517 ·2	0521 ·4	0525 ·6	0529 ·8	0534 ·1	0538 ·3	0542 ·5	0546 ·7	0550 ·9	4 ·2
178	4 0555 ·1	0559 ·3	0563 ·5	0567 ·7	0571 ·9	0576 ·1	0580 ·3	0584 ·4	0588 ·6	0592 ·8	4 ·2
179	0596 ·9	0601 ·1	0605 ·3	0609 ·5	0613 ·6	0617 ·8	0622 ·0	0626 ·1	0630 ·3	0634 ·5	4 ·2
180	0638 ·6	0642 ·8	0646 ·9	0651 ·1	0655 ·2	0659 ·4	0663 ·5	0667 ·7	0671 ·8	0676 ·0	4 ·2
181	4 0680 ·1	0684 ·2	0688 ·4	0692 ·5	0696 ·6	0700 ·8	0704 ·9	0709 ·0	0713 ·1	0717 ·3	4 ·1
182	0721 ·4	0725 ·5	0729 ·6	0733 ·7	0737 ·9	0742 ·0	0746 ·1	0750 ·2	0754 ·4	0758 ·5	4 ·1
183	0762 ·6	0766 ·7	0770 ·8	0774 ·9	0779 ·0	0783 ·1	0787 ·2	0791 ·3	0795 ·3	0799 ·4	4 ·1
184	4 0803 ·5	0807 ·6	0811 ·7	0815 ·8	0819 ·9	0824 ·0	0828 ·1	0832 ·2	0836 ·2	0840 ·3	4 ·1
185	0844 ·3	0848 ·4	0852 ·4	0856 ·5	0860 ·6	0864 ·6	0868 ·7	0872 ·8	0876 ·8	0880 ·9	4 ·1
186	0884 ·9	0889 ·0	0893 ·0	0897 ·1	0901 ·1	0905 ·1	0909 ·2	0913 ·2	0917 ·2	0921 ·3	4 ·0
187	4 0925 ·3	0929 ·3	0933 ·4	0937 ·4	0941 ·4	0945 ·5	0949 ·5	0953 ·5	0957 ·6	0961 ·6	4 ·0
188	0965 ·6	0969 ·6	0973 ·6	0977 ·6	0981 ·6	0985 ·6	0989 ·6	0993 ·6	0997 ·6	1001 ·6	4 ·0
189	1005 ·6	1009 ·6	1013 ·6	1017 ·6	1021 ·6	1025 ·6	1029 ·5	1033 ·5	1037 ·5	1041 ·5	4 ·0
190	4 1045 ·5	1049 ·5	1053 ·5	1057 ·5	1061 ·5	1065 ·5	1069 ·4	1073 ·4	1077 ·4	1081 ·4	4 ·0
191	1085 ·3	1089 ·3	1093 ·2	1097 ·2	1101 ·2	1105 ·1	1109 ·1	1113 ·1	1117 ·0	1121 ·0	4 ·0
192	1124 ·9	1128 ·9	1132 ·8	1136 ·8	1140 ·7	1144 ·6	1148 ·6	1152 ·5	1156 ·4	1160 ·4	3 ·9
193	4 1164 ·3	1168 ·2	1172 ·1	1176 ·0	1180 ·0	1183 ·9	1187 ·8	1191 ·7	1195 ·7	1199 ·6	3 ·9
194	1203 ·5	1207 ·4	1211 ·3	1215 ·2	1219 ·1	1223 ·1	1227 ·0	1230 ·9	1234 ·8	1238 ·7	3 ·9
195	1242 ·6	1246 ·5	1250 ·4	1254 ·3	1258 ·2	1262 ·1	1266 ·0	1269 ·9	1273 ·8	1277 ·7	3 ·9
196	4 1281 ·6	1285 ·5	1289 ·4	1293 ·3	1297 ·2	1301 ·0	1304 ·9	1308 ·8	1312 ·6	1316 ·5	3 ·9
197	1320 ·3	1324 ·2	1328 ·1	1332 ·0	1335 ·9	1339 ·8	1343 ·6	1347 ·5	1351 ·3	1355 ·2	3 ·9
198	1359 ·0	1362 ·9	1366 ·7	1370 ·6	1374 ·4	1378 ·2	1382 ·1	1385 ·9	1389 ·7	1393 ·6	3 ·8

199	4 1397·4	1401·2	1405·0	1408·9	1412·7	1416·5	1420·4	1424·2	1428·0	1431·9	3·8
200	1435·7	1439·5	1443·3	1447·1	1450·9	1454·8	1458·6	1462·4	1466·2	1470·1	3·8
201	1473·9	1477·7	1481·5	1485·3	1489·1	1492·9	1496·7	1500·5	1504·4	1508·2	3·8
202	4 1512·0	1515·8	1519·6	1523·4	1527·2	1531·0	1534·8	1538·6	1542·4	1546·2	3·8
203	1550·0	1553·8	1557·6	1561·4	1565·2	1569·0	1572·8	1576·6	1580·4	1584·1	3·8
204	1587·9	1591·7	1595·5	1599·3	1603·1	1606·9	1610·6	1614·4	1618·2	1622·0	3·8
205	4 1625·7	1629·5	1633·3	1637·0	1640·8	1644·6	1648·3	1652·1	1655·9	1659·6	3·8
206	1663·4	1667·1	1670·9	1674·6	1678·4	1682·2	1685·9	1689·7	1693·5	1697·2	3·8
207	1701·0	1704·7	1708·5	1712·2	1716·0	1719·7	1723·5	1727·2	1731·0	1734·7	3·7
208	4 1738·5	1742·2	1746·0	1749·7	1753·5	1757·2	1761·0	1764·7	1768·5	1772·2	3·7
209	1776·0	1779·7	1783·4	1787·2	1790·9	1794·6	1798·4	1802·1	1805·8	1809·6	3·7
210	1813·3	1817·0	1820·8	1824·5	1828·2	1831·9	1835·7	1839·4	1843·1	1846·9	3·7
211	4 1850·6	1854·3	1858·0	1861·7	1865·4	1869·1	1872·8	1876·5	1880·3	1884·0	3·7
212	1887·7	1891·4	1895·1	1898·8	1902·5	1906·2	1910·0	1913·7	1917·4	1921·1	3·7
213	1924·8	1928·5	1932·2	1935·9	1939·6	1943·3	1947·0	1950·7	1954·4	1958·1	3·7
214	4 1961·8	1965·5	1969·2	1972·9	1976·6	1980·3	1984·0	1987·7	1991·4	1995·0	3·7
215	1998·7	2002·4	2006·1	2009·8	2013·5	2017·2	2020·9	2024·6	2028·2	2031·9	3·7
216	2035·6	2039·3	2043·0	2046·7	2050·3	2054·0	2057·7	2061·3	2065·0	2068·7	3·7
217	4 2072·3	2076·0	2079·7	2083·4	2087·0	2090·7	2094·4	2098·0	2101·7	2105·4	3·7
218	2109·0	2112·7	2116·3	2120·0	2123·6	2127·3	2130·9	2134·6	2138·2	2141·9	3·7
219	2145·5	2149·2	2152·8	2156·5	2160·1	2163·8	2167·4	2171·1	2174·7	2178·4	3·7
220	4 2182·0	2185·7	2189·3	2193·0	2196·6	2200·2	2203·9	2207·5	2211·1	2214·8	3·6
221	2218·4	2222·1	2225·7	2229·4	2233·0	2236·6	2240·3	2243·9	2247·5	2251·2	3·6
222	2254·8	2258·4	2262·0	2265·7	2269·3	2272·9	2276·5	2280·1	2283·8	2287·4	3·6
223	4 2291·0	2294·6	2298·2	2301·9	2305·5	2309·1	2312·7	2316·3	2320·0	2323·6	3·6
224	2327·2	2330·8	2334·4	2338·0	2341·6	2345·2	2348·8	2352·4	2356·0	2359·6	3·6
225	2363·2	2366·8	2370·4	2374·0	2377·6	2381·2	2384·8	2388·4	2392·0	2395·6	3·6
226	4 2399·2	2402·8	2406·4	2410·0	2413·6	2417·2	2420·8	2424·4	2428·0	2431·6	3·6
227	2435·2	2438·8	2442·4	2446·0	2449·6	2453·1	2456·7	2460·3	2463·9	2467·4	3·6
228	2471·0	2474·6	2478·2	2481·8	2485·3	2488·9	2492·5	2496·1	2499·6	2503·2	3·6
229	4 2506·8	2510·3	2513·9	2517·5	2521·1	2524·6	2528·2	2531·8	2535·4	2538·9	3·6
230	2542·5	2546·0	2549·6	2553·1	2556·7	2560·3	2563·8	2567·4	2571·0	2574·5	3·6
231	2578·1	2581·6	2585·2	2588·7	2592·3	2595·8	2599·4	2602·9	2606·5	2610·0	3·5
232	4 2613·6	2617·1	2620·7	2624·2	2627·8	2631·3	2634·9	2638·4	2642·0	2645·5	3·5
233	2649·1	2652·6	2656·1	2659·7	2663·2	2666·7	2670·3	2673·8	2677·3	2680·9	3·5
234	2684·4	2687·9	2691·5	2695·0	2698·5	2702·1	2705·6	2709·1	2712·7	2716·2	3·5

Table III—*continued.*

$s = C\,[S\,(V) - S\,(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
235	4 2719 ·7	2723 ·2	2726 ·8	2730 ·3	2733 ·8	2737 ·3	2740 ·9	2744 ·4	2747 ·9	2751 ·5	3 ·5
236	2755 ·0	2758 ·5	2762 ·0	2765 ·5	2769 ·0	2772 ·6	2776 ·1	2779 ·6	2783 ·1	2786 ·6	3 ·5
237	2790 ·1	2793 ·6	2797 ·1	2800 ·6	2804 ·1	2807 ·7	2811 ·2	2814 ·7	2818 ·2	2821 ·7	3 ·5
238	4 2825 ·2	2828 ·7	2832 ·2	2835 ·7	2839 ·2	2842 ·7	2846 ·2	2849 ·7	2853 ·2	2856 ·7	3 ·5
239	2860 ·2	2863 ·7	2867 ·2	2870 ·7	2874 ·2	2877 ·7	2881 ·2	2884 ·7	2888 ·1	2891 ·6	3 ·5
240	2895 ·1	2898 ·6	2902 ·1	2905 ·6	2909 ·1	2912 ·6	2916 ·1	2919 ·6	2923 ·0	2926 ·5	3 ·5
241	4 2930 ·0	2933 ·5	2937 ·0	2940 ·5	2944 ·0	2947 ·5	2950 ·9	2954 ·4	2957 ·9	2961 ·4	3 ·5
242	2964 ·8	2968 ·3	2971 ·8	2975 ·2	2978 ·7	2982 ·2	2985 ·6	2989 ·1	2992 ·6	2996 ·0	3 ·5
243	2999 ·5	3002 ·9	3006 ·4	3009 ·8	3013 ·3	3016 ·8	3020 ·2	3023 ·7	3027 ·2	3030 ·6	3 ·5
244	4 3034 ·1	3037 ·5	3041 ·0	3044 ·5	3047 ·9	3051 ·4	3054 ·9	3058 ·3	3061 ·8	3065 ·3	3 ·5
245	3068 ·7	3072 ·2	3075 ·6	3079 ·1	3082 ·5	3086 ·0	3089 ·4	3092 ·9	3096 ·3	3099 ·8	3 ·5
246	3103 ·2	3106 ·7	3110 ·1	3113 ·6	3117 ·0	3120 ·5	3123 ·9	3127 ·4	3130 ·8	3134 ·3	3 ·5
247	4 3137 ·7	3141 ·1	3144 ·6	3148 ·0	3151 ·4	3154 ·9	3158 ·3	3161 ·7	3165 ·2	3168 ·6	3 ·4
248	3172 ·0	3175 ·4	3178 ·9	3182 ·3	3185 ·7	3189 ·2	3192 ·6	3196 ·0	3199 ·5	3202 ·9	3 ·4
249	3206 ·3	3209 ·7	3213 ·1	3216 ·5	3220 ·0	3223 ·4	3226 ·8	3230 ·3	3233 ·7	3237 ·1	3 ·4
250	4 3240 ·5	3243 ·9	3247 ·3	3250 ·8	3254 ·2	3257 ·6	3261 ·0	3264 ·4	3267 ·8	3271 ·3	3 ·4
251	3274 ·7	3278 ·1	3281 ·5	3284 ·9	3288 ·3	3291 ·8	3295 ·2	3298 ·6	3302 ·0	3305 ·4	3 ·4
252	3308 ·8	3312 ·2	3315 ·6	3319 ·0	3322 ·4	3325 ·8	3329 ·2	3332 ·6	3336 ·0	3339 ·4	3 ·4
253	4 3342 ·8	3346 ·2	3349 ·6	3353 ·0	3356 ·4	3359 ·8	3363 ·2	3366 ·6	3370 ·0	3373 ·4	3 ·4
254	3376 ·8	3380 ·2	3383 ·6	3387 ·0	3390 ·4	3393 ·7	3397 ·1	3400 ·5	3403 ·9	3407 ·2	3 ·4
255	3410 ·6	3414 ·0	3417 ·4	3420 ·8	3424 ·2	3427 ·6	3431 ·0	3434 ·4	3437 ·7	3441 ·1	3 ·4
256	4 3444 ·5	3447 ·9	3451 ·3	3454 ·7	3458 ·0	3461 ·4	3464 ·8	3468 ·1	3471 ·5	3474 ·9	3 ·4
257	3478 ·2	3481 ·6	3485 ·0	3488 ·3	3491 ·7	3495 ·1	3498 ·5	3501 ·8	3505 ·2	3508 ·6	3 ·4
258	3511 ·9	3515 ·3	3518 ·6	3522 ·0	3525 ·4	3528 ·7	3532 ·1	3535 ·5	3538 ·8	3542 ·2	3 ·4
259	4 3545 ·5	3548 ·9	3552 ·2	3555 ·6	3559 ·0	3562 ·3	3565 ·7	3569 ·1	3572 ·4	3575 ·8	3 ·4
260	3579 ·1	3582 ·5	3585 ·8	3589 ·2	3592 ·5	3595 ·9	3599 ·2	3602 ·6	3605 ·9	3609 ·3	3 ·4
261	3612 ·6	3616 ·0	3619 ·3	3622 ·7	3626 ·0	3629 ·3	3632 ·7	3636 ·0	3639 ·3	3642 ·7	3 ·3
262	4 3646 ·0	3649 ·3	3652 ·7	3656 ·0	3659 ·3	3662 ·7	3666 ·0	3669 ·3	3672 ·7	3676 ·0	3 ·3
263	3679 ·3	3682 ·6	3685 ·9	3689 ·3	3692 ·6	3695 ·9	3699 ·2	3702 ·6	3705 ·9	3709 ·2	3 ·3
264	3712 ·5	3715 ·8	3719 ·1	3722 ·5	3725 ·8	3729 ·1	3732 ·4	3735 ·8	3739 ·1	3742 ·4	3 ·3

265	4 3745 ·7	3749 ·0	3752 ·3	3755 ·6	3758 ·9	3762 ·2	3765 ·5	3768 ·8	3772 ·1	3775 ·4	3 ·3
266	3778 ·7	3782 ·0	3785 ·3	3788 ·6	3791 ·9	3795 ·2	3798 ·5	3801 ·8	3805 ·1	3808 ·4	3 ·3
267	3811 ·7	3815 ·0	3818 ·3	3821 ·6	3824 ·9	3828 ·2	3831 ·5	3834 ·8	3838 ·0	3841 ·3	3 ·3
268	4 3844 ·6	3847 ·9	3851 ·2	3854 ·4	3857 ·7	3861 ·0	3864 ·3	3867 ·5	3870 ·8	3874 ·1	3 ·3
269	3877 ·4	3880 ·7	3883 ·9	3887 ·2	3890 ·5	3893 ·7	3897 ·0	3900 ·3	3903 ·6	3906 ·8	3 ·3
270	3910 ·1	3913 ·4	3916 ·6	3919 ·9	3923 ·2	3926 ·4	3929 ·7	3933 ·0	3936 ·3	3939 ·5	3 ·3
271	4 3942 ·8	3946 ·1	3949 ·3	3952 ·6	3955 ·9	3959 ·1	3962 ·4	3965 ·6	3968 ·9	3972 ·2	3 ·3
272	3975 ·4	3978 ·7	3981 ·9	3985 ·2	3988 ·4	3991 ·7	3994 ·9	3998 ·2	4001 ·4	4004 ·7	3 ·3
273	4007 ·9	4011 ·1	4014 ·4	4017 ·6	4020 ·9	4024 ·1	4027 ·4	4030 ·6	4033 ·8	4037 ·1	3 ·2
274	4 4040 ·3	4043 ·6	4046 ·8	4050 ·0	4053 ·2	4056 ·5	4059 ·7	4062 ·9	4066 ·1	4069 ·4	3 ·2
275	4072 ·6	4075 ·8	4079 ·1	4082 ·3	4085 ·5	4088 ·7	4091 ·9	4095 ·2	4098 ·4	4101 ·6	3 ·2
276	4104 ·8	4108 ·0	4111 ·2	4114 ·5	4117 ·7	4120 ·9	4124 ·1	4127 ·4	4130 ·6	4133 ·8	3 ·2
277	4 4137 ·0	4140 ·2	4143 ·4	4146 ·6	4149 ·8	4153 ·1	4156 ·3	4159 ·5	4162 ·7	4165 ·9	3 ·2
278	4169 ·1	4172 ·3	4175 ·5	4178 ·7	4181 ·9	4185 ·1	4188 ·3	4191 ·5	4194 ·7	4197 ·9	3 ·2
279	4201 ·1	4204 ·3	4207 ·5	4210 ·7	4213 ·9	4217 ·1	4220 ·3	4223 ·5	4226 ·7	4229 ·9	3 ·2
280	4 4233 ·1	4236 ·3	4239 ·5	4242 ·7	4245 ·9	4249 ·0	4252 ·2	4255 ·4	4258 ·6	4261 ·7	3 ·2
281	4264 ·9	4268 ·1	4271 ·3	4274 ·5	4277 ·7	4280 ·8	4284 ·0	4287 ·2	4290 ·4	4293 ·5	3 ·2
282	4296 ·7	4299 ·9	4303 ·1	4306 ·3	4309 ·5	4312 ·6	4315 ·8	4319 ·0	4322 ·1	4325 ·3	3 ·2
283	4 4328 ·4	4331 ·6	4334 ·8	4338 ·0	4341 ·2	4344 ·3	4347 ·5	4350 ·7	4353 ·8	4357 ·0	3 ·2
284	4360 ·1	4363 ·3	4366 ·4	4369 ·6	4372 ·7	4375 ·9	4379 ·0	4382 ·2	4385 ·3	4388 ·5	3 ·2
285	4391 ·6	4394 ·8	4397 ·9	4401 ·1	4404 ·2	4407 ·4	4410 ·5	4413 ·7	4416 ·8	4420 ·0	3 ·2
286	4 4423 ·1	4426 ·3	4429 ·4	4432 ·6	4435 ·7	4438 ·9	4442 ·0	4445 ·2	4448 ·3	4451 ·5	3 ·2
287	4454 ·6	4457 ·7	4460 ·9	4464 ·0	4467 ·1	4470 ·3	4473 ·4	4476 ·5	4479 ·6	4482 ·8	3 ·1
288	4485 ·9	4489 ·0	4492 ·2	4495 ·3	4498 ·4	4501 ·6	4504 ·7	4507 ·8	4510 ·9	4514 ·1	3 ·1
289	4 4517 ·2	4520 ·3	4523 ·4	4526 ·6	4529 ·7	4532 ·8	4535 ·9	4539 ·1	4542 ·2	4545 ·3	3 ·1
290	4548 ·4	4551 ·5	4554 ·6	4557 ·7	4560 ·8	4564 ·0	4567 ·1	4570 ·2	4573 ·3	4576 ·4	3 ·1
291	4579 ·5	4582 ·6	4585 ·7	4588 ·8	4591 ·9	4595 ·0	4598 ·1	4601 ·2	4604 ·3	4607 ·4	3 ·1
292	4 4610 ·5	4613 ·6	4616 ·7	4619 ·8	4622 ·9	4626 ·0	4629 ·1	4632 ·2	4635 ·3	4638 ·4	3 ·1
293	4641 ·5	4644 ·6	4647 ·7	4650 ·8	4653 ·9	4656 ·9	4660 ·0	4663 ·1	4666 ·2	4669 ·3	3 ·1
294	4672 ·4	4675 ·5	4678 ·6	4681 ·7	4684 ·8	4687 ·8	4690 ·9	4694 ·0	4697 ·1	4700 ·2	3 ·1
295	4 4703 ·3	4706 ·4	4709 ·5	4712 ·5	4715 ·6	4718 ·7	4721 ·8	4724 ·8	4727 ·9	4731 ·0	3 ·1
296	4734 ·0	4737 ·1	4740 ·2	4743 ·3	4746 ·3	4749 ·4	4752 ·5	4755 ·5	4758 ·6	4761 ·7	3 ·1
297	4764 ·8	4767 ·9	4770 ·9	4774 ·0	4777 ·1	4780 ·1	4783 ·2	4786 ·3	4789 ·3	4792 ·4	3 ·1
298	4 4795 ·4	4798 ·5	4801 ·5	4804 ·6	4807 ·6	4810 ·7	4813 ·7	4816 ·8	4819 ·8	4822 ·9	3 ·1
299	4825 ·9	4829 ·0	4832 ·0	4835 ·1	4838 ·1	4841 ·2	4844 ·2	4847 ·3	4850 ·3	4853 ·4	3 ·1
300	4856 ·4	4859 ·5	4862 ·5	4865 ·6	4868 ·6	4871 ·7	4874 ·7	4877 ·8	4880 ·8	4883 ·9	3 ·1

Table III—*continued.*

$s = C[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
301	4 4886 ·9	4890 ·0	4893 ·0	4896 ·0	4899 ·1	4902 ·1	4905 ·1	4908 ·1	4911 ·2	4914 ·2	3 ·0
302	4917 ·2	4920 ·3	4923 ·3	4926 ·3	4929 ·4	4932 ·4	4935 ·4	4938 ·4	4941 ·5	4944 ·5	3 ·0
303	4947 ·5	4950 ·6	4953 ·6	4956 ·6	4959 ·6	4962 ·7	4965 ·7	4968 ·7	4971 ·7	4974 ·7	3 ·0
304	4 4977 ·7	4980 ·8	4983 ·8	4986 ·8	4989 ·8	4992 ·9	4995 ·9	4998 ·9	5001 ·9	5004 ·9	3 ·0
305	5007 ·9	5011 ·0	5014 ·0	5017 ·0	5020 ·0	5023 ·0	5026 ·0	5029 ·0	5032 ·0	5035 ·0	3 ·0
306	5038 ·0	5041 ·0	5044 ·0	5047 ·0	5050 ·0	5053 ·0	5056 ·0	5059 ·0	5062 ·0	5065 ·0	3 ·0
307	4 5068 ·0	5071 ·0	5074 ·0	5077 ·0	5080 ·0	5083 ·0	5086 ·0	5089 ·0	5092 ·0	5095 ·0	3 ·0
308	5097 ·9	5100 ·9	5103 ·9	5106 ·9	5109 ·9	5112 ·9	5115 ·9	5118 ·9	5121 ·9	5124 ·9	3 ·0
309	5127 ·9	5130 ·9	5133 ·9	5136 ·9	5139 ·9	5142 ·8	5141 ·8	5148 ·8	5151 ·8	5154 ·8	3 ·0
310	4 5157 ·7	5160 ·7	5163 ·7	5166 ·7	5169 ·7	5172 ·6	5175 ·6	5178 ·6	5181 ·6	5184 ·6	3 ·0
311	5187 ·5	5190 ·5	5193 ·5	5196 ·5	5199 ·4	5202 ·4	5205 ·4	5208 ·3	5211 ·3	5214 ·3	3 ·0
312	5217 ·2	5220 ·2	5223 ·1	5226 ·1	5229 ·1	5232 ·0	5235 ·0	5238 ·0	5240 ·9	5243 ·9	3 ·0
313	4 5246 ·8	5249 ·8	5252 ·7	5255 ·7	5258 ·7	5261 ·6	5264 ·6	5267 ·6	5270 ·5	5273 ·5	3 ·0
314	5276 ·4	5279 ·4	5282 ·3	5285 ·3	5288 ·2	5291 ·2	5294 ·1	5297 ·1	5300 ·0	5303 ·0	3 ·0
315	5305 ·9	5308 ·9	5311 ·8	5314 ·8	5317 ·7	5320 ·7	5323 ·6	5326 ·6	5329 ·5	5332 ·5	3 ·0
316	4 5335 ·4	5338 ·4	5341 ·3	5344 ·3	5347 ·2	5350 ·2	5353 ·1	5356 ·0	5359 ·0	5361 ·9	2 ·9
317	5364 ·8	5367 ·8	5370 ·7	5373 ·6	5376 ·6	5379 ·5	5382 ·4	5385 ·4	5388 ·3	5391 ·2	2 ·9
318	5394 ·1	5397 ·1	5400 ·0	5402 ·9	5405 ·9	5408 ·8	5411 ·7	5414 ·7	5417 ·6	5420 ·5	2 ·9
319	4 5423 ·4	5426 ·4	5429 ·3	5432 ·2	5435 ·1	5438 ·1	5441 ·0	5443 ·9	5446 ·8	5449 ·7	2 ·9
320	5452 ·6	5455 ·6	5458 ·5	5461 ·4	5464 ·3	5467 ·2	5470 ·1	5473 ·0	5475 ·9	5478 ·8	2 ·9
321	5481 ·7	5484 ·7	5487 ·6	5490 ·5	5493 ·4	5496 ·3	5499 ·2	5502 ·1	5505 ·0	5507 ·9	2 ·9
322	4 5510 ·8	5513 ·7	5516 ·6	5519 ·5	5522 ·4	5525 ·3	5528 ·2	5531 ·1	5534 ·0	5536 ·9	2 ·9
323	5539 ·8	5542 ·7	5545 ·6	5548 ·5	5551 ·4	5554 ·3	5557 ·2	5560 ·1	5563 ·0	5565 ·9	2 ·9
324	5568 ·8	5571 ·7	5574 ·6	5577 ·5	5580 ·4	5583 ·3	5586 ·2	5589 ·1	5592 ·0	5594 ·9	2 ·9
325	4 5597 ·7	5600 ·6	5603 ·5	5606 ·4	5609 ·3	5612 ·2	5615 ·1	5618 ·0	5620 ·8	5623 ·7	2 ·9
326	5626 ·5	5629 ·4	5632 ·3	5635 ·2	5638 ·1	5641 ·0	5643 ·9	5646 ·8	5649 ·6	5652 ·5	2 ·9
327	5655 ·3	5658 ·2	5661 ·1	5664 ·0	5666 ·8	5669 ·7	5672 ·6	5675 ·4	5678 ·3	5681 ·2	2 ·9
328	4 5684 ·0	5686 ·9	5689 ·8	5692 ·7	5695 ·5	5698 ·4	5701 ·3	5704 ·1	5707 ·0	5709 ·9	2 ·9
329	5712 ·7	5715 ·6	5718 ·5	5721 ·3	5724 ·2	5727 ·1	5729 ·9	5732 ·8	5735 ·6	5738 ·5	2 ·9
330	5741 ·3	5744 ·2	5747 ·0	5749 ·9	5752 ·7	5755 ·6	5758 ·4	5761 ·3	5764 ·1	5767 ·0	2 ·9

331	4 5769·8	5772·7	5775·5	5778·4	5781·2	5784·1	5786·9	5789·8	5792·6	5795·5	2·9
332	5798·3	5801·2	5804·0	5806·9	5809·7	5812·6	5815·4	5818·3	5821·1	5824·0	2·9
333	5826·8	5829·7	5832·5	5835·3	5838·2	5841·0	5843·8	5846·7	5849·5	5852·3	2·8
334	4 5855·1	5858·0	5860·8	5863·7	5866·5	5869·3	5872·2	5875·0	5877·8	5880·7	2·8
335	5883·5	5886·4	5889·2	5892·0	5894·8	5897·7	5900·5	5903·3	5906·1	5908·9	2·8
336	5911·7	5914·6	5917·4	5920·2	5923·0	5925·9	5928·7	5931·5	5934·3	5937·1	2·8
337	4 5939·9	5942·8	5945·6	5948·4	5951·2	5954·1	5956·9	5959·7	5962·5	5965·3	2·8
338	5968·1	5971·0	5973·8	5976·6	5979·4	5982·2	5985·0	5987·8	5990·6	5993·4	2·8
339	5996·2	5999·0	6001·8	6004·6	6007·4	6010·2	6013·0	6015·8	6018·6	6021·4	2·8
340	4 6024·2	6027·0	6029·8	6032·6	6035·4	6038·2	6041·0	6043·8	6046·6	6049·4	2·8
341	6052·2	6055·0	6057·8	6060·6	6063·4	6066·2	6069·0	6071·8	6074·6	6077·4	2·8
342	6080·1	6082·9	6085·7	6088·5	6091·3	6094·1	6096·9	6099·7	6102·5	6105·3	2·8
343	4 6108·0	6110·8	6113·6	6116·4	6119·2	6122·0	6124·7	6127·5	6130·3	6133·1	2·8
344	6135·8	6138·6	6141·4	6144·2	6147·0	6149·8	6152·5	6155·3	6158·1	6160·9	2·8
345	6163·6	6166·4	6169·2	6172·0	6174·7	6177·5	6180·3	6183·1	6185·8	6188·6	2·8
346	4 6191·3	6194·1	6196·9	6199·7	6202·4	6205·2	6208·0	6210·8	6213·5	6216·3	2·8
347	6219·0	6221·8	6224·6	6227·3	6230·1	6232·9	6235·6	6238·4	6241·1	6243·9	2·8
348	6246·6	6249·4	6252·1	6254·9	6257·6	6260·4	6263·1	6265·9	6268·6	6271·4	2·8
349	4 6274·1	6276·9	6279·6	6282·4	6285·1	6287·9	6290·6	6293·4	6296·1	6298·9	2·8
350	6301·6	6304·4	6307·1	6309·9	6312·6	6315·4	6318·1	6320·9	6323·6	6326·4	2·8
351	6329·1	6331·9	6334·6	6337·4	6340·1	6342·9	6345·6	6348·4	6351·1	6353·8	2·7
352	4 6356·5	6359·3	6362·0	6364·8	6367·5	6370·3	6373·0	6375·7	6378·4	6381·1	2·7
353	6383·8	6386·6	6389·3	6392·1	6394·8	6397·6	6400·3	6403·0	6405·7	6408·4	2·7
354	6411·1	6413·9	6416·6	6419·4	6422·1	6424·8	6427·5	6430·2	6432·9	6435·6	2·7
355	4 6438·3	6441·1	6443·8	6446·6	6449·3	6452·0	6454·7	6457·4	6460·1	6462·8	2·7
356	6465·5	6468·3	6471·0	6473·7	6476·4	6479·1	6481·8	6484·5	6487·2	6489·9	2·7
357	6492·6	6495·4	6498·1	6500·8	6503·5	6506·2	6508·9	6511·6	6514·3	6517·0	2·7
358	4 6519·7	6522·5	6525·2	6527·9	6530·6	6533·3	6536·0	6538·7	6541·4	6544·1	2·7
359	6546·8	6549·5	6552·2	6554·9	6557·6	6560·3	6563·0	6565·7	6568·4	6571·1	2·7
360	6573·8	6576·5	6579·2	6581·9	6584·6	6587·3	6590·0	6592·7	6595·4	6598·1	2·7
361	4 6600·7	6603·4	6606·1	6608·8	6611·5	6614·2	6616·9	6619·6	6622·3	6625·0	2·7
362	6627·6	6630·3	6633·0	6635·7	6638·4	6641·1	6643·8	6646·4	6649·1	6651·8	2·7
363	6654·4	6657·1	6659·8	6662·5	6665·2	6667·9	6670·6	6673·2	6675·9	6678·6	2·7
364	4 6681·2	6683·9	6686·6	6689·3	6691·9	6694·6	6697·3	6699·9	6702·6	6705·3	2·7
365	6707·9	6710·6	6713·3	6716·0	6718·6	6721·3	6724·0	6726·6	6729·3	6732·0	2·7
366	6734·6	6737·3	6740·0	6742·6	6745·3	6748·0	6750·6	6753·3	6755·9	6758·6	2·7

Table III—*continued.*

$s = C[S(V) - S(v)]$.

v.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
367	4 6761·2	6763·9	6766·6	6769·2	6771·9	6774·6	6777·2	6779·9	6782·5	6785·2	2·7
368	6787·8	6790·5	6793·2	6795·8	6798·5	6801·2	6803·8	6806·5	6809·1	6811·8	2·7
369	6814·4	6817·1	6819·7	6822·4	6825·0	6827·7	6830·3	6833·0	6835·6	6838·3	2·7
370	4 6840·9	6843·6	6846·2	6848·9	6851·5	6854·2	6856·8	6859·4	6862·1	6864·7	2·6
371	6867·3	6870·0	6872·6	6875·3	6877·9	6880·6	6883·2	6885·8	6888·5	6891·1	2·6
372	6893·7	6896·4	6899·0	6901·7	6904·3	6907·0	6909·6	6912·2	6914·9	6917·5	2·6
373	4 6920·1	6922·8	6925·4	6928·0	6930·7	6933·3	6935·9	6938·6	6941·2	6943·8	2·6
374	6946·4	6949·1	6951·7	6954·3	6956·9	6959·5	6962·2	6964·8	6967·4	6970·0	2·6
375	6972·6	6975·3	6977·9	6980·5	6983·1	6985·7	6988·4	6991·0	6993·6	6996·2	2·6
376	4 6998·8	7001·4	7004·1	7006·7	7009·3	7011·9	7014·6	7017·2	7019·8	7022·4	2·6
377	7025·0	7027·7	7030·3	7032·9	7035·5	7038·1	7040·7	7043·3	7045·9	7048·5	2·6
378	7051·1	7053·8	7056·4	7059·0	7061·6	7064·2	7066·8	7069·4	7072·0	7074·6	2·6
379	4 7077·2	7079·8	7082·4	7085·0	7087·6	7090·2	7092·8	7095·4	7098·0	7100·6	2·6
380	7103·2	7105·8	7108·4	7111·0	7113·6	7116·2	7118·8	7121·4	7124·0	7126·6	2·6
381	7129·2	7131·8	7134·4	7137·0	7139·6	7142·2	7144·8	7147·4	7150·0	7152·6	2·6
382	4 7155·1	7157·7	7160·3	7162·9	7165·5	7168·1	7170·7	7173·3	7175·9	7178·5	2·6
383	7181·0	7183·6	7186·2	7188·8	7191·4	7194·0	7196·5	7199·1	7201·7	7204·3	2·6
384	7206·8	7209·4	7212·0	7214·6	7217·2	7219·8	7222·3	7224·9	7227·5	7230·1	2·6
385	4 7232·6	7235·2	7237·8	7240·4	7243·0	7245·6	7248·2	7250·7	7253·3	7255·9	2·6
386	7258·4	7261·0	7263·6	7266·1	7268·7	7271·3	7273·8	7276·4	7279·0	7281·5	2·6
387	7284·1	7286·7	7289·3	7291·8	7294·4	7297·0	7299·5	7302·1	7304·7	7307·2	2·6
388	4 7309·8	7312·4	7314·9	7317·5	7320·0	7322·6	7325·1	7327·7	7330·2	7332·8	2·6
389	7335·4	7338·0	7340·5	7343·1	7345·6	7348·2	7350·7	7353·3	7355·8	7358·4	2·6
390	7360·9	7363·5	7366·1	7368·7	7371·2	7373·8	7376·3	7378·9	7381·4	7384·0	2·6
391	4 7386·5	7389·1	7391·6	7394·2	7396·7	7399·3	7401·8	7404·4	7406·9	7409·5	2·6
392	7412·0	7414·6	7417·1	7419·7	7422·2	7424·8	7427·3	7429·9	7432·4	7434·9	2·5
393	7437·4	7440·0	7442·5	7445·1	7447·6	7450·2	7452·7	7455·3	7457·8	7460·3	2·5
394	4 7462·8	7465·4	7467·9	7470·5	7473·0	7475·6	7478·1	7480·7	7483·2	7485·7	2·5
395	7488·2	7490·8	7493·3	7495·8	7498·3	7500·9	7503·4	7505·9	7508·4	7511·0	2·5
396	7513·5	7516·1	7518·6	7521·2	7523·7	7526·2	7528·7	7531·2	7533·7	7536·2	2·5

397	4 7538·7	7541·3	7543·8	7546·3	7548·9	7551·4	7553·9	7556·4	7558·9	7561·4	2·5
398	7564·0	7566·5	7569·0	7571·6	7574·1	7576·6	7579·1	7581·6	7584·1	7586·6	2·5
399	7589·2	7591·7	7594·2	7596·7	7599·2	7601·7	7604·3	7606·8	7609·3	7611·8	2·5
400	4 7614·3										

TABLE IV.

$$\tan\phi - \tan\theta = C\,[I(V) - I(v)], \quad \textit{or} \quad \phi^\circ - \theta^\circ = 57\cdot 3\ C\,[I(V) - I(v)].$$

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
50	0·00000	0·00415	0·00827	0·01238	0·01646	0·02052	0·02456	0·02858	0·03258	0·03655	406
51	·04051	·04445	·04837	·05226	·05614	·06000	·06384	·06766	·07146	·07524	386
52	·07900	·08275	·08647	·09018	·09387	·09754	·10119	·10483	·10844	·11204	367
53	0·11562	·11919	·12273	·12626	·12977	·13327	·13674	·14020	·14365	·14707	350
54	·15049	·15388	·15726	·16062	·16397	·16730	·17061	·17391	·17719	·18046	333
55	·18371	·18695	·19017	·19337	·19656	·19974	·20290	·20605	·20918	·21229	318
56	0·21540	·21849	·22156	·22462	·22767	·23070	·23372	·23672	·23971	·24269	303
57	·24565	·24860	·25154	·25446	·25737	·26027	·26315	·26602	·26888	·27172	290
58	·27455	·27737	·28018	·28298	·28576	·28853	·29128	·29403	·29676	·29948	277
59	0·30219	·30489	·30757	·31025	·31291	·31556	·31820	·32082	·32344	·32604	265
60	·32864	·33122	·33379	·33635	·33890	·34143	·34396	·34648	·34898	·35147	253
61	·35396	·35643	·35889	·36135	·36379	·36622	·36864	·37105	·37345	·37584	243
62	0·37822	·38060	·38296	·38531	·38765	·38998	·39230	·39461	·39691	·39921	233
63	·40149	·40377	·40603	·40829	·41053	·41277	·41500	·41722	·41942	·42163	224
64	·42382	·42600	·42818	·43034	·43250	·43465	·43679	·43892	·44104	·44316	215
65	0·44526	·44736	·44945	·45153	·45360	·45566	·45772	·45977	·46181	·46384	206
66	·46586	·46788	·46988	·47188	·47387	·47586	·47784	·47980	·48176	·48372	199
67	·48566	·48760	·48953	·49146	·49337	·49528	·49718	·49907	·50096	·50284	191
68	0·50471	·50658	·50844	·51029	·51213	·51397	·51580	·51762	·51943	·52124	184
69	·52305	·52484	·52663	·52841	·53019	·53196	·53372	·53548	·53722	·53897	177
70	·54070	·54243	·54416	·54587	·54758	·54929	·55099	·55268	·55436	·55604	171
71	0·55772	·55938	·56104	·56270	·56435	·56599	·56763	·56926	·57088	·57250	164
72	·57412	·57572	·57733	·57892	·58051	·58210	·58368	·58525	·58682	·58838	159
73	·58994	·59149	·59303	·59457	·59611	·59764	·59916	·60068	·60219	·60370	153
74	0·60520	·60670	·60819	·60968	·61116	·61264	·61411	·61557	·61703	·61849	148
75	·61994	·62139	·62283	·62426	·62569	·62712	·62854	·62996	·63137	·63277	143
76	·63418	·63557	·63697	·63835	·63974	·64111	·64249	·64386	·64522	·64658	137

77	0·64793	·64928	·65063	·65197	·65331	·65464	·65597	·65729	·65861	·65992	133
78	·66123	·66254	·66384	·66514	·66643	·66772	·66900	·67028	·67156	·67283	129
79	·67410	·67536	·67662	·67788	·67913	·68037	·68162	·68286	·68409	·68532	124
80	0·68655	·68777	·68899	·69020	·69141	·69262	·69382	·69502	·69622	·69741	121
81	·69860	·69978	·70096	·70214	·70331	·70448	·70564	·70681	·70796	·70912	117
82	·71027	·71141	·71256	·71370	·71483	·71596	·71709	·71822	·71934	·72046	113
83	0·72157	·72268	·72379	·72490	·72600	·72709	·72819	·72928	·73036	·73145	109
84	·73253	·73360	·73468	·73574	·73680	·73786	·73891	·74000	·74100	·74203	106
85	·74306	·74409	·74511	·74613	·74714	·74815	·74915	·75015	·75114	·75213	101
86	0·75311	·75409	·75507	·75604	·75701	·75797	·75892	·75987	·76082	·76177	96
87	·76271	·76364	·76457	·76550	·76642	·76734	·76826	·76917	·77007	·77097	92
88	·77187	·77276	·77365	·77454	·77542	·77630	·77717	·77804	·77891	·77977	88
89	0·78063	·78148	·78233	·78318	·78402	·78486	·78569	·78652	·78735	·78818	84
90	·78900	·78981	·79063	·79144	·79224	·79304	·79384	·79464	·79543	·79622	80
91	·79701	·79779	·79856	·79934	·80011	·80088	·80164	·80240	·80316	·80392	77
92	0·80467	·80542	·80616	·80690	·80764	·80838	·80911	·80984	·81056	·81128	74
93	·81200	·81272	·81343	·81414	·81485	·81556	·81626	·81696	·81765	·81834	71
94	·81903	·81972	·82040	·82108	·82176	·82244	·82311	·82378	·82444	·82511	68
95	0·82577	·82643	·82708	·82773	·82838	·82903	·82967	·83032	·83096	·83159	65
96	·83223	·83286	·83348	·83411	·83473	·83535	·83597	·83659	·83720	·83781	62
97	·83842	·83902	·83963	·84023	·84082	·84142	·84201	·84260	·84319	·84378	60
98	0·84436	·84494	·84552	·84610	·84667	·84724	·84781	·84838	·84894	·84951	57
99	·85007	·85063	·85118	·85174	·85229	·85284	·85338	·85393	·85447	·85501	55
100	·85555	·85608	·85662	·85715	·85768	·85821	·85873	·85925	·85977	·86029	53
101	0·86081	·86133	·86184	·86235	·86286	·86337	·86387	·86438	·86488	·86538	51
102	·86587	·86637	·86686	·86735	·86784	·86833	·86881	·86930	·86978	·87026	49
103	·87074	·87121	·87169	·87216	·87263	·87310	·87357	·87403	·87450	·87500	47
104	0·87542	·87588	·87633	·87678	·87723	·87767	·87811	·87855	·87899	·87942	44
105	·87985	·88028	·88070	·88112	·88154	·88195	·88236	·88277	·88317	·88358	41
106	·88398	·88437	·88477	·88516	·88555	·88593	·88632	·88670	·88701	·88745	38
107	0·88782	·88819	·88856	·88893	·88929	·88965	·89001	·89036	·89071	·89106	36
108	·89141	·89176	·89210	·89244	·89278	·89311	·89345	·89378	·89411	·89444	33
109	·89476	·89508	·89540	·89572	·89604	·89635	·89666	·89697	·89728	·89759	31
110	0·89789	·89819	·89849	·89878	·89908	·89938	·89967	·89995	·90024	·90053	30
111	·90081	·90109	·90137	·90165	·90193	·90220	·90247	·90274	·90301	·90328	27
112	·90355	·90381	·90407	·90433	·90459	·90484	·90510	·90535	·90560	·90585	25

Table IV—*continued.*

$\tan\phi - \tan\theta = C\,[I(V) - I(v)]$, *or* $\phi^{\circ} - \theta^{\circ} = 57 \cdot 3\,C\,[I(V) - I(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
113	0 ·90610	·90635	·90659	·90683	·90708	·90732	·90756	·90780	·90803	·90826	24
114	·90850	·90873	·90896	·90919	·90941	·90964	·90986	·91008	·91030	·91052	23
115	·91074	·91096	·91117	·91138	·91160	·91181	·91202	·91222	·91243	·91264	21
116	0 ·91284	·91304	·91325	·91345	·91365	·91384	·91404	·91423	·91443	·91462	19
117	·91481	·91500	·91519	·91538	·91557	·91575	·91594	·91612	·91630	·91648	18
118	·91666	·91684	·91702	·91720	·91737	·91755	·91772	·91789	·91806	·91823	17
119	0 ·91840	·91857	·91873	·91890	·91907	·91923	·91940	·91956	·91973	·91989	16
120	·92005	·92021	·92038	·92054	·92070	·92086	·92102	·92118	·92134	·92149	16
121	·92165	·92181	·92197	·92212	·92228	·92243	·92259	·92274	·92290	·92305	15
122	0 ·92320	·92335	·92351	·92366	·92381	·92396	·92411	·92426	·92440	·92455	15
123	·92470	·92485	·92499	·92514	·92529	·92543	·92558	·92572	·92586	·92601	14
124	·92615	·92629	·92644	·92658	·92672	·92686	·92700	·92714	·92728	·92742	14
125	0 ·92756	·92769	·92783	·92797	·92810	·92824	·92838	·92851	·92865	·92878	14
126	·92892	·92905	·92918	·92932	·92945	·92958	·92971	·92984	·92997	·93010	13
127	·93023	·93036	·93049	·93062	·93075	·93088	·93100	·93113	·93126	·93139	13
128	0 ·93151	·93164	·93176	·93189	·93201	·93213	·93226	·93238	·93250	·93263	12
129	·93275	·93287	·93299	·93311	·93323	·93335	·93347	·93359	·93371	·93383	12
130	·93395	·93407	·93418	·93430	·93442	·93453	·93465	·93477	·93488	·93500	12
131	0 ·93511	·93522	·93534	·93545	·93556	·93568	·93579	·93590	·93601	·93613	12
132	·93624	·93635	·93646	·93657	·93668	·93679	·93690	·93701	·93712	·93722	11
133	·93733	·93744	·93755	·93765	·93776	·93787	·93797	·93808	·93818	·93829	11
134	0 ·93840	·93850	·93861	·93871	·93881	·93892	·93902	·93912	·93922	·93933	11
135	·93943	·93953	·93963	·93973	·93983	·93993	·94003	·94013	·94023	·94033	10
136	·94043	·94053	·94063	·94072	·94082	·94092	·94102	·94111	·94121	·94130	10
137	0 ·94140	·94150	·94159	·94169	·94178	·94188	·94197	·94206	·94216	·94225	10
138	·94234	·94244	·94253	·94262	·94271	·94281	·94290	·94299	·94308	·94317	9
139	·94326	·94335	·94344	·94353	·94362	·94371	·94380	·94389	·94397	·94406	9
140	0 ·94415	·94424	·94433	·94441	·94450	·94459	·94467	·94476	·94485	·94493	9
141	·94502	·94510	·94519	·94527	·94536	·94544	·94553	·94561	·94569	·94578	8
142	·94586	·94594	·94602	·94611	·94619	·94627	·94635	·94643	·94652	·94660	8

143	0·94668	·94676	·94684	·94692	·94700	·94708	·94716	·94724	·94732	·94739	8
144	·94747	·94755	·94763	·94771	·94778	·94786	·94794	·94802	·94809	·94817	8
145	·94825	·94832	·94840	·94848	·94855	·94863	·94870	·94878	·94885	·94893	8
146	0·94900	·94907	·94915	·94922	·94930	·94937	·94944	·94952	·94959	·94966	7
147	·94974	·94981	·94988	·94995	·95003	·95010	·95017	·95024	·95031	·95039	7
148	·95046	·95053	·95060	·95067	·95074	·95081	·95088	·95095	·95103	·95110	7
149	0·95117	·95124	·95131	·95138	·95145	·95151	·95158	·95165	·95172	·95179	7
150	·95186	·95193	·95200	·95207	·95214	·95220	·95227	·95234	·95241	·95248	7
151	·95254	·95261	·95268	·95274	·95281	·95288	·95294	·95301	·95308	·95314	7
152	0·95321	·95328	·95334	·95341	·95348	·95354	·95361	·95367	·95374	·95380	7
153	·95387	·95393	·95400	·95406	·95413	·95419	·95426	·95432	·95438	·95445	6
154	·95451	·95458	·95464	·95470	·95477	·95483	·95489	·95496	·95502	·95508	6
155	0·95515	·95521	·95527	·95534	·95540	·95546	·95552	·95559	·95565	·95571	6
156	·95577	·95583	·95589	·95596	·95602	·95608	·95614	·95620	·95626	·95632	6
157	·95638	·95644	·95650	·95656	·95662	·95668	·95674	·95680	·95686	·95692	6
158	0·95698	·95704	·95710	·95716	·95722	·95728	·95734	·95739	·95745	·95751	6
159	·95757	·95763	·95769	·95775	·95780	·95786	·95792	·95798	·95804	·95810	6
160	·95815	·95821	·95827	·95833	·95838	·95844	·95850	·95855	·95861	·95867	6
161	0·95872	·95878	·95883	·95889	·95895	·95900	·95906	·95911	·95917	·95923	6
162	·95928	·95934	·95939	·95945	·95950	·95956	·95961	·95967	·95972	·95978	6
163	·95983	·95989	·95994	·95999	·96005	·96010	·96016	·96021	·96026	·96032	5
164	0·96037	·96042	·96048	·96053	·96058	·96064	·96069	·96074	·96080	·96085	5
165	·96090	·96095	·96101	·96106	·96111	·96116	·96122	·96127	·96132	·96137	5
166	·96143	·96148	·96153	·96158	·96163	·96168	·96174	·96179	·96184	·96189	5
167	0·96194	·96199	·96204	·96209	·96214	·96220	·96225	·96230	·96235	·96240	5
168	·96245	·96250	·96255	·96260	·96265	·96270	·96275	·96279	·96284	·96289	5
169	·96294	·96299	·96304	·96309	·96314	·96319	·96324	·96328	·96333	·96338	5
170	0·96343	·96348	·96353	·96358	·96362	·96367	·96372	·96377	·96382	·96387	5
171	·96391	·96396	·96401	·96406	·96410	·96415	·96420	·96425	·96429	·96434	5
172	·96439	·96444	·96448	·96453	·96457	·96462	·96467	·96471	·96476	·96481	5
173	0·96485	·96490	·96494	·96499	·96504	·96508	·96513	·96518	·96522	·96527	5
174	·96531	·96536	·96540	·96545	·96549	·96554	·96558	·96563	·96567	·96572	5
175	·96576	·96580	·96585	·96589	·96594	·96598	·96602	·96607	·96611	·96616	4
176	0·96620	·96624	·96629	·96633	·96638	·96642	·96646	·96651	·96655	·96660	4
177	·96664	·96668	·96673	·96677	·96681	·96686	·96690	·96694	·96699	·96703	4
178	·96707	·96711	·96716	·96720	·96724	·96728	·96733	·96737	·96741	·96745	4

Table IV—*continued.*

$\tan\phi - \tan\theta = C\,[I\,(V) - I\,(v)]$, *or* $\phi^\circ - \theta^\circ = 57 \cdot 3\,C\,[I\,(V) - I\,(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
179	0 ·96749	·96754	·96758	·96762	·96766	·96770	·96774	·96779	·96783	·96787	4
180	·96791	·96795	·96799	·96804	·96808	·96812	·96816	·96820	·96824	·96828	4
181	·96832	·96836	·96840	·96844	·96848	·96852	·96856	·96860	·96864	·96868	4
182	0 ·96872	·96876	·96880	·96884	·96888	·96892	·96896	·96900	·96904	·96908	4
183	·96912	·96916	·96920	·96924	·96928	·96932	·96936	·96940	·96944	·96947	4
184	·96951	·96955	·96959	·96963	·96967	·96971	·96974	·96978	·96982	·96986	4
185	0 ·96990	·96994	·96997	·97001	·97005	·97009	·97013	·97017	·97020	·97024	4
186	·97028	·97032	·97035	·97039	·97043	·97047	·97050	·97054	·97058	·97062	4
187	·97065	·97069	·97073	·97076	·97080	·97084	·97087	·97091	·97095	·97098	4
188	0 ·97102	·97106	·97109	·97113	·97116	·97120	·97124	·97127	·97131	·97134	4
189	·97138	·97142	·97145	·97149	·97152	·97156	·97160	·97163	·97167	·97170	4
190	·97174	·97178	·97181	·97185	·97188	·97192	·97196	·97199	·97203	·97206	4
191	0 ·97210	·97213	·97217	·97220	·97224	·97227	·97230	·97234	·97237	·97241	3
192	·97244	·97248	·97251	·97254	·97258	·97261	·97265	·97268	·97271	·97275	3
193	·97278	·97282	·97285	·97288	·97292	·97295	·97299	·97302	·97305	·97309	3
194	0 ·97312	·97315	·97319	·97322	·97325	·97329	·97332	·97335	·97339	·97342	3
195	·97345	·97348	·97352	·97355	·97358	·97362	·97365	·97368	·97372	·97375	3
196	·97378	·97381	·97385	·97388	·97391	·97394	·97398	·97401	·97404	·97407	3
197	0 ·97411	·97414	·97417	·97420	·97423	·97427	·97430	·97433	·97436	·97439	3
198	·97442	·97446	·97449	·97452	·97455	·97458	·97461	·97464	·97467	·97471	3
199	·97474	·97477	·97480	·97483	·97486	·97489	·97492	·97495	·97498	·97502	3
200	0 ·97505	·97508	·97511	·97514	·97517	·97520	·97523	·97526	·97529	·97532	3
201	·97535	·97538	·97541	·97544	·97547	·97551	·97554	·97557	·97560	·97563	3
202	·97566	·97569	·97572	·97575	·97578	·97581	·97584	·97587	·97590	·97592	3
203	0 ·97595	·97598	·97601	·97604	·97607	·97610	·97613	·97616	·97619	·97622	3
204	·97625	·97628	·97631	·97634	·97637	·97639	·97642	·97645	·97648	·97651	3
205	·97654	·97657	·97660	·97663	·97666	·97669	·97671	·97674	·97677	·97680	3
206	0 ·97683	·97686	·97689	·97691	·97694	·97697	·97700	·97703	·97706	·97708	3
207	·97711	·97714	·97717	·97720	·97723	·97725	·97728	·97731	·97734	·97737	3
208	·97739	·97742	·97745	·97748	·97750	·97753	·97756	·97759	·97761	·97764	3

209	0·97767	·97770	·97772	·97775	·97778	·97781	·97783	·97786	·97789	·97792	3
210	·97794	·97797	·97800	·97803	·97805	·97808	·97811	·97813	·97816	·97819	3
211	·97821	·97824	·97827	·97830	·97832	·97835	·97838	·97840	·97843	·97846	3
212	0·97848	·97851	·97854	·97856	·97859	·97862	·97864	·97867	·97870	·97872	3
213	·97875	·97877	·97880	·97883	·97885	·97888	·97890	·97893	·97896	·97898	3
214	·97901	·97903	·97906	·97909	·97911	·97914	·97916	·97919	·97921	·97924	3
215	0·97927	·97929	·97932	·97934	·97937	·97940	·97942	·97945	·97947	·97950	3
216	·97952	·97955	·97957	·97960	·97962	·97965	·97967	·97970	·97972	·97975	3
217	·97977	·97980	·97982	·97985	·97987	·97990	·97992	·97995	·97997	·98000	3
218	0·98002	·98005	·98007	·98010	·98012	·98015	·98017	·98020	·98022	·98024	2
219	·98027	·98029	·98032	·98034	·98037	·98039	·98041	·98044	·98046	·98049	2
220	·98051	·98054	·98056	·98058	·98061	·98063	·98066	·98068	·98070	·98073	2
221	0·98075	·98078	·98080	·98082	·98085	·98087	·98090	·98092	·98094	·98097	2
222	·98099	·98102	·98104	·98106	·98109	·98111	·98113	·98116	·98118	·98120	2
223	·98123	·98125	·98127	·98130	·98132	·98134	·98137	·98139	·98141	·98144	2
224	0·98146	·98148	·98151	·98153	·98155	·98158	·98160	·98162	·98165	·98167	2
225	·98169	·98171	·98174	·98176	·98178	·98180	·98183	·98185	·98187	·98189	2
226	·98192	·98194	·98196	·98198	·98201	·98203	·98205	·98207	·98210	·98212	2
227	0·98214	·98216	·98219	·98221	·98223	·98225	·98228	·98230	·98232	·98234	2
228	·98237	·98239	·98241	·98243	·98246	·98248	·98250	·98252	·98254	·98256	2
229	·98259	·98261	·98263	·98265	·98268	·98270	·98272	·98274	·98276	·98279	2
230	0·98281	·98283	·98285	·98287	·98289	·98292	·98294	·98296	·98298	·98300	2
231	·98302	·98304	·98307	·98309	·98311	·98313	·98315	·98317	·98319	·98321	2
232	·98324	·98326	·98328	·98330	·98332	·98334	·98336	·98338	·98340	·98343	2
233	0·98345	·98347	·98349	·98351	·98353	·98355	·98357	·98359	·98361	·98364	2
234	·98366	·98368	·98370	·98372	·98374	·98376	·98378	·98380	·98382	·98384	2
235	·98386	·98388	·98390	·98392	·98394	·98396	·98399	·98401	·98403	·98405	2
236	0·98407	·98409	·98411	·98413	·98415	·98417	·98419	·98421	·98423	·98425	2
237	·98427	·98429	·98431	·98433	·98435	·98437	·98439	·98441	·98443	·98445	2
238	·98447	·98449	·98451	·98453	·98455	·98457	·98459	·98461	·98463	·98465	2
239	0·98467	·98469	·98471	·98473	·98475	·98476	·98478	·98480	·98482	·98484	2
240	·98486	·98488	·98490	·98492	·98494	·98496	·98498	·98500	·98502	·98504	2
241	·98506	·98508	·98509	·98511	·98513	·98515	·98517	·98519	·98521	·98523	2
242	0·98525	·98527	·98528	·98530	·98532	·98534	·98536	·98538	·98540	·98542	2
243	·98544	·98546	·98547	·98549	·98551	·98553	·98555	·98557	·98559	·98561	2
244	·98563	·98565	·98566	·98568	·98570	·98572	·98574	·98576	·98577	·98579	2

Table IV—*continued.*

$\tan\phi - \tan\theta = C\,[I(V) - I(v)]$, *or* $\phi^\circ - \theta^\circ = 57 \cdot 3\, C\,[I(V) - I(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
245	0·98581	·98583	·98585	·98587	·98589	·98591	·98592	·98594	·98596	·98598	2
246	·98600	·98602	·98604	·98605	·98607	·98609	·98611	·98613	·98615	·98616	2
247	·98618	·98620	·98622	·98624	·98625	·98627	·98629	·98631	·98633	·98634	2
248	0·98636	·98638	·98640	·98642	·98643	·98645	·98647	·98649	·98650	·98652	2
249	·98654	·98656	·98658	·98659	·98661	·98663	·98665	·98666	·98668	·98670	2
250	·98672	·98673	·98675	·98677	·98679	·98680	·98682	·98684	·98686	·98687	2
251	0·98689	·98691	·98693	·98694	·98696	·98698	·98700	·98701	·98703	·98705	2
252	·98707	·98708	·98710	·98712	·98714	·98715	·98717	·98719	·98720	·98722	2
253	·98724	·98725	·98727	·98729	·98731	·98732	·98734	·98736	·98738	·98739	2
254	0·98741	·98743	·98744	·98746	·98748	·98749	·98751	·98753	·98754	·98756	2
255	·98758	·98759	·98761	·98763	·98764	·98766	·98768	·98769	·98771	·98773	2
256	·98774	·98776	·98778	·98779	·98781	·98783	·98784	·98786	·98788	·98789	2
257	0·98791	·98792	·98794	·98796	·98797	·98799	·98801	·98802	·98804	·98805	2
258	·98807	·98809	·98810	·98812	·98813	·98815	·98817	·98818	·98820	·98821	2
259	·98823	·98825	·98826	·98828	·98829	·98831	·98833	·98834	·98836	·98837	2
260	0·98839	·98841	·98842	·98844	·98845	·98847	·98849	·98850	·98852	·98853	2
261	·98855	·98857	·98858	·98860	·98861	·98863	·98865	·98866	·98868	·98869	2
262	·98871	·98873	·98874	·98876	·98877	·98879	·98881	·98882	·98884	·98885	2
263	0·98887	·98888	·98890	·98891	·98893	·98895	·98896	·98898	·98899	·98901	2
264	·98902	·98904	·98905	·98907	·98908	·98910	·98911	·98913	·98914	·98916	2
265	·98917	·98919	·98920	·98922	·98923	·98925	·98926	·98928	·98929	·98931	2
266	0·98932	·98934	·98935	·98937	·98938	·98940	·98941	·98943	·98944	·98946	2
267	·98947	·98949	·98950	·98952	·98953	·98955	·98956	·98958	·98959	·98961	2
268	·98962	·98964	·98965	·98967	·98968	·98970	·98971	·98973	·98974	·98975	1
269	0·98977	·98978	·98980	·98981	·98983	·98984	·98986	·98987	·98988	·98990	1
270	·98991	·98993	·98994	·98996	·98997	·98999	·99000	·99001	·99003	·99004	1
271	·99006	·99007	·99009	·99010	·99012	·99013	·99014	·99016	·99017	·99019	1
272	0·99020	·99021	·99023	·99024	·99026	·99027	·99028	·99030	·99031	·99033	1
273	·99034	·99035	·99037	·99038	·99040	·99041	·99042	·99044	·99045	·99047	1
274	·99048	·99049	·99051	·99052	·99053	·99055	·99056	·99058	·99059	·99060	1

275	0 ·99062	·99063	·99064	·99066	·99067	·99068	·99070	·99071	·99072	·99074	1
276	·99075	·99076	·99078	·99079	·99081	·99082	·99083	·99085	·99086	·99087	1
277	·99089	·99090	·99092	·99093	·99094	·99096	·99097	·99098	·99100	·99101	1
278	0 ·99102	·99104	·99105	·99106	·99108	·99109	·99110	·99112	·99113	·99114	1
279	·99116	·99117	·99118	·99120	·99121	·99122	·99124	·99125	·99126	·99127	1
280	·99129	·99130	·99131	·99133	·99134	·99135	·99137	·99138	·99139	·99141	1
281	0 ·99142	·99143	·99144	·99146	·99147	·99148	·99150	·99151	·99152	·99153	1
282	·99155	·99156	·99157	·99158	·99160	·99161	·99162	·99163	·99165	·99166	1
283	·99167	·99168	·99170	·99171	·99172	·99174	·99175	·99176	·99178	·99179	1
284	0 ·99180	·99181	·99183	·99184	·99185	·99186	·99188	·99189	·99190	·99191	1
285	·99193	·99194	·99195	·99196	·99198	·99199	·99200	·99201	·99203	·99204	1
286	·99205	·99206	·99208	·99209	·99210	·99211	·99213	·99214	·99215	·99216	1
287	0 ·99217	·99219	·99220	·99221	·99222	·99224	·99225	·99226	·99227	·99229	1
288	·99230	·99231	·99232	·99234	·99235	·99236	·99237	·99238	·99239	·99241	1
289	·99242	·99243	·99244	·99245	·99247	·99248	·99249	·99250	·99252	·99253	1
290	0 ·99254	·99255	·99256	·99258	·99259	·99260	·99261	·99262	·99263	·99265	1
291	·99266	·99267	99268	·99269	·99270	·99272	·99273	·99274	·99275	·99276	1
292	·99277	·99279	·99280	·99281	·99282	·99283	·99285	·99286	·99287	·99288	1
293	0 ·99289	·99290	·99292	·99293	·99294	·99295	·99296	·99297	·99299	·99300	1
294	·99301	·99302	·99303	·99304	·99305	·99307	·99308	·99309	·99310	·99311	1
295	·99312	·99313	·99314	·99316	·99317	·99318	·99319	·99320	·99321	·99322	1
296	0 ·99323	·99325	·99326	·99327	·99328	·99329	·99330	·99331	·99333	·99334	1
297	·99335	·99336	·99337	·99338	·99339	·99340	·99341	·99343	·99344	·99345	1
298	·99346	·99347	·99348	·99349	·99350	·99351	·99352	·99354	·99355	·99356	1
299	0 ·99357	·99358	·99359	·99360	·99361	·99362	·99363	·99365	·99366	·99367	1
300	·99368	·99369	·99370	·99371	·99372	·99373	·99374	·99375	·99376	·99378	1
301	·99379	·99380	·99381	·99382	·99383	·99384	·99385	·99386	·99387	·99388	1
302	0 ·99389	·99391	·99392	·99393	·99394	·99395	·99396	·99397	·99398	·99399	1
303	·99400	·99401	·99402	·99403	·99404	·99406	·99407	·99408	·99409	·99410	1
304	·99411	·99412	·99413	·99414	·99415	·99416	·99417	·99418	·99419	·99420	1
305	0 ·99421	·99422	·99423	·99424	·99425	·99426	·99427	·99428	·99429	·99430	1
306	·99431	·99432	·99433	·99434	·99435	·99437	·99438	·99439	·99440	·99441	1
307	·99442	·99443	·99444	·99445	·99446	·99447	·99448	·99449	·99450	·99451	1
308	0 ·99452	·99453	·99454	·99455	·99456	·99457	·99458	·99459	·99460	·99461	1
309	·99462	·99463	·99464	·99465	·99466	·99467	·99468	·99469	·99470	·99471	1
310	·99472	·99473	·99474	·99475	·99476	·99477	·99478	·99479	·99480	·99481	1

Table IV—*continued.*

$\tan\phi - \tan\theta = C\,[I\,(V) - I\,(v)]$, *or* $\phi^\circ - \theta^\circ = 57 \cdot 3\,C\,[I\,(V) - I\,(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
311	0 ·99482	·99483	·99484	·99485	·99486	·99487	·99488	·99489	·99490	·99491	1
312	·99492	·99493	·99494	·99495	·99496	·99497	·99498	·99499	·99500	·99501	1
313	·99502	·99503	·99504	·99505	·99506	·99506	·99507	·99508	·99509	·99510	1
314	0 ·99511	·99512	·99513	·99514	·99515	·99516	·99517	·99518	·99519	·99520	1
315	·99521	·99522	·99523	·99524	·99525	·99525	·99526	·99527	·99528	·99529	1
316	·99530	·99531	·99532	·99533	·99534	·99535	·99536	·99537	·99538	·99539	1
317	0 ·99540	·99541	·99542	·99543	·99544	·99544	·99545	·99546	·99547	·99548	1
318	·99549	·99550	·99551	·99552	·99553	·99554	·99555	·99556	·99557	·99558	1
319	·99559	·99560	·99561	·99562	·99563	·99563	·99564	·99565	·99566	·99567	1
320	0 ·99568	·99569	·99570	·99571	·99572	·99572	·99573	·99574	·99575	·99576	1
321	·99577	·99578	·99579	·99580	·99581	·99581	·99582	·99583	·99584	·99585	1
322	·99586	·99587	·99588	·99589	·99590	·99590	·99591	·99592	·99593	·99594	1
323	0 ·99595	·99596	·99597	·99598	·99599	·99599	·99600	·99601	·99602	·99603	1
324	·99604	·99605	·99606	·99607	·99607	·99608	·99609	·99610	·99611	·99612	1
325	·99613	·99614	·99614	·99615	·99616	·99617	·99618	·99619	·99620	·99621	1
326	0 ·99621	·99622	·99623	·99624	·99625	·99626	·99627	·99628	·99628	·99629	1
327	·99630	·99631	·99632	·99633	·99634	·99634	·99635	·99636	·99637	·99638	1
328	·99639	·99640	·99640	·99641	·99642	·99643	·99644	·99645	·99645	·99646	1
329	0 ·99647	·99648	·99649	·99650	·99651	·99652	·99652	·99653	·99654	·99655	1
330	·99656	·99657	·99658	·99658	·99659	·99660	·99661	·99662	·99663	·99664	1
331	·99664	·99665	·99666	·99667	·99668	·99669	·99669	·99670	·99671	·99672	1
332	0 ·99673	·99673	·99674	·99675	·99676	·99677	·99678	·99678	·99679	·99680	1
333	·99681	·99682	·99683	·99683	·99684	·99685	·99686	·99687	·99688	·99688	1
334	·99689	·99690	·99691	·99692	·99692	·99693	·99694	·99695	·99696	·99696	1
335	0 ·99697	·99698	·99699	·99700	·99700	·99701	·99702	·99703	·99704	·99704	1
336	·99705	·99706	·99707	·99708	·99708	·99709	·99710	·99711	·99712	·99712	1
337	·99713	·99714	·99715	·99716	·99716	·99717	·99718	·99719	·99720	·99720	1
338	0 ·99721	·99722	·99723	·99724	·99724	·99725	·99726	·99727	·99728	·99728	1
339	·99729	·99730	·99731	·99731	·99732	·99733	·99734	·99734	·99735	·99736	1
340	·99737	·99737	·99738	·99739	·99740	·99741	·99742	·99742	·99743	·99744	1

341	0 ·99745	·99745	·99746	·99747	·99748	·99748	·99749	·99750	·99751	·99751	1
342	·99752	·99753	·99754	·99755	·99755	·99756	·99757	·99758	·99759	·99759	1
343	·99760	·99761	·99762	·99762	·99763	·99764	·99765	·99765	·99766	·99767	1
344	0 ·99768	·99768	·99769	·99770	·99771	·99771	·99772	·99773	·99774	·99774	1
345	·99775	·99776	·99777	·99777	·99778	·99779	·99780	·99780	·99781	·99782	1
346	·99783	·99783	·99784	·99785	·99786	·99786	·99787	·99788	·99789	·99789	1
347	0 ·99790	·99791	·99792	·99792	·99793	·99794	·99795	·99795	·99796	·99797	1
348	·99797	·99798	·99799	·99799	·99800	·99801	·99802	·99802	·99803	·99804	1
349	·99805	·99805	·99806	·99807	·99808	·99808	·99809	·99810	·99810	·99811	1
350	0 ·99812	·99812	·99813	·99814	·99815	·99815	·99816	·99817	·99818	·99818	1
351	·99819	·99820	·99821	·99821	·99822	·99823	·99824	·99824	·99825	·99826	1
352	·99826	·99827	·99828	·99828	·99829	·99830	·99831	·99831	·99832	·99833	1
353	0 ·99833	·99834	·99835	·99835	·99836	·99837	·99838	·99838	·99839	·99840	1
354	·99840	·99841	·99842	·99842	·99843	·99844	·99845	·99845	·99846	·99847	1
355	·99847	·99848	·99849	·99849	·99850	·99851	·99852	·99852	·99853	·99854	1
356	0 ·99854	·99855	·99856	·99856	·99857	·99858	·99859	·99859	·99860	·99861	1
357	·99861	·99862	·99863	·99864	·99864	·99865	·99866	·99866	·99867	·99868	1
358	·99868	·99869	·99870	·99870	·99871	·99872	·99872	·99873	·99874	·99874	1
359	0 ·99875	·99876	·99876	·99877	·99878	·99878	·99879	·99880	·99880	·99881	1
360	·99882	·99882	·99883	·99884	·99884	·99885	·99886	·99886	·99887	·99888	1
361	·99888	·99889	·99890	·99890	·99891	·99892	·99892	·99893	·99894	·99894	1
362	0 ·99895	·99895	·99896	·99897	·99897	·99898	·99899	·99899	·99900	·99901	1
363	·99901	·99902	·99903	·99903	·99904	·99905	·99905	·99906	·99907	·99907	1
364	·99908	·99908	·99909	·99910	·99910	·99911	·99912	·99912	·99913	·99914	1
365	0 ·99914	·99915	·99916	·99916	·99917	·99918	·99918	·99919	·99920	·99920	1
366	·99921	·99921	·99922	·99923	·99923	·99924	·99925	·99925	·99926	·99927	1
367	·99927	·99928	·99929	·99929	·99930	·99931	·99931	·99932	·99932	·99933	1
368	0 ·99934	·99934	·99935	·99936	·99936	·99937	·99937	·99938	·99939	·99939	1
369	·99940	·99491	·99941	·99942	·99943	·99943	·99944	·99944	·99945	·99946	1
370	·99946	·99947	·99948	·99948	·99949	·99949	·99950	·99951	·99951	·99952	1
371	0 ·99952	·99953	·99954	·99954	·99955	·99955	·99956	·99957	·99957	·99958	1
372	·99958	·99959	·99960	·99960	·99961	·99962	·99962	·99963	·99964	·99964	1
373	·99965	·99965	·99966	·99967	·99967	·99968	·99968	·99969	·99970	·99970	1
374	0 ·99971	·99971	·99972	·99973	·99973	·99974	·99974	·99975	·99976	·99976	1
375	·99977	·99977	·99978	·99979	·99979	·99980	·99980	·99981	·99982	·99982	1
376	·99983	·99983	·99984	·99985	·99985	·99986	·99986	·99987	·99987	·99988	1

Table IV—*continued.*

$\tan\phi - \tan\theta = C[I(V) - I(v)]$, *or* $\phi^\circ - \theta^\circ = 57 \cdot 3\, C[I(V) - I(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
377	0 ·99989	·99989	·99990	·99990	·99991	·99991	·99992	·99993	·99993	·99994	1
378	·99994	·99995	·99996	·99996	·99997	·99997	·99998	·99999	·99999	1 ·00000	1
379	1 ·00000	1 ·00001	1 ·00002	1 ·00002	1 ·00003	1 ·00003	1 ·00004	1 ·00004	1 ·00005	1 ·00006	1
380	1 ·00006	·00007	·00007	·00008	·00008	·00009	·00009	·00010	·00011	·00011	1
381	·00012	·00012	·00013	·00014	·00014	·00015	·00015	·00016	·00017	·00017	1
382	·00018	·00018	·00019	·00019	·00020	·00021	·00021	·00022	·00022	·00023	1
383	1 ·00023	·00024	·00025	·00025	·00026	·00026	·00027	·00028	·00028	·00029	1
384	·00029	·00030	·00030	·00031	·00031	·00032	·00033	·00033	·00034	·00034	1
385	·00035	·00035	·00036	·00036	·00037	·00038	·00038	·00039	·00039	·00040	1
386	1 ·00040	·00041	·00041	·00042	·00042	·00043	·00044	·00044	·00045	·00045	1
387	·00046	·00046	·00047	·00047	·00048	·00049	·00049	·00050	·00050	·00051	1
388	·00051	·00052	·00052	·00053	·00053	·00054	·00055	·00055	·00056	·00056	1
389	1 ·00057	·00057	·00058	·00058	·00059	·00059	·00060	·00061	·00061	·00062	1
390	·00062	·00063	·00063	·00064	·00064	·00065	·00066	·00066	·00067	·00067	1
391	·00068	·00068	·00069	·00069	·00070	·00070	·00071	·00071	·00072	·00072	1
392	1 ·00073	·00073	·00074	·00074	·00075	·00076	·00076	·00077	·00077	·00078	1
393	·00078	·00079	·00079	·00080	·00080	·00081	·00081	·00082	·00082	·00083	1
394	·00083	·00084	·00084	·00085	·00085	·00086	·00087	·00087	·00088	·00088	1
395	1 ·00089	·00089	·00090	·00090	·00091	·00091	·00092	·00092	·00093	·00093	1
396	·00094	·00094	·00095	·00095	·00096	·00097	·00097	·00098	·00098	·00099	1
397	·00099	·00100	·00100	·00101	·00101	·00102	·00102	·00103	·00103	·00104	1
398	1 ·00104	·00105	·00105	·00106	·00106	·00107	·00107	·00108	·00108	·00109	1
399	·00109	·00110	·00110	·00111	·00111	·00112	·00112	·00113	·00113	·00114	1
400	·00114										

TABLE V.

Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
50	0 ·00	0 ·05	0 ·26	0 ·59	1 ·06	1 ·65	2 ·37	3 ·22	4 ·20	5 ·31	0 ·59
51	6 ·54	7 ·89	9 ·37	10 ·98	12 ·70	14 ·53	16 ·49	18 ·58	20 ·78	23 ·10	1 ·83
52	25 ·54	28 ·10	30 ·74	33 ·52	36 ·41	39 ·41	42 ·52	45 ·74	49 ·07	52 ·52	3 ·00
53	56 ·07	59 ·73	63 ·50	67 ·37	71 ·34	75 ·41	79 ·59	83 ·88	88 ·27	92 ·76	4 ·07
54	97 ·35	102 ·03	106 ·82	111 ·70	116 ·68	121 ·76	126 ·95	132 ·23	137 ·58	143 ·04	5 ·08
55	148 ·60	154 ·24	159 ·97	165 ·80	171 ·72	177 ·74	183 ·84	190 ·03	196 ·31	202 ·67	6 ·02
56	209 ·12	215 ·66	222 ·28	228 ·98	235 ·78	242 ·67	249 ·63	256 ·67	263 ·80	271 ·01	6 ·89
57	278 ·30	285 ·66	293 ·11	300 ·65	308 ·27	315 ·96	323 ·71	331 ·55	339 ·47	347 ·47	7 ·69
58	355 ·54	363 ·68	371 ·89	380 ·19	388 ·56	397 ·00	405 ·51	414 ·09	422 ·75	431 ·47	8 ·44
59	440 ·27	449 ·14	458 ·07	467 ·08	476 ·16	485 ·30	494 ·51	503 ·78	513 ·13	522 ·54	9 ·14
60	532 ·01	541 ·54	551 ·14	560 ·81	570 ·54	580 ·33	590 ·19	600 ·11	610 ·10	620 ·16	9 ·79
61	630 ·29	640 ·48	650 ·72	661 ·01	671 ·36	681 ·77	692 ·24	702 ·76	713 ·34	723 ·98	10 ·41
62	734 ·68	745 ·43	756 ·23	767 ·09	778 ·01	788 ·99	800 ·03	811 ·13	822 ·28	833 ·49	10 ·98
63	844 ·76	856 ·08	867 ·45	878 ·87	890 ·34	901 ·86	913 ·43	925 ·05	936 ·72	948 ·44	11 ·52
64	960 ·20	972 ·01	983 ·87	995 ·78	*007 ·74	*019 ·76	*031 ·83	*043 ·95	*056 ·12	*068 ·34	12 ·02
65	1 080 ·61	092 ·92	105 ·28	117 ·68	130 ·12	142 ·61	155 ·14	167 ·71	180 ·32	192 ·97	12 ·49
66	205 ·67	218 ·41	231 ·20	244 ·04	256 ·92	269 ·84	282 ·81	295 ·82	308 ·87	321 ·97	12 ·92
67	335 ·11	348 ·29	361 ·51	374 ·77	388 ·06	401 ·39	414 ·76	428 ·17	441 ·61	455 ·00	13 ·33
68	1 468 ·61	482 ·17	495 ·77	509 ·41	523 ·08	536 ·79	550 ·54	564 ·33	578 ·16	592 ·03	13 ·71
69	605 ·94	619 ·88	633 ·86	647 ·87	661 ·91	675 ·99	690 ·10	704 ·24	718 ·41	732 ·62	14 ·08
70	746 ·86	761 ·14	775 ·45	789 ·79	804 ·17	818 ·58	833 ·02	847 ·50	862 ·01	876 ·55	14 ·41
71	1 891 ·13	905 ·74	920 ·38	935 ·05	949 ·75	964 ·48	979 ·23	994 ·01	*008 ·82	*023 ·66	14 ·73
72	2 038 ·53	053 ·43	068 ·36	083 ·32	098 ·31	113 ·33	128 ·38	143 ·46	158 ·57	173 ·71	15 ·02
73	188 ·88	204 ·07	219 ·29	234 ·54	249 ·81	265 ·11	280 ·43	295 ·78	311 ·15	326 ·54	15 ·30
74	2 341 ·95	357 ·39	372 ·85	388 ·34	403 ·86	419 ·41	434 ·99	450 ·60	466 ·24	481 ·91	15 ·55
75	497 ·61	513 ·33	529 ·07	544 ·83	560 ·61	576 ·41	592 ·23	608 ·07	623 ·93	639 ·80	15 ·80
76	655 ·69	671 ·60	687 ·53	703 ·49	719 ·48	735 ·50	751 ·55	767 ·63	783 ·73	799 ·86	16 ·02

Table V—*continued.*

Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
77	2 816·02	832·20	848·40	864·61	880·83	897·07	913·32	929·58	945·86	962·15	16·24
78	978·46	994·79	*011·15	*027·53	*043·94	*060·37	*076·83	*093·31	*109·81	*126·34	16·43
79	3 142·89	159·46	176·04	192·63	209·24	225·86	242·49	259·14	275·80	292·47	16·62
80	3 309·16	325·87	342·60	359·35	376·12	392·91	409·72	426·55	443·40	460·27	16·79
81	477·16	494·06	510·98	527·91	544·85	561·80	578·77	595·75	612·74	629·75	16·95
82	646·77	663·81	680·86	697·93	715·02	732·12	749·24	766·38	783·53	800·70	17·10
83	3 817·89	835·09	852·30	869·53	886·77	904·02	921·28	938·55	955·83	973·12	17·25
84	990·42	*007·72	*025·01	*042·28	*059·54	*076·78	*094·01	*111·23	*128·43	*145·62	17·24
85	4 162·80	179·96	197·10	214·22	231·32	248·41	265·48	282·53	299·56	316·57	17·09
86	4 333·56	350·54	367·50	384·44	401·37	418·28	435·17	452·04	468·90	485·74	16·91
87	502·56	519·36	536·15	552·92	569·67	586·40	603·11	619·80	636·47	653·13	16·73
88	669·77	686·39	702·99	719·57	736·13	752·67	769·20	785·71	802·20	818·67	16·54
89	4 835·12	851·55	867·96	484·35	900·72	917·08	933·42	949·74	966·04	982·32	16·36
90	998·58	*014·82	*031·04	*047·24	*063·42	*079·58	*095·73	*111·86	*127·97	*144·06	16·16
91	5 160·13	176·18	192·21	208·22	224·21	240·18	256·13	272·06	287·97	303·86	15·97
92	5 319·72	335·56	351·39	367·20	382·99	398·76	414·51	430·24	445·96	461·66	15·77
93	477·34	493·00	508·64	524·26	539·86	555·44	570·99	586·52	602·03	617·52	15·58
94	632·99	648·45	663·89	679·31	694·71	710·08	725·43	740·76	756·07	771·36	15·37
95	5 786·63	801·88	817·11	832·32	847·51	862·68	877·84	892·98	908·10	923·20	15·17
96	938·28	953·34	968·38	983·40	998·40	*013·38	*028·34	*043·27	*058·18	*073·07	14·98
97	6 087·94	102·79	117·62	132·43	147·23	162·01	176·77	191·51	206·23	220·93	14·78
98	6 235·61	250·27	264·91	279·53	294·13	308·71	323·27	337·81	352·33	366·83	14·58
99	381·30	395·76	410·20	424·62	439·02	453·40	467·76	482·10	496·42	510·73	14·38
100	525·02	539·29	553·54	567·77	581·98	596·17	610·34	624·48	638·60	652·70	14·19
101	6 666·78	680·85	694·90	708·93	722·94	736·93	750·90	764·85	778·78	792·70	13·99
102	806·60	820·48	834·34	848·18	862·00	875·80	889·58	903·34	917·08	930·80	13·80
103	944·49	958·17	971·83	985·47	999·10	*012·71	*026·30	*039·87	*053·42	*066·95	13·61
104	7 080·46	093·92	107·32	120·66	133·94	147·16	160·32	173·42	186·46	199·44	13·22
105	212·36	225·22	238·02	250·76	263·44	276·05	288·60	301·09	313·52	325·89	12·61
106	338·20	350·46	362·66	374·81	386·90	398·93	410·91	422·83	434·70	446·51	12·03

107	7 458 ·27	469 ·97	481 ·62	493 ·21	504 ·75	516 ·24	527 ·67	539 ·05	550 ·37	561 ·64	11 ·49
108	572 ·86	584 ·03	595 ·15	606 ·21	617 ·22	628 ·18	639 ·09	649 ·95	660 ·76	671 ·52	10 ·96
109	682 ·23	692 ·89	703 ·50	714 ·06	724 ·58	735 ·05	745 ·47	755 ·84	766 ·16	776 ·43	10 ·47
110	7 786 ·65	796 ·83	806 ·96	817 ·05	827 ·09	837 ·08	847 ·03	856 ·93	866 ·78	876 ·59	9 ·99
111	886 ·35	896 ·07	905 ·74	915 ·37	924 ·96	934 ·50	944 ·00	953 ·46	962 ·87	972 ·24	9 ·54
112	981 ·57	990 ·85	*000 ·09	*009 ·29	*018 ·45	*027 ·57	*036 ·65	*045 ·69	*054 ·68	*063 ·63	9 ·12
113	8 072 ·54	081 ·41	090 ·24	099 ·03	107 ·78	116 ·49	125 ·16	133 ·79	142 ·39	150 ·95	8 ·71
114	159 ·47	167 ·95	176 ·39	184 ·79	193 ·15	201 ·48	209 ·77	218 ·02	226 ·23	234 ·40	8 ·33
115	242 ·54	250 ·64	258 ·78	266 ·73	274 ·72	282 ·68	290 ·60	298 ·49	306 ·35	314 ·17	7 ·96
116	8 321 ·96	329 ·71	337 ·43	345 ·11	352 ·76	360 ·37	367 ·95	375 ·49	383 ·00	390 ·47	7 ·61
117	397 ·90	405 ·30	412 ·67	420 ·01	427 ·32	434 ·60	441 ·85	449 ·07	456 ·26	463 ·41	7 ·28
118	470 ·53	477 ·62	484 ·68	491 ·71	498 ·71	505 ·68	512 ·61	519 ·51	526 ·38	533 ·22	6 ·97
119	8 540 ·03	546 ·83	553 ·62	560 ·40	567 ·16	573 ·91	580 ·65	587 ·38	594 ·10	600 ·81	6 ·75
120	607 ·51	614 ·20	620 ·88	627 ·55	634 ·21	640 ·86	647 ·50	654 ·13	660 ·75	667 ·37	6 ·65
121	673 ·98	680 ·58	687 ·17	693 ·75	700 ·32	706 ·87	713 ·41	719 ·94	726 ·46	732 ·97	6 ·55
122	8 739 ·47	745 ·96	752 ·45	758 ·93	765 ·40	771 ·86	778 ·31	784 ·75	791 ·18	797 ·60	6 ·46
123	804 ·01	810 ·41	816 ·80	823 ·18	829 ·55	835 ·91	842 ·26	848 ·61	854 ·95	861 ·28	6 ·36
124	867 ·60	873 ·91	880 ·21	886 ·50	892 ·78	890 ·05	905 ·31	911 ·56	917 ·80	924 ·04	6 ·27
125	8 930 ·27	936 ·49	942 ·70	948 ·90	955 ·09	961 ·27	967 ·44	973 ·60	979 ·76	985 ·91	6 ·18
126	992 ·05	998 ·18	*004 ·30	*010 ·41	*016 ·51	*022 ·60	*028 ·68	*034 ·75	*040 ·82	*046 ·88	6 ·09
127	9 052 ·93	058 ·97	065 ·00	071 ·03	077 ·05	083 ·06	089 ·06	095 ·05	101 ·03	107 ·00	6 ·01
128	9 112 ·96	118 ·91	124 ·86	130 ·80	136 ·73	142 ·65	148 ·56	154 ·46	160 ·35	166 ·23	5 ·92
129	172 ·10	177 ·97	183 ·83	189 ·68	195 ·53	201 ·37	207 ·20	213 ·03	218 ·85	224 ·66	5 ·84
130	230 ·46	236 ·25	242 ·03	247 ·80	253 ·57	259 ·33	265 ·08	270 ·82	276 ·55	282 ·27	5 ·76
131	9 287 ·98	293 ·69	299 ·39	305 ·08	310 ·76	316 ·44	322 ·11	327 ·77	333 ·42	339 ·06	5 ·68
132	344 ·69	350 ·32	355 ·94	361 ·55	367 ·15	372 ·75	378 ·34	383 ·92	389 ·49	395 ·06	5 ·60
133	400 ·62	406 ·17	411 ·71	417 ·24	422 ·77	428 ·29	433 ·80	439 ·31	444 ·81	450 ·30	5 ·52
134	9 455 ·78	461 ·26	466 ·73	472 ·19	477 ·64	483 ·08	488 ·51	493 ·94	499 ·36	504 ·77	5 ·44
135	510 ·18	515 ·58	520 ·97	526 ·35	531 ·73	537 ·10	542 ·46	547 ·82	553 ·17	558 ·51	5 ·37
136	563 ·84	569 ·17	574 ·49	579 ·80	585 ·10	590 ·40	595 ·69	600 ·97	606 ·24	611 ·51	5 ·30
137	9 616 ·77	622 ·02	627 ·27	632 ·51	637 ·74	642 ·96	648 ·18	653 ·39	658 ·59	663 ·79	5 ·22
138	668 ·98	674 ·16	679 ·34	684 ·51	689 ·67	694 ·82	699 ·97	705 ·11	710 ·25	715 ·38	5 ·15
139	720 ·50	725 ·61	730 ·72	735 ·82	740 ·91	746 ·00	751 ·08	756 ·15	761 ·21	766 ·27	5 ·09
140	9 771 ·32	776 ·36	781 ·40	786 ·43	791 ·45	796 ·47	801 ·48	806 ·49	811 ·49	816 ·49	5 ·02
141	821 ·48	826 ·46	831 ·43	836 ·40	841 ·36	846 ·31	851 ·26	856 ·20	861 ·13	866 ·06	4 ·95
142	870 ·98	875 ·89	880 ·80	885 ·70	890 ·59	895 ·48	900 ·36	905 ·23	910 ·10	914 ·96	4 ·89

Table V—*continued.*

Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
143	9 919 ·82	924 ·67	929 ·51	934 ·35	939 ·18	944 ·00	948 ·82	953 ·63	958 ·44	963 ·24	4 ·82
144	963 ·03	972 ·82	977 ·60	982 ·37	987 ·14	991 ·90	996 ·65	*001 ·40	*006 ·14	*010 ·88	4 ·76
145	10 015 ·61	020 ·33	025 ·05	029 ·76	034 ·47	039 ·17	043 ·87	048 ·56	053 ·24	057 ·92	4 ·70
146	10 062 ·59	067 ·26	071 ·93	076 ·59	081 ·25	085 ·91	090 ·56	095 ·21	099 ·86	104 ·51	4 ·66
147	109 ·15	113 ·79	118 ·43	123 ·07	127 ·71	132 ·34	136 ·97	141 ·60	146 ·23	150 ·86	4 ·63
148	155 ·48	160 ·10	164 ·72	169 ·34	173 ·96	178 ·57	183 ·18	187 ·79	192 ·40	197 ·00	4 ·61
149	10 201 ·60	206 ·20	210 ·80	215 ·40	219 ·99	224 ·58	229 ·17	233 ·76	238 ·35	242 ·93	4 ·59
150	247 ·51	252 ·09	256 ·67	261 ·25	265 ·82	270 ·39	274 ·96	279 ·53	284 ·09	288 ·65	4 ·57
151	293 ·21	297 ·77	302 ·33	306 ·88	311 ·43	315 ·98	320 ·53	325 ·08	329 ·62	334 ·16	4 ·55
152	10 338 ·70	343 ·24	347 ·78	352 ·31	356 ·84	361 ·37	365 ·90	370 ·42	374 ·94	379 ·46	4 ·53
153	383 ·98	388 ·50	393 ·01	397 ·52	402 ·03	406 ·54	411 ·05	415 ·55	420 ·05	424 ·55	4 ·51
154	429 ·05	433 ·55	438 ·04	442 ·53	447 ·02	451 ·51	456 ·00	460 ·48	464 ·96	469 ·44	4 ·49
155	10 473 ·92	478 ·40	482 ·87	487 ·34	491 ·81	496 ·28	500 ·75	505 ·21	509 ·67	514 ·13	4 ·47
156	518 ·59	523 ·05	527 ·50	531 ·95	536 ·40	540 ·85	545 ·30	549 ·74	554 ·18	558 ·62	4 ·45
157	563 ·06	567 ·50	571 ·93	576 ·36	580 ·79	585 ·22	589 ·65	594 ·08	598 ·50	602 ·92	4 ·43
158	10 607 ·34	611 ·76	616 ·18	620 ·59	625 ·00	629 ·41	633 ·82	638 ·22	642 ·62	647 ·02	4 ·41
159	651 ·42	655 ·82	660 ·21	664 ·60	668 ·99	673 ·38	677 ·77	682 ·16	686 ·54	690 ·92	4 ·39
160	695 ·30	699 ·68	704 ·06	708 ·44	712 ·81	717 ·18	721 ·55	725 ·92	730 ·28	734 ·64	4 ·37
161	10 739 ·00	743 ·36	747 ·72	752 ·07	756 ·42	760 ·77	765 ·12	769 ·47	773 ·81	778 ·15	4 ·35
162	782 ·49	786 ·83	791 ·17	795 ·51	799 ·84	804 ·17	808 ·50	812 ·83	817 ·16	821 ·48	4 ·33
163	825 ·80	830 ·12	834 ·44	838 ·76	843 ·08	847 ·39	851 ·70	856 ·01	860 ·32	864 ·62	4 ·31
164	10 868 ·92	873 ·22	877 ·52	881 ·82	886 ·12	890 ·41	894 ·70	898 ·99	903 ·28	907 ·57	4 ·29
165	911 ·85	916 ·13	920 ·41	924 ·69	928 ·97	933 ·25	937 ·53	941 ·80	946 ·07	950 ·34	4 ·28
166	954 ·61	958 ·88	963 ·14	967 ·40	971 ·66	975 ·92	980 ·18	984 ·43	988 ·68	992 ·93	4 ·26
167	10 997 ·18	*001 ·43	*005 ·68	*009 ·92	*014 ·16	*018 ·40	*022 ·64	*026 ·88	*031 ·11	*035 ·34	4 ·24
168	11 039 ·57	043 ·80	048 ·03	052 ·26	056 ·48	060 ·70	064 ·92	069 ·14	073 ·36	077 ·57	4 ·22
169	081 ·78	085 ·99	090 ·20	094 ·41	098 ·61	102 ·81	107 ·01	111 ·21	115 ·41	119 ·61	4 ·20
170	12 123 ·80	127 ·99	132 ·18	136 ·37	140 ·56	144 ·75	148 ·94	153 ·12	157 ·30	161 ·48	4 ·19
171	165 ·66	169 ·84	174 ·01	178 ·18	182 ·35	186 ·52	190 ·69	194 ·86	199 ·02	203 ·18	4 ·17
172	207 ·34	211 ·50	215 ·66	219 ·82	223 ·97	228 ·12	232 ·27	236 ·42	240 ·57	244 ·71	4 ·15

173	11 248 ·85	252 ·99	257 ·13	261 ·27	265 ·41	269 ·54	273 ·67	277 ·80	281 ·93	286 ·06	4 ·13
174	290 ·19	294 ·31	298 ·43	302 ·55	306 ·67	310 ·79	314 ·91	319 ·02	323 ·13	327 ·24	4 ·12
175	331 ·35	335 ·46	339 ·57	343 ·67	347 ·77	351 ·87	355 ·97	360 ·07	364 ·16	368 ·25	4 ·10
176	11 372 ·34	376 ·43	380 ·52	384 ·61	388 ·69	392 ·77	396 ·85	400 ·93	405 ·01	409 ·09	4 ·08
177	413 ·17	417 ·24	421 ·31	425 ·38	429 ·45	433 ·52	437 ·59	441 ·66	445 ·72	449 ·78	4 ·07
178	453 ·84	457 ·90	461 ·96	466 ·01	470 ·06	474 ·11	478 ·16	482 ·21	486 ·25	490 ·29	4 ·05
179	11 494 ·33	498 ·37	502 ·41	506 ·45	510 ·49	514 ·52	518 ·55	522 ·58	526 ·61	530 ·64	4 ·03
180	534 ·67	538 ·69	542 ·71	546 ·73	550 ·75	554 ·77	558 ·79	562 ·81	566 ·82	570 ·83	4 ·02
181	574 ·84	578 ·85	582 ·86	586 ·87	590 ·87	594 ·87	598 ·87	602 ·87	606 ·87	610 ·86	4 ·00
182	11 614 ·85	618 ·84	622 ·83	626 ·82	630 ·81	634 ·80	638 ·79	642 ·77	646 ·75	650 ·73	3 ·99
183	654 ·71	658 ·69	662 ·67	666 ·64	670 ·61	674 ·58	678 ·55	682 ·52	686 ·49	690 ·45	3 ·97
184	694 ·41	698 ·37	702 ·33	706 ·29	710 ·25	714 ·20	718 ·15	722 ·10	726 ·05	730 ·00	3 ·95
185	11 733 ·95	737 ·90	741 ·84	745 ·78	749 ·72	753 ·66	757 ·60	761 ·54	765 ·47	769 ·40	3 ·94
186	773 ·33	777 ·26	781 ·19	785 ·12	789 ·04	792 ·96	796 ·88	800 ·80	804 ·72	808 ·64	3 ·92
187	812 ·56	816 ·48	820 ·39	824 ·30	828 ·21	832 ·12	836 ·03	839 ·94	843 ·84	847 ·74	3 ·91
188	11 851 ·64	855 ·54	859 ·44	863 ·34	867 ·23	871 ·12	875 ·01	878 ·90	882 ·79	886 ·68	3 ·89
189	890 ·57	894 ·45	898 ·33	902 ·21	906 ·09	909 ·97	913 ·85	917 ·73	921 ·60	925 ·47	3 ·88
190	929 ·34	933 ·21	937 ·08	940 ·95	944 ·82	948 ·68	952 ·54	956 ·40	960 ·26	964 ·12	3 ·86
191	11 967 ·97	971 ·82	975 ·67	979 ·52	983 ·37	987 ·22	991 ·07	994 ·92	998 ·77	*002 ·62	3 ·85
192	12 006 ·46	010 ·30	014 ·14	017 ·98	021 ·82	025 ·65	029 ·48	033 ·31	037 ·14	040 ·97	3 ·83
193	044 ·79	048 ·62	052 ·44	056 ·26	060 ·08	063 ·90	067 ·72	071 ·54	075 ·36	079 ·17	3 ·82
194	12 082 ·98	086 ·79	090 ·60	094 ·41	098 ·22	102 ·02	105 ·82	109 ·62	113 ·42	117 ·22	3 ·80
195	121 ·02	124 ·82	128 ·61	132 ·40	136 ·19	139 ·98	143 ·77	147 ·56	151 ·35	155 ·14	3 ·79
196	158 ·92	162 ·70	166 ·48	170 ·26	174 ·04	177 ·82	181 ·60	185 ·38	189 ·15	192 ·92	3 ·78
197	12 196 ·69	200 ·46	204 ·23	208 ·00	211 ·76	215 ·52	219 ·28	223 ·04	226 ·80	230 ·56	3 ·76
198	234 ·32	238 ·07	241 ·82	245 ·57	249 ·32	253 ·07	256 ·82	260 ·57	264 ·31	268 ·05	3 ·75
199	271 ·79	275 ·53	279 ·27	283 ·01	286 ·75	290 ·48	294 ·21	297 ·94	301 ·67	305 ·40	3 ·73
200	12 309 ·13	312 ·86	316 ·59	320 ·32	324 ·05	327 ·77	331 ·49	335 ·21	338 ·93	342 ·65	3 ·72
201	346 ·37	350 ·09	353 ·81	357 ·53	361 ·25	364 ·96	368 ·67	372 ·38	376 ·09	379 ·80	3 ·71
202	383 ·51	387 ·22	390 ·93	394 ·64	398 ·35	402 ·06	405 ·77	409 ·48	413 ·18	416 ·88	3 ·71
203	12 420 ·58	424 ·28	427 ·98	431 ·68	435 ·38	439 ·08	442 ·78	446 ·48	450 ·18	453 ·88	3 ·70
204	457 ·57	461 ·26	464 ·95	468 ·64	472 ·33	476 ·02	479 ·71	483 ·40	487 ·09	490 ·78	3 ·69
205	494 ·47	498 ·16	501 ·85	505 ·54	509 ·23	512 ·92	516 ·60	520 ·28	523 ·96	527 ·64	3 ·69
206	12 531 ·32	535 ·00	538 ·68	542 ·36	546 ·04	549 ·72	553 ·40	557 ·07	560 ·74	564 ·41	3 ·68
207	568 ·08	571 ·75	575 ·42	579 ·09	582 ·76	586 ·43	590 ·10	593 ·76	597 ·42	601 ·08	3 ·67
208	604 ·74	608 ·40	612 ·06	615 ·72	619 ·38	623 ·04	626 ·70	630 ·36	634 ·02	637 ·68	3 ·66

Table V—*continued.*

Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
209	12 641·34	645·00	648·66	652·31	655·96	659·61	663·26	666·91	670·56	674·21	3·65
210	677·86	681·51	685·16	688·81	692·46	696·11	699·75	703·39	707·03	710·67	3·65
211	714·31	717·95	721·59	725·23	728·87	732·51	736·15	739·79	743·42	747·05	3·64
212	12 750·68	754·31	757·94	761·57	765·20	768·83	772·46	776·09	779·72	783·35	3·63
213	786·98	790·61	794·23	797·85	801·47	805·09	808·71	812·33	815·95	819·57	3·62
214	823·19	826·81	830·43	834·05	837·66	841·27	844·88	848·49	852·10	855·71	3·61
215	12 859·32	862·93	866·54	870·15	873·76	877·37	880·98	884·59	888·20	891·80	3·61
216	895·40	899·00	902·60	906·20	909·80	913·40	917·00	920·60	924·20	927·80	3·60
217	931·40	935·00	938·59	942·18	945·77	949·36	952·95	956·54	960·13	963·72	3·59
218	12 967·31	970·90	974·49	978·08	981·67	985·25	988·83	992·41	995·99	999·57	3·58
219	13 003·15	006·73	010·31	013·89	017·47	021·05	024·63	028·21	031·79	035·36	3·58
220	038·93	042·50	046·07	049·64	053·21	056·78	060·35	063·92	067·49	071·06	3·57
221	13 074·63	078·20	081·77	085·34	088·91	092·47	096·03	099·59	103·15	106·71	3·56
222	110·27	113·83	117·39	120·95	124·51	128·07	131·63	135·18	138·73	142·28	3·56
223	145·83	149·38	152·93	156·48	160·03	163·58	167·13	170·68	174·23	177·77	3·55
224	13 181·31	184·85	188·39	191·93	195·47	199·01	202·55	206·09	209·63	213·17	3·54
225	216·71	220·25	223·79	227·33	230·87	234·41	237·95	241·48	245·01	248·54	3·54
226	252·07	255·60	258·13	261·66	265·19	268·72	272·25	275·78	279·31	282·83	3·53
227	13 286·35	289·87	293·39	296·91	300·43	303·95	307·47	310·99	314·51	318·03	3·52
228	322·55	326·07	329·59	333·11	336·62	340·13	343·64	347·15	350·66	354·17	3·51
229	357·68	361·19	364·70	368·21	371·72	375·23	378·74	382·25	385·75	389·25	3·51
230	13 392·75	396·25	399·75	403·25	406·75	410·25	413·75	417·25	420·75	424·25	3·50
231	427·75	431·25	434·75	438·25	441·75	445·24	448·73	452·22	455·71	459·20	3·49
232	462·69	466·18	469·67	473·16	476·65	480·14	483·63	487·11	490·59	494·07	3·49
233	13 497·55	501·03	504·51	507·99	511·47	514·95	518·43	521·91	525·39	528·87	3·48
234	532·35	535·83	539·31	542·78	546·25	549·72	553·19	556·66	560·13	563·60	3·47
235	567·07	570·54	574·01	577·48	580·95	584·42	587·89	591·36	594·82	598·28	3·47
236	13 601·74	605·20	608·66	612·12	615·58	619·04	622·50	625·96	629·42	632·88	3·46
237	636·34	639·80	643·26	646·72	650·17	653·62	657·07	660·52	663·97	667·42	3·45
238	670·87	674·32	677·77	681·22	684·67	688·12	691·57	695·02	698·46	701·90	3·45

239	13 705·34	708·78	712·22	715·66	719·10	722·54	725·98	729·42	732·86	736·30	3·44
240	739·74	743·18	746·62	750·06	753·49	756·92	760·35	763·78	767·21	770·64	3·43
241	774·07	777·50	780·93	784·36	787·79	791·22	794·65	798·08	801·51	804·93	3·43
242	13 808·35	811·77	815·19	818·61	822·03	825·45	828·87	832·29	835·71	839·13	3·42
243	842·55	845·97	849·39	852·81	856·23	859·64	863·05	866·46	869·87	873·28	3·41
244	876·69	880·10	883·51	886·92	890·33	893·74	897·15	900·56	903·97	907·38	3·41
245	13 910·78	914·18	917·58	920·98	924·38	927·78	931·18	934·58	937·98	941·38	3·40
246	944·78	948·18	951·58	954·98	958·38	961·78	965·17	968·56	971·95	975·34	3·40
247	978·73	982·12	985·51	988·90	992·29	995·68	999·07	*002·46	*005·85	*009·24	3·39
248	14 012·63	016·02	019·41	022·79	026·17	029·55	032·93	036·31	039·69	043·07	3·38
249	046·45	049·83	053·21	056·59	059·97	063·35	066·73	070·11	073·49	076·86	3·38
250	080·23	083·60	086·97	090·34	093·71	097·08	100·45	103·82	107·19	110·56	3·37
251	14 113·93	117·30	120·67	124·04	127·41	130·78	134·14	137·50	140·86	144·22	3·37
252	147·58	150·94	154·30	157·66	161·02	164·38	167·74	171·10	174·46	177·82	3·36
253	181·18	184·54	187·89	191·24	194·59	197·94	201·29	204·64	207·99	211·34	3·35
254	14 214·69	218·04	221·39	224·74	228·09	231·44	234·79	238·14	241·48	244·82	3·35
255	248·16	251·50	254·84	258·18	261·52	264·86	268·20	271·54	274·88	278·22	3·34
256	281·56	284·90	288·24	291·58	294·91	298·24	301·57	304·90	308·23	311·56	3·33
257	14 314·89	318·22	321·55	324·88	328·21	331·54	334·87	338·20	341·53	344·86	3·33
258	348·18	351·50	354·82	358·14	361·46	364·78	368·10	371·42	374·74	378·06	3·32
259	381·38	384·70	388·02	391·34	394·66	397·98	401·30	404·62	407·94	411·26	3·32
260	14 414·58	417·89	421·20	424·51	427·82	431·13	434·44	437·75	441·06	444·37	3·31
261	447·68	450·99	454·30	457·61	460·91	464·21	467·51	470·81	474·11	477·41	3·30
262	480·71	484·01	487·31	490·61	493·91	497·20	500·49	503·78	507·07	510·36	3·29
263	14 513·65	516·94	520·23	523·52	526·81	530·10	533·39	536·67	539·95	543·23	3·29
264	546·51	549·79	553·07	556·35	559·63	562·91	566·19	569·47	572·75	576·02	3·28
265	579·29	582·56	585·83	589·10	592·37	595·64	598·91	602·18	605·45	608·72	3·27
266	14 611·99	615·26	618·53	621·79	625·05	628·31	631·57	634·83	638·09	641·35	3·26
267	644·61	647·87	651·13	654·39	657·65	660·91	664·17	667·43	670·68	673·93	3·26
268	677·18	680·43	683·68	686·93	690·18	693·43	696·68	699·92	703·16	706·40	3·25
269	14 709·64	712·88	716·12	719·36	722·60	725·84	729·08	732·32	735·56	738·80	3·24
270	742·04	745·28	748·51	751·74	754·97	758·20	761·43	764·66	767·89	771·12	3·23
271	774·35	777·58	780·81	784·04	787·27	790·50	793·73	796·96	800·18	803·40	3·23
272	14 806·62	809·84	813·06	816·28	819·50	822·72	825·94	829·16	832·37	835·58	3·22
273	838·79	842·00	845·21	848·42	851·63	854·84	858·05	861·26	864·47	867·68	3·21
274	870·89	874·10	877·31	880·51	883·71	886·91	890·11	893·31	896·51	899·71	3·20

Table V—*continued.*

Altitude Function, $A'(v)$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
275	14 902 ·91	906 ·11	909 ·31	912 ·51	915 ·71	918 ·91	922 ·10	925 ·29	928 ·48	931 ·67	3 ·20
276	934 ·86	938 ·05	941 ·24	944 ·43	947 ·62	950 ·81	954 ·00	957 ·19	960 ·38	963 ·57	3 ·19
277	966 ·75	969 ·93	973 ·11	976 ·29	979 ·47	982 ·65	985 ·83	989 ·01	992 ·19	995 ·37	3 ·18
278	14 998 ·55	*001 ·73	*004 ·91	*008 ·08	*011 ·25	*014 ·42	*017 ·59	*020 ·76	*023 ·93	*027 ·10	3 ·17
279	15 030 ·27	033 ·44	036 ·61	039 ·78	042 ·95	046 ·12	049 ·29	052 ·45	055 ·61	058 ·77	3 ·17
280	061 ·93	065 ·09	068 ·25	071 ·41	074 ·57	077 ·73	080 ·89	084 ·05	087 ·21	090 ·37	3 ·16
281	15 093 ·53	096 ·69	099 ·84	102 ·99	106 ·14	109 ·29	112 ·44	115 ·59	118 ·74	121 ·89	3 ·15
282	125 ·04	128 ·19	131 ·34	134 ·49	137 ·64	140 ·79	143 ·93	147 ·07	150 ·21	153 ·35	3 ·15
283	156 ·49	159 ·63	162 ·77	165 ·91	169 ·05	172 ·19	175 ·33	178 ·47	181 ·61	184 ·74	3 ·14
284	15 187 ·87	191 ·00	194 ·13	197 ·26	200 ·39	203 ·52	206 ·65	209 ·78	212 ·91	216 ·04	3 ·13
285	219 ·17	222 ·30	225 ·43	228 ·56	231 ·69	234 ·81	237 ·93	241 ·05	244 ·17	247 ·29	3 ·12
286	250 ·41	253 ·53	256 ·65	259 ·77	262 ·89	266 ·01	269 ·13	272 ·25	275 ·36	278 ·47	3 ·12
287	15 281 ·58	284 ·69	287 ·80	290 ·91	294 ·02	297 ·13	300 ·24	303 ·35	306 ·46	309 ·57	3 ·11
288	312 ·68	315 ·79	318 ·90	322 ·01	325 ·11	328 ·21	331 ·31	334 ·41	337 ·51	340 ·61	3 ·10
289	343 ·71	346 ·81	349 ·91	353 ·01	356 ·11	359 ·21	362 ·31	365 ·40	368 ·49	371 ·58	3 ·10
290	15 374 ·67	377 ·76	380 ·85	383 ·94	387 ·03	390 ·12	393 ·21	396 ·30	399 ·39	402 ·48	3 ·09
291	405 ·57	408 ·66	411 ·75	414 ·84	417 ·93	421 ·01	424 ·09	427 ·17	430 ·25	433 33	3 ·08
292	436 ·41	439 ·49	442 ·57	445 ·65	448 ·73	451 ·81	454 ·88	457 ·95	461 ·02	464 ·09	3 ·08
293	15 467 ·16	470 ·23	473 ·30	476 ·37	479 ·44	482 ·51	485 ·58	488 ·65	491 ·72	494 ·79	3 ·07
294	497 ·86	500 ·93	503 ·99	507 ·05	510 ·11	513 ·17	516 ·23	519 ·29	522 ·35	525 ·41	3 ·06
295	528 ·47	531 ·53	534 ·59	537 ·65	540 ·71	543 ·77	546 ·83	549 ·89	552 ·94	555 ·99	3 ·06
296	15 559 ·04	562 ·09	565 ·14	568 ·19	571 ·24	574 ·29	577 ·34	580 ·39	583 ·44	586 ·49	3 ·05
297	589 ·54	592 ·59	595 ·64	598 ·69	601 ·73	604 ·77	607 ·81	610 ·85	613 ·89	616 ·93	3 ·04
298	619 ·97	623 ·01	626 ·05	629 ·09	632 ·13	635 ·17	638 ·21	641 ·25	644 ·28	647 ·31	3 ·04
299	15 650 ·34	653 ·37	656 ·40	659 ·43	662 ·46	665 ·49	668 ·52	671 ·55	674 ·58	677 ·61	3 ·03
300	680 ·64	683 ·67	686 ·70	689 ·72	692 ·74	695 ·76	698 ·78	701 ·80	704 ·82	707 ·84	3 ·02
301	710 ·86	713 ·88	716 ·90	719 ·92	722 ·94	725 ·96	728 ·98	732 ·00	735 ·02	738 ·03	3 ·02
302	15 741 ·04	744 ·05	747 ·06	750 ·07	753 ·08	756 ·09	759 ·10	762 ·11	765 ·12	768 ·13	3 ·01
303	771 ·14	774 ·15	777 ·16	780 ·17	783 ·18	786 ·19	789 ·20	792 ·20	795 ·20	798 ·20	3 ·01
304	801 ·20	804 ·20	807 ·20	810 ·20	813 ·20	816 ·20	819 ·20	822 ·20	825 ·20	828 ·20	3 ·00

305	15 831 ·20	834 ·20	837 ·19	840 ·18	843 ·17	846 ·16	849 ·15	852 ·14	855 ·13	858 ·12	2 ·99
306	861 ·11	864 ·10	867 ·09	870 ·08	873 ·07	876 ·06	879 ·05	882 ·03	885 ·01	887 ·99	2 ·99
307	890 ·97	893 ·95	896 ·93	899 ·91	902 ·89	905 ·87	908 ·85	911 ·83	914 ·81	917 ·79	2 ·98
308	15 920 ·77	923 ·75	926 ·73	929 ·71	932 ·68	935 ·65	938 ·62	941 ·59	944 ·56	947 ·53	2 ·97
309	950 ·50	953 ·47	956 ·44	959 ·41	962 ·38	965 ·35	968 ·32	971 ·29	974 ·26	977 ·22	2 ·97
310	980 ·18	983 ·14	986 ·10	989 ·06	992 ·02	994 ·98	997 ·94	*000 ·90	*003 ·86	*006 ·82	2 ·96
311	16 009 ·78	012 ·74	015 ·70	018 ·66	021 ·62	024 ·58	027 ·54	030 ·49	033 ·44	036 ·39	2 ·96
312	039 ·34	042 ·29	045 ·24	048 ·19	051 ·14	054 ·09	057 ·04	059 ·99	062 ·94	065 ·89	2 ·95
313	068 ·84	071 ·79	074 ·73	077 ·67	080 ·61	083 ·55	086 ·49	089 ·43	092 ·37	095 ·31	2 ·94
314	16 098 ·25	101 ·19	104 ·13	107 ·07	110 ·01	112 ·95	115 ·89	118 ·83	121 ·77	124 ·71	2 ·94
315	127 ·64	130 ·57	133 ·50	136 ·43	139 ·36	142 ·29	145 ·22	148 ·15	151 ·08	154 ·01	2 ·93
316	156 ·94	159 ·87	162 ·80	165 ·73	168 ·66	171 ·59	174 ·51	177 ·43	180 ·35	183 ·27	2 ·93
317	16 186 ·19	189 ·11	192 ·03	194 ·95	197 ·87	200 ·79	203 ·71	206 ·63	209 ·55	212 ·47	2 ·92
318	215 ·39	218 ·31	221 ·23	224 ·15	227 ·06	229 ·97	232 ·88	235 ·79	238 ·70	241 ·61	2 ·91
319	244 ·52	247 ·43	250 ·34	253 ·25	256 ·16	259 ·07	261 ·98	264 ·89	267 ·80	270 ·70	2 ·91
320	16 273 ·60	276 ·50	279 ·40	282 ·30	285 ·20	288 ·10	291 ·00	293 ·90	296 ·80	299 ·70	2 ·90
321	302 ·60	305 ·50	308 ·40	311 ·30	314 ·20	317 ·10	320 ·00	322 ·90	325 ·79	328 ·68	2 ·90
322	331 ·57	334 ·46	337 ·35	340 ·24	343 ·13	346 ·02	348 ·91	351 ·80	354 ·69	357 ·58	2 ·89
323	16 360 ·47	363 ·36	366 ·25	369 ·14	372 ·03	374 ·92	377 ·80	380 ·68	383 ·56	386 ·44	2 ·89
324	389 ·32	392 ·20	395 ·08	397 ·96	400 ·84	403 ·72	406 ·60	409 ·48	412 ·36	415 ·24	2 ·88
325	418 ·12	421 ·00	423 ·88	426 ·75	429 ·62	432 ·49	435 ·36	438 ·23	441 ·10	443 ·97	2 ·87
326	16 446 ·84	449 ·71	452 ·58	455 ·45	458 ·32	461 ·19	464 ·06	466 ·93	469 ·80	472 ·67	2 ·87
327	475 ·53	478 ·39	481 ·25	484 ·11	486 ·97	489 ·83	492 ·69	495 ·55	498 ·41	501 ·27	2 ·86
328	504 ·13	506 ·99	509 ·85	512 ·71	515 ·57	518 ·43	521 ·29	524 ·14	526 ·99	529 ·84	2 ·86
329	16 532 ·69	535 ·54	538 ·39	541 ·24	544 ·09	546 ·94	549 ·79	552 ·64	555 ·49	558 ·34	2 ·85
330	561 ·19	564 ·04	566 ·89	569 ·74	572 ·59	575 ·44	578 ·29	581 ·13	583 ·97	586 ·81	2 ·85
331	689 ·65	592 ·49	595 ·33	598 ·17	601 ·01	603 ·85	606 ·69	609 ·53	612 ·37	615 ·21	2 ·84
332	16 618 ·05	620 ·89	623 ·73	626 ·57	629 ·40	632 ·23	635 ·06	637 ·89	640 ·72	643 ·55	2 ·83
333	646 ·38	649 ·21	652 ·04	654 ·87	657 ·70	660 ·53	663 ·36	666 ·19	669 ·02	671 ·85	2 ·83
334	674 ·68	677 ·51	680 ·34	683 ·16	685 ·98	688 ·80	691 ·62	694 ·44	697 ·26	700 ·08	2 ·82
335	16 702 ·90	705 ·72	708 ·54	711 ·36	714 ·18	717 ·00	719 ·82	722 ·64	725 ·46	728 ·28	2 ·82
336	731 ·09	733 ·90	736 ·71	739 ·52	742 ·33	745 ·14	747 ·95	750 ·76	753 ·57	756 ·38	2 ·81
337	759 ·19	762 ·00	764 ·81	767 ·62	770 ·43	773 ·24	776 ·05	778 ·86	781 ·67	784 ·48	2 ·81
338	16 787 ·28	790 ·08	792 ·88	795 ·68	798 ·48	801 ·28	804 ·08	806 ·88	809 ·68	812 ·48	2 ·80
339	815 ·28	818 ·08	820 ·88	823 ·68	826 ·48	829 ·28	832 ·08	834 ·88	837 ·68	840 ·47	2 ·80
340	843 ·26	846 ·05	848 ·84	851 ·63	854 ·42	857 ·21	860 ·00	862 ·79	865 ·58	868 ·37	2 ·79

Table V—*continued.*

Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ.
f/s.											+
341	16 871 ·16	873 ·95	876 ·74	879 ·53	882 ·32	885 ·11	887 ·90	890 ·69	893 ·48	896 ·26	2 ·79
342	899 ·04	901 ·82	904 ·60	907 ·38	910 ·16	912 ·94	915 ·72	918 ·50	921 ·28	924 ·06	2 ·78
343	926 ·84	929 ·62	932 ·40	935 ·18	937 ·96	940 ·74	943 ·52	946 ·29	949 ·06	951 ·83	2 ·78
344	16 954 ·60	957 ·37	960 ·14	962 ·91	965 ·68	968 ·45	971 ·22	973 ·99	976 ·76	979 ·53	2 ·77
345	982 ·30	985 ·07	987 ·84	990 ·61	993 ·38	996 ·15	998 ·91	*001 ·67	*004 ·43	*007 ·19	2 ·77
346	17 009 ·95	012 ·71	015 ·47	018 ·23	020 ·99	023 ·75	026 ·51	029 ·27	032 ·03	034 ·79	2 ·76
347	17 037 ·55	040 ·31	043 ·07	045 ·83	048 ·59	051 ·35	054 ·10	056 ·85	059 ·60	062 ·35	2 ·76
348	065 ·10	067 ·85	070 ·60	073 ·35	076 ·10	078 ·85	081 ·60	084 ·35	087 ·10	089 ·85	2 ·75
349	092 ·60	095 ·35	098 ·10	100 ·85	103 ·60	106 ·35	109 ·09	111 ·83	114 ·57	117 ·31	2 ·75
350	17 120 ·05	122 ·79	125 ·53	128 ·27	131 ·01	133 ·75	136 ·49	139 ·23	141 ·97	144 ·71	2 ·74
351	147 ·45	150 ·19	152 ·93	155 ·67	158 ·41	161 ·14	163 ·87	166 ·60	169 ·33	172 ·06	2 ·73
352	174 ·79	177 ·52	180 ·25	182 ·98	185 ·71	188 ·44	191 ·17	193 ·90	196 ·63	199 ·36	2 ·73
353	17 202 ·09	204 ·82	207 ·55	210 ·28	213 ·01	215 ·73	218 ·45	221 ·17	223 ·89	226 ·61	2 ·72
354	229 ·33	232 ·05	234 ·77	237 ·49	240 ·21	242 ·93	245 ·65	248 ·37	251 ·09	253 ·81	2 ·72
355	256 ·53	259 ·25	261 ·97	264 ·69	267 ·41	270 ·12	272 ·83	275 ·54	278 ·25	280 ·96	2 ·71
356	17 283 ·67	286 ·38	289 ·09	291 ·80	294 ·51	297 ·22	299 ·93	302 ·64	305 ·35	308 ·06	2 ·71
357	310 ·77	313 ·48	316 ·19	318 ·90	321 ·61	324 ·32	327 ·02	329 ·72	332 ·42	335 ·12	2 ·71
358	337 ·82	340 ·52	343 ·22	345 ·92	348 ·62	351 ·32	354 ·02	356 ·72	359 ·42	362 ·12	2 ·70
359	17 364 ·82	367 ·52	370 ·22	372 ·92	375 ·62	378 ·32	381 ·01	383 ·70	386 ·39	389 ·08	2 ·70
360	391 ·77	394 ·46	397 ·15	399 ·84	402 ·53	405 ·22	407 ·91	410 ·60	413 ·29	415 ·98	2 ·69
361	418 ·67	421 ·36	424 ·05	426 ·74	429 ·43	432 ·12	434 ·80	437 ·48	440 ·16	442 ·84	2 ·69
362	17 445 ·52	448 ·20	450 ·88	453 ·56	456 ·24	458 ·92	461 ·60	464 ·28	466 ·96	469 ·64	2 ·68
363	472 ·32	475 ·00	477 ·68	480 ·36	483 ·04	485 ·72	488 ·40	491 ·08	493 ·75	496 ·42	2 ·68
364	499 ·09	501 ·76	504 ·43	507 ·10	509 ·77	512 ·44	515 ·11	517 ·78	520 ·45	523 ·12	2 ·67
365	17 525 ·79	528 ·46	531 ·13	533 ·80	536 ·47	539 ·14	541 ·81	544 ·48	547 ·14	549 ·80	2 ·67
366	552 ·46	555 ·12	557 ·78	560 ·44	563 ·10	565 ·76	568 ·42	571 ·08	573 ·74	576 ·40	2 ·66
367	579 ·06	581 ·72	584 ·38	587 ·04	589 ·70	592 ·36	595 ·02	597 ·68	600 ·34	603 ·00	2 ·66
368	17 605 ·65	608 ·31	610 ·96	613 ·61	616 ·26	618 ·91	621 ·56	624 ·21	626 ·86	629 ·51	2 ·65
369	632 ·16	634 ·81	637 ·46	640 ·11	642 ·76	645 ·41	648 ·06	650 ·71	653 ·36	656 ·01	2 ·65
370	658 ·66	661 ·31	663 ·96	666 ·60	669 ·24	671 ·88	674 ·52	677 ·16	679 ·80	682 ·44	2 ·64

371	17 685 ·08	687 ·72	690 ·36	693 ·00	695 ·64	698 ·28	700 ·92	703 ·56	706 ·20	708 ·84	2 ·64
372	711 ·48	714 ·12	716 ·76	719 ·40	722 ·03	724 ·66	727 ·29	729 ·92	732 ·55	735 ·18	2 ·63
373	737 ·81	740 ·44	743 ·07	745 ·70	748 ·33	750 ·96	753 ·59	756 ·22	758 ·85	761 ·48	2 ·63
374	17 764 ·11	766 ·74	769 ·37	772 ·00	774 ·63	777 ·26	779 ·88	782 ·50	785 ·12	787 ·74	2 ·63
375	790 ·36	792 ·98	795 ·60	798 ·22	800 ·84	803 ·46	806 ·08	808 ·70	811 ·32	813 ·94	2 ·62
376	816 ·56	819 ·18	821 ·80	824 ·42	827 ·04	829 ·66	832 ·28	834 ·90	837 ·51	840 ·12	2 ·62
377	17 842 ·73	845 ·34	847 ·95	850 ·56	853 ·17	855 ·78	858 ·39	861 ·00	863 ·61	866 ·22	2 ·61
378	868 ·83	871 ·44	874 ·05	876 ·66	879 ·27	881 ·88	884 ·49	887 ·10	889 ·71	892 ·32	2 ·61
379	894 ·92	897 ·52	900 ·12	902 ·72	905 ·32	907 ·92	910 ·52	913 ·12	915 ·72	918 ·32	2 ·60
380	17 920 ·92	923 ·52	926 ·12	928 ·72	931 ·32	933 ·92	936 ·52	939 ·12	941 ·72	944 ·32	2 ·60
381	946 ·92	949 ·52	952 ·11	954 ·70	957 ·29	959 ·88	962 ·47	965 ·06	967 ·65	970 ·24	2 ·59
382	972 ·83	975 ·42	978 ·01	980 ·60	983 ·19	985 ·78	988 ·37	990 ·96	993 ·55	996 ·14	2 ·59
383	17 998 ·73	*001 ·32	*003 ·91	*006 ·50	*009 ·09	*011 ·67	*014 ·25	*016 ·83	*019 ·41	*021 ·99	2 ·58
384	18 024 ·57	027 ·15	029 ·73	032 ·31	034 ·89	037 ·47	040 ·05	042 ·63	045 ·21	047 ·79	2 ·58
385	050 ·37	052 ·95	055 ·53	058 ·11	060 ·69	063 ·27	065 ·85	068 ·43	071 ·00	073 ·57	2 ·58
386	18 076 ·14	078 ·71	081 ·28	083 ·85	086 ·42	088 ·99	091 ·56	094 ·13	096 ·70	099 ·27	2 ·57
387	101 ·84	104 ·41	106 ·98	109 ·55	112 ·12	114 ·69	117 ·26	119 ·83	122 ·40	124 ·97	2 ·57
388	127 ·53	130 ·10	132 ·67	135 ·23	137 ·79	140 ·35	142 ·91	145 ·47	148 ·03	150 ·59	2 ·56
389	18 153 ·15	155 ·71	158 ·27	160 ·83	163 ·39	165 ·95	168 ·51	171 ·07	173 ·63	176 ·19	2 ·56
390	178 ·75	181 ·31	183 ·87	186 ·43	188 ·99	191 ·55	194 ·10	196 ·65	199 ·20	201 ·75	2 ·56
391	204 ·30	206 ·85	209 ·40	211 ·95	214 ·50	217 ·05	219 ·60	222 ·15	224 ·70	227 ·25	2 ·55
392	18 229 ·80	232 ·35	234 ·90	237 ·45	240 ·00	242 ·55	245 ·10	247 ·65	250 ·20	252 ·74	2 ·55
393	255 ·28	257 ·82	260 ·36	262 ·90	265 ·44	267 ·98	270 ·52	273 ·06	275 ·60	278 ·14	2 ·54
394	280 ·68	283 ·22	285 ·76	288 ·30	290 ·84	293 ·38	295 ·92	298 ·46	301 ·00	303 ·54	2 ·54
395	18 306 ·08	308 ·62	311 ·15	313 ·68	316 ·21	318 ·74	321 ·27	323 ·80	326 ·33	328 ·86	2 ·53
396	331 ·39	333 ·92	336 ·45	338 ·98	341 ·51	344 ·04	346 ·57	349 ·10	351 ·63	354 ·16	2 ·53
397	356 ·69	359 ·22	361 ·75	364 ·28	366 ·81	369 ·34	371 ·86	374 ·38	376 ·90	379 ·42	2 ·53
398	18 381 ·94	384 ·46	386 ·98	389 ·50	392 ·02	394 ·54	397 ·06	399 ·58	402 ·10	404 ·62	2 ·52
399	407 ·14	409 ·66	412 ·18	414 ·70	417 ·22	419 ·74	422 ·26	424 ·78	427 ·30	429 ·81	2 ·52
400	432 ·32										

TABLE VI.

$$(\theta) = \int \sec^3\theta \, d\theta = \tfrac{1}{2} \sec\theta \tan\theta + \tfrac{1}{2} \log_e (\sec\theta + \tan\theta).$$

θ.	(θ).	tan θ.	θ.	(θ).	tan θ.	θ.	(θ).	tan θ.
° ′			° ′			° ′		
0 0	0·00000	0·00000	24 0	0·45953	0·44523	48 0	1·30863	1·11061
0 30	0·00873	0·00873	24 30	0·47104	0·45573	48 30	1·33818	1·13029
1 0	0·01746	0·01746	25 0	0·48269	0·46631	49 0	1·36863	1·15037
1 30	0·02619	0·02619	25 30	0·49449	0·47698	49 30	1·40001	1·17085
2 0	0·03493	0·03492	26 0	0·50643	0·48773	50 0	1·43236	1·19175
2 30	0·04367	0·04366	26 30	0·51853	0·49858	50 30	1·46574	1·21310
3 0	0·05243	0·05241	27 0	0·53078	0·50953	51 0	1·50020	1·23490
3 30	0·06120	0·06116	27 30	0·54320	0·52057	51 30	1·53579	1·25717
4 0	0·06998	0·06993	28 0	0·55580	0·53171	52 0	1·57257	1·27994
4 30	0·07878	0·07870	28 30	0·56856	0·54296	52 30	1·61060	1·30323
5 0	0·08760	0·08749	29 0	0·58151	0·55431	53 0	1·64995	1·32704
5 30	0·09644	0·09629	29 30	0·59465	0·56577	53 30	1·69070	1·35142
6 0	0·10530	0·10510	30 0	0·60799	0·57735	54 0	1·73291	1·37638
6 30	0·11418	0·11394	30 30	0·62152	0·58904	54 30	1·77667	1·40195
7 0	0·12309	0·12278	31 0	0·63527	0·60086	55 0	1·82207	1·42815
7 30	0·13203	0·13165	31 30	0·64924	0·61280	55 30	1·86919	1·45501
8 0	0·14100	0·14054	32 0	0·66343	0·62487	56 0	1·91815	1·48256
8 30	0·15001	0·14945	32 30	0·67786	0·63707	56 30	1·9[illegible]905	1·51084
9 0	0·15904	0·15838	33 0	0·69253	0·64941	57 0	2·02199	1·53986
9 30	0·16812	0·16734	33 30	0·70744	0·66189	57 30	2·07712	1·56969
10 0	0·17724	0·17633	34 0	0·72263	0·67451	58 0	2·13456	1·60033
10 30	0·18639	0·18534	34 30	0·73808	0·68728	58 30	2·19446	1·63185
11 0	0·19560	0·19438	35 0	0·75382	0·70021	59 0	2·25697	1·66428
11 30	0·20485	0·20345	35 30	0·76984	0·71329	59 30	2·32226	1·69766
12 0	0·21415	0·21256	36 0	0·78617	0·72654	60 0	2·39053	1·73205
12 30	0·22350	0·22169	36 30	0·80280	0·73996	60 30	2·46196	1·76749
13 0	0·23290	0·23087	37 0	0·81977	0·75355	61 0	2·53678	1·80405
13 30	0·24237	0·24008	37 30	0·83707	0·76733	61 30	2·61521	1·84177
14 0	0·25189	0·24933	38 0	0·85473	0·78129	62 0	2·69752	1·88073
14 30	0·26147	0·25862	38 30	0·87275	0·79544	62 30	2·78398	1·92098
15 0	0·27112	0·26795	39 0	0·89114	0·80978	63 0	2·87490	1·96261
15 30	0·28084	0·27732	39 30	0·90994	0·82434	63 30	2·97062	2·00569
16 0	0·29063	0·28675	40 0	0·92914	0·83910	64 0	3·07150	2·05030
16 30	0·30049	0·29621	40 30	0·94877	0·85408	64 30	3·17794	2·09654
17 0	0·31043	0·30573	41 0	0·96884	0·86929	65 0	3·29039	2·14451
17 30	0·32045	0·31530	41 30	0·98937	0·88473	65 30	3·40934	2·19430
18 0	0·33055	0·32492	42 0	1·01039	0·90040	66 0	3·53532	2·24604
18 30	0·34074	0·33460	42 30	1·03191	0·91633	66 30	3·66893	2·29984
19 0	0·35102	0·34433	43 0	1·05395	0·93252	67 0	3·81083	2·35585
19 30	0·36139	0·35412	43 30	1·07653	0·94896	67 30	3·96177	2·41422
20 0	0·37185	0·36397	44 0	1·09968	0·96569	68 0	4·12255	2·47509
20 30	0·38242	0·37388	44 30	1·12343	0·98270	68 30	4·29410	2·53865
21 0	0·39309	0·38386	45 0	1·14779	1·00000	69 0	4·47744	2·60509
21 30	0·40387	0·39391	45 30	1·17280	1·01761	69 30	4·67372	2·67462
22 0	0·41476	0·40403	46 0	1·19849	1·03553	70 0	4·88425	2·74748
22 30	0·42577	0·41421	46 30	1·22488	1·05378			
23 0	0·43690	0·42447	47 0	1·25201	1·07237			
23 30	0·44815	0·43481	47 30	1·27991	1·09131			

TABLE VII.

EXTRACT FROM GLAISHER'S HYGROMETRICAL TABLES.

SHOWING the weight in grains of a cubic foot of air at 29-inch barometric height and temperature between 80° and 32° F. Standard density 534·22 grains/ft.³

Reading of Thermometer—Wet Bulb—Degrees F. (wet bulb 80–65)

Difference for 1 inch in Barometer.	Dry Bulb—Degrees F.	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65
17·0	80	493·0	493·4	493·7	494·0	494·3	494·6	494·8	495·1	495·3	495·6	495·8	496·0	496·2	496·4	496·6	496·7
17·0	79		494·1	494·5	494·8	495·1	495·4	495·7	495·9	496·2	496·4	496·6	496·8	497·0	497·2	497·4	497·6
17·1	78			495·2	495·5	495·8	496·1	496·4	496·7	496·9	497·2	497·4	497·6	497·8	498·0	498·2	498·4
17·1	77				496·3	496·6	496·9	497·2	497·5	497·8	498·0	498·3	498·5	498·7	498·9	499·1	499·3
17·2	76					497·4	497·7	498·0	498·3	498·5	498·8	499·0	499·3	499·5	499·7	499·9	500·1
17·2	75						498·5	498·8	499·1	499·4	499·6	499·9	500·1	500·4	500·6	500·8	501·0
17·2	74							499·6	499·9	500·2	500·5	500·7	501·0	501·2	501·5	501·7	501·9
17·3	73								500·7	501·0	501·3	501·5	501·8	502·0	502·3	502·5	502·7
17·3	72									501·8	502·1	502·4	502·6	502·9	503·1	503·4	503·6
17·4	71										502·9	503·2	503·4	503·7	503·9	504·2	504·4
17·4	70											504·0	504·3	504·6	504·8	505·0	505·3
17·4	69												505·2	505·4	505·7	505·9	506·2
17·5	68													506·2	506·5	506·7	507·0
17·5	67														507·4	507·6	507·9
17·5	66															508·5	508·8
17·6	65																509·5

Reading of Thermometer—Wet Bulb—Degrees F. (wet bulb 64–49)

Difference for 1 inch in Barometer.	Dry Bulb—Degrees F.	64	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49
17·0	80	496·9	497·0	497·2	497·3	497·4	497·6	497·7	497·8								
17·0	79	497·8	497·9	498·1	498·2	498·3	498·5	498·6	498·7	498·8							
17·1	78	498·5	498·7	498·8	499·0	499·1	499·3	499·4	499·5	499·6	499·7						
17·1	77	499·4	499·6	499·8	499·9	500·1	500·2	500·3	500·4	500·5	500·6	500·7					
17·2	76	500·3	500·4	500·6	500·7	500·9	501·0	501·2	501·3	501·4	501·5	501·6	501·7				
17·2	75	501·2	501·3	501·5	501·6	501·8	501·9	502·1	502·2	502·3	502·4	502·5	502·6				
17·2	74	502·1	502·2	502·4	502·6	502·7	502·9	503·0	503·1	503·2	503·3	503·4	503·5	503·6	503·7		
17·3	73	502·9	503·1	503·2	503·4	503·5	503·7	503·8	504·0	504·1	504·2	504·3	504·4	504·5	504·6	504·7	
17·3	72	503·8	503·9	504·1	504·3	504·5	504·6	504·8	504·9	505·0	505·1	505·3	505·4	505·5	505·6	505·6	505·7
17·4	71	504·6	504·7	504·9	505·1	505·3	505·5	505·6	505·7	505·9	506·0	506·1	506·2	506·3	506·4	506·5	506·6
17·4	70	505·5	505·7	505·9	506·0	506·2	506·4	506·5	506·7	506·8	506·9	507·1	507·2	507·3	507·4	507·5	507·5
17·4	69	506·4	506·6	506·8	507·0	507·1	507·3	507·5	507·6	507·7	507·9	508·0	508·1	508·2	508·3	508·4	508·5
17·5	68	507·2	507·4	507·6	507·8	508·0	508·2	508·3	508·5	508·6	508·7	508·9	509·0	509·1	509·2	509·3	509·4
17·5	67	508·1	508·3	508·5	508·7	508·9	509·1	509·2	509·4	509·5	509·7	509·8	509·9	510·1	510·2	510·3	510·4
17·5	66	509·0	509·2	509·4	509·6	509·8	510·0	510·2	510·4	510·5	510·6	510·8	510·9	511·0	511·1	511·2	511·3
17·6	65	509·8	510·0	510·2	510·4	510·6	510·8	511·0	511·2	511·3	511·5	511·6	511·8	511·9	512·0	512·1	512·2
17·6	64	510·7	510·9	511·1	511·4	511·6	511·8	511·9	512·1	512·3	512·4	512·6	512·7	512·8	513·0	513·1	513·2
17·7	63		511·7	512·0	512·2	512·4	512·6	512·8	513·0	513·1	513·3	513·4	513·6	513·7	513·8	514·0	514·1
17·7	62			512·9	513·1	513·3	513·5	513·7	513·9	514·1	514·2	514·4	514·5	514·7	514·8	514·9	515·0
17·7	61				514·0	514·2	514·4	514·6	514·8	515·0	515·2	515·3	515·5	515·6	515·8	515·9	516·0
17·8	60					515·0	515·3	515·5	515·7	515·9	516·0	516·2	516·4	516·5	516·6	516·8	516·9
17·8	59						516·2	516·4	516·6	516·8	517·0	517·2	517·3	517·5	517·6	517·8	517·9
17·8	58							517·4	517·6	517·8	518·0	518·1	518·3	518·5	518·6	518·7	518·9
17·9	57								518·4	518·6	518·8	519·0	519·2	519·3	519·4	519·6	519·7
17·9	56									519·6	519·8	519·9	520·1	520·3	520·5	520·6	520·7
18·0	55										520·6	520·8	521·0	521·2	521·3	521·5	521·6
18·0	54											521·8	522·0	522·1	522·3	522·5	522·6
18·0	53												522·9	523·1	523·3	523·4	523·6
18·1	52													524·0	524·2	524·4	524·6
18·1	51														525·1	525·3	525·3
18·2	50															526·2	526·3
18·2	49																527·3

Reading of Thermometer—Wet Bulb—Degrees F. (wet bulb 48–32)

Difference for 1 inch in Barometer.	Dry Bulb—Degrees F.	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	Dry Bulb—Degrees F.
17·4	71	506·7																	71
17·4	69	508·6																	69
17·5	68	509·5	509·6	509·6															68
17·5	67	510·4	510·5	510·6															67
17·5	66	511·4	511·5	511·6	511·7														66
17·6	65	512·3	512·4	512·5	512·6	512·6													65
17·6	64	513·3	513·4	513·5	513·5														64
17·7	63	514·2	514·3	514·3	514·4	514·5													63
17·7	62	515·1	515·2	515·3	515·4	515·5	515·6												62
17·7	61	516·1	516·2	516·3	516·4	516·5	516·6	516·6											61
17·8	60	517·0	517·1	517·2	517·3	517·4	517·5	517·6	517·6										60
17·8	59	518·0	518·1	518·2	518·3	518·4	518·5	518·6	518·6	518·7									59
17·8	58	519·0	519·1	519·2	519·3	519·4	519·5	519·6	519·6	519·7									58
17·9	57	519·9	520·0	520·1	520·2	520·3	520·4	520·5	520·6	520·6	520·7								57
17·9	56	520·9	521·0	521·1	521·2	521·3	521·4	521·5	521·6	521·7	521·7								56
18·0	55	521·8	521·9	522·0	522·1	522·2	522·3	522·4	522·5	522·6	522·7	522·7							55
18·0	54	522·7	522·9	523·0	523·1	523·2	523·3	523·4	523·5	523·6	523·7	523·8	523·8						54
18·0	53	523·7	523·9	524·0	524·1	524·2	524·3	524·4	524·5	524·6	524·7	524·8	524·9	524·9					53
18·1	52	524·7	524·9	525·0	525·1	525·3	525·4	525·5	525·6	525·7	525·7	525·8	525·9	526·0	526·0				52
18·1	51	525·6	525·8	525·9	526·0	526·2	526·3	526·4	526·5	526·6	526·7	526·8	526·8	526·9	527·0	527·0			51
18·2	50	526·5	526·7	526·8	527·0	527·1	527·2	527·3	527·4	527·5	527·6	527·7	527·8	527·9	527·9	528·0			50
18·2	49	527·5	527·7	527·8	528·0	528·1	528·2	528·3	528·4	528·5	528·6	528·7	528·8	528·9	529·0	529·0			49
18·2	48	528·5	528·7	528·8	529·0	529·1	529·2	529·4	529·5	529·6	529·7	529·8	529·9	529·9	530·0	530·1			48
18·3	47		529·6	529·7	529·9	530·0	530·1	530·3	530·4	530·5	530·6	530·7	530·8	530·9	531·0	531·0	531·1		47
18·3	46			530·7	530·9	531·0	531·2	531·3	531·4	531·5	531·6	531·8	531·9	532·0	532·0	532·1	532·2	532·2	46
18·3	45				531·9	532·1	532·2	532·3	532·5	532·6	532·7	532·8	532·9	533·0	533·1	533·2	533·2	533·3	45
18·4	44					533·0	533·1	533·3	533·4	533·5	533·6	533·7	533·8	533·9	534·0	534·1	534·2	534·3	44
18·4	43						534·1	534·3	534·4	534·5	534·7	534·8	534·9	535·0	535·1	535·2	535·3	535·3	43
18·5	42							535·2	535·3	535·5	535·6	535·7	535·8	535·9	536·0	536·1	536·2	536·3	42
18·5	41								536·4	536·5	536·7	536·8	536·9	537·0	537·1	537·2	537·3	537·4	41
18·5	40									537·5	537·7	537·8	538·0	538·1	538·2	538·3	538·4	538·5	40
18·6	39										538·6	538·8	538·9	539·0	539·1	539·2	539·3	539·4	39
18·6	38											539·8	540·0	540·1	540·2	540·3	540·4	540·5	38
18·7	37												540·9	541·0	541·2	541·3	541·4	541·5	37
18·7	36													542·1	542·2	542·4	542·5	542·6	36
18·7	35														543·3	543·4	543·6	543·7	35
[illegible]·8	34															544·4	544·5	544·7	34
[illegible]·8	33																545·6	545·7	33
[illegible]·9	32																	546·7	32

(Rows for dry bulb 80–72 and 70 have no entries in wet bulb columns 48–32; the right-hand "Dry Bulb—Degrees F." column reads 80 to 32.)

Inch.	Proportional Parts for Difference from 29-inch Barometric Height.																			
1·00	17·0	17·1	17·2	17·3	17·4	17·5	17·6	17·7	17·8	17·9	18·0	18·1	18·2	18·3	18·4	18·5	18·6	18·7	18·8	18·9
0·05	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9
0·1	1·7	1·7	1·7	1·7	1·7	1·8	1·8	1·8	1·8	1·8	1·8	1·8	1·8	1·8	1·8	1·9	1·9	1·9	1·9	1·9
0·2	3·4	3·4	3·4	3·5	3·5	3·5	3·5	3·5	3·6	3·6	3·6	3·6	3·6	3·7	3·7	3·7	3·7	3·8	3·8	3·8
0·3	5,1	5·1	5·2	5·2	5·2	5·3	5·3	5·3	5·3	5·4	5·4	5·4	5·5	5·5	5·5	5·6	5·6	5·6	5·6	5·7
0·4	6·8	6·8	6·9	6·9	7·0	7·0	7·0	7·1	7·1	7·2	7·2	7·2	7·3	7·3	7·4	7·4	7·4	7·5	7·5	7·6
0·5	8·5	8·6	8·6	8·7	8·7	8·8	8·8	8·9	8·9	9·0	9·0	9·1	9·1	9·1	9·2	9·3	9·3	9·4	9·4	9·5
0·6	10·2	10·3	10·3	10·4	10·4	10·5	10·6	10·6	10·7	10·7	10·8	10·9	10·9	11·0	11·0	11·1	11·2	11·2	11·3	11·3
0·7	11·9	12·0	12·0	12·1	12·2	12·3	12·3	12·4	12·5	12·5	12·6	12·7	12·7	12·8	12·9	13·0	13·0	13·1	13·2	13·2
0·8	13·6	13·7	13·8	13·8	13·9	14·0	14·1	14·2	14·2	14·3	14·4	14·5	14·6	14·6	14·7	14·8	14·9	15·0	15·0	15·1
0·9	15·3	15·4	15·5	15·6	15·7	15·8	15·8	15·9	16·0	16·1	16·2	16·3	16·4	16·5	16·6	16·7	16·7	16·8	16·9	17·0

TABLE VII.b

CORRECTION FOR T, AND PERCENTAGE CORRECTION OF BALLISTIC COEFFICIENTS FOR VARIATIONS OF THERMOMETER AND BAROMETER.

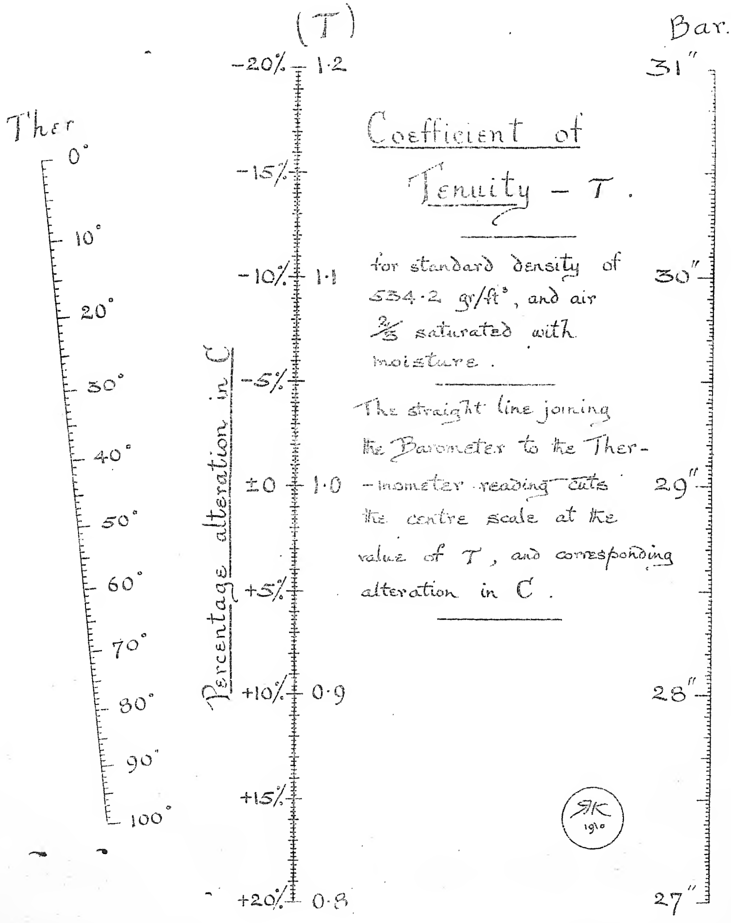

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TABLE VIIa.

Tenuity correction τ for Temperature and Pressure of Atmosphere two-thirds saturated with Moisture.

(From the Rev. F. Bashforth's paper, *Proc. R.A.I.*, Vol. XIII, No. 10.)

F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +	F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +
0°	·983	1·021	1·059	1·097	1·134	1·172	38	50°	·884	919	·953	·987	1·021	1·055	34
1	·981	1·019	1·056	1·094	1·132	1·170	38	51	883	·917	·951	·985	1·019	1·053	34
2	·979	1·017	1·054	1·092	1·130	1·167	38	52	·881	·915	·949	·983	1·017	1·051	34
3	·977	1·015	1·052	1·090	1·127	1·165	38	53	·879	·913	·947	·981	1·015	1·048	34
4	·975	1·012	1·050	1·087	1·125	1·162	38	54	·877	·911	·945	·978	1·012	1·046	34
5	·973	1·010	1·047	1·085	1·122	1·160	37	55	·875	·909	·943	·976	1·010	1·044	34
6	·971	1·008	1·045	1·083	1·120	1·157	37	56	·874	·907	·941	·974	1·008	1·042	34
7	·969	1·006	1·043	1·080	1·118	1·155	37	57	·872	·905	·939	·972	1·006	1·039	34
8	·966	1·004	1·041	1·078	1·115	1·152	37	58	·870	·904	·937	·970	1·004	1·037	34
9	·964	1·001	1·039	1·076	1·113	1·150	37	59	·868	·902	·935	·968	1·002	1·035	33
10	·962	·999	1·036	1·073	1·110	1·147	37	60	·866	·900	·933	·966	1·000	1·033	33
11	·960	·997	1·034	1·071	1·108	1·145	37	61	·865	·898	·931	·964	·998	1·031	33
12	·958	·995	1·032	1·069	1·105	1·142	37	62	·863	·896	·929	·962	·996	1·029	33
13	·956	·993	1·029	1·066	1·103	1·140	37	63	·861	·894	·927	·960	·993	1·027	33
14	·954	·991	1·027	1·064	1·101	1·137	37	64	·859	·892	·925	·958	·991	1·024	33
15	·952	·989	1·025	1·062	1·098	1·135	37	65	·857	·890	·923	·956	·989	1·022	33
16	·950	·986	1·023	1·060	1·096	1·133	37	66	·856	·889	·921	·954	·987	1·020	33
17	·948	.984	1·021	1·057	1·094	1·130	37	67	·854	·887	·919	·952	·985	1·018	33
18	·946	·982	1·019	1·055	1·091	1·128	36	68	·852	·885	·918	·950	·983	1·016	33
19	·944	·980	1·017	1·053	1·089	1·125	36	69	·850	·883	·916	·949	·981	1·014	33
20	·942	·978	1·014	1·051	1·087	1·123	36	70	·849	·881	·914	·946	·979	1·012	33
21	·940	·976	1·012	1·048	1·084	1·121	36	71	·847	·879	·912	·944	·977	1·010	33
22	·938	·974	1·010	1·046	1·082	1·118	36	72	·845	·878	·910	·943	·975	1·008	33
23	·936	·972	1·008	1·044	1·080	1·116	36	73	·843	·876	·908	·941	·973	1·006	32
24	·934	·970	1·006	1·042	1·078	1·114	36	74	·842	·874	·906	·939	·971	1·004	32
25	·932	·968	1·004	1·039	1·075	1·111	36	75	·840	·872	·904	·937	·969	1·001	32
26	·930	·966	1·001	1·037	1·073	1·109	36	76	·838	·870	·902	·935	·967	·999	32
27	·928	·964	·999	1·035	1·071	1·106	36	77	·836	·868	·901	·933	·965	·997	32
28	·926	·962	·997	1·033	1·069	1·104	36	78	·834	·867	·899	·931	·963	·995	32
29	·924	·960	·995	1·031	1·066	1·102	36	79	·833	·865	·897	·929	·961	·993	32
30	·922	·958	·993	1·028	1·064	1·099	36	80	·831	·863	·895	·927	·959	·991	32
31	·920	·956	·991	1·026	1·062	1·097	35	81	·829	·861	·893	·925	·957	·989	32
32	·918	·954	·989	1·024	1·059	1·095	35	82	·827	·859	·891	·923	·955	·987	32
33	·916	·952	·987	1·022	1·057	1·093	35	83	·826	·858	·889	·921	·953	·985	32
34	·914	·950	·985	1·020	1·055	1·090	35	84	·824	·856	·887	·919	·951	·983	32
35	·913	·948	·983	1·018	1·053	1·088	35	85	·822	·854	·885	·917	·949	·980	32
36	·911	·946	·981	1·016	1·051	1·086	35	86	·821	·852	·884	·915	·947	·978	32
37	·909	·944	·979	1·013	1·048	1·083	35	87	·819	·850	·882	·913	·945	·976	32
38	·907	·942	·977	1·011	1·046	1·081	35	88	·817	·848	·880	·911	·943	·974	31
39	·905	·940	·974	1·009	1·044	1·079	35	89	·815	·847	·878	·909	·941	·972	31
40	·903	·938	·973	1·007	1·042	1·077	35	90	·814	·845	·876	·908	·939	·970	31
41	·901	·936	·971	1·005	1·040	1·075	35	91	·812	·843	·874	·905	·937	·968	31
42	·899	·934	·968	1·003	1·038	1·072	35	92	·810	·841	·872	·903	·935	·966	31
43	·898	·932	·967	1·001	1·036	1·070	35	93	·808	·839	·870	·902	·933	·964	31
44	·896	·930	·964	0·999	1·033	1·068	34	94	·806	·837	·868	·900	·931	·962	31
45	·894	·928	·963	0·997	1·031	1·066	34	95	·805	·836	·867	·898	·929	·960	31
46	·892	·926	·960	0·995	1·029	1·063	34	96	·803	·834	·865	·896	·926	·957	31
47	·890	·924	·958	0·993	1·027	1·061	34	97	·801	·832	·863	·893	·924	·955	31
48	·888	·923	·957	·991	1·025	1·059	34	98	·799	·830	·861	·891	·922	·953	31
49	·886	·920	·955	·989	1·023	1·057	34	99	·797	·828	·859	·889	·920	·951	31
50	·884	·919	·953	·987	1·021	1·055	34	100	·796	·826	·857	·888	·918	·949	31

TABLE VIII.

Ballistic Table for Spherical Shot.

(Recalculated by Mr. Hadcock, late R.A., from Bashforth's data, and extended to low velocities.)

For lower velocities this table is provisional, pending the results of further experiments.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
300	1·2232	0·0000	366·91	0·00	7·5191	0·0000
310	1·1505	1·2232	356·67	366·91	6·8154	7·5191
320	1·0824	2·3737	346·37	723·58	6·2387	14·3645
330	1·0217	3·4561	337·22	1069·95	5·7113	20·6032
340	0·9647	4·4778	328·01	1407·17	5·2335	26·3145
350	0·9137	5·4425	319·78	1735·18	4·8148	31·5480
360	0·8653	6·3562	311·51	2054·96	4·4333	36·3628
370	0·8218	7·2215	304·07	2366·47	4·0967	40·7961
380	0·7805	8·0433	296·60	2670·54	3·7884	44·8928
390	0·7432	8·8238	289·84	2967·14	3·5147	48·6812
400	0·7076	9·5670	283·05	3256·98	3·2629	52·1959
410	0·6753	10·2746	276·88	3540·03	3·0380	55·4588
420	0·6445	10·9499	270·69	3816·91	2·8303	58·4968
430	0·6151	11·5944	264·51	4087·60	2·6385	61·3271
440	0·5763	12·2095	253·59	4352·11	2·4159	63·9656
450	0·5508	12·7858	247·86	4605·70	2·2575	66·3815
460	0·5265	13·3366	242·20	4853·56	2·1111	68·6390
470	0·5035	13·8631	236·64	5095·76	1·9758	70·7501
480	0·4816	14·3666	231·18	5332·40	1·8506	72·7259
490	0·4609	14·8482	225·84	5563·58	1·7349	74·5765
500	0·4413	15·3091	220·63	5789·42	1·6277	76·3114
510	0·4227	15·7504	215·55	6010·05	1·5285	77·9391
520	0·4050	16·1731	210·61	6225·60	1·4366	79·4676
530	0·3883	16·5781	205·80	6436·21	1·3513	80·9042
540	0.3725	16·9664	201·14	6642·01	1·2722	82·2555
550	0·3575	17·3389	196·61	6843·15	1·1988	83·5277
560	0·3429	17·6964	192·01	7039·76	1·1293	84·7265
570	0·3291	18·0393	187·57	7231·77	1·0648	85·8558
580	0·3157	18·3684	183·11	7419·34	1·0039	86·9206
590	0·3028	18·6841	178·64	7602·45	0·9465	87·9245
600	0·2903	18·9869	174·19	7781·09	0·8925	88·8710
610	0·2786	19·2773	169·95	7955·28	0·8424	89·7635
620	0·2673	19·5558	165·75	8125·23	0·7953	90·6059
630	0·2567	19·8231	161·74	8290·98	0·7516	91·4012
640	0·2467	20·0798	157·92	8452·72	0·7111	92·1528
650	0·2371	20·3265	154·14	8610·64	0·6729	92·8639
660	0·2281	20·5636	150·53	8764·78	0·6374	93·5368
670	0·2195	20·7917	147·09	8915·31	0·6044	94·1742
680	0·2115	21·0112	143·80	9062·40	0·5736	94·7786
690	0·2038	21·2227	140·65	9206·20	0·5449	95·3522
700	0·1966	21·4265	137·63	9346·85	0·5180	95·8971
710	0·1898	21·6231	134·73	9484·48	0·4930	96·4151
720	0·1832	21·8129	131·88	9619·21	0·4692	96·9081
730	0·1770	21·9961	129·22	9751·09	0·4472	97·3773
740	0·1711	22·1731	126·59	9880·31	0·4264	97·8245
750	0·1653	22·3442	123·99	10006·90	0·4066	98·2509
760	0·1600	22·5095	121·57	10130·89	0·3882	98·6575
770	0·1547	22·6695	119·12	10252·46	0·3706	99·0457
780	0·1496	22·8242	116·72	10371·58	0·3539	99·4163
790	0·1447	22·9738	114·30	10488·30	0·3378	99·7702
800	0·1399	23·1185	111·89	10602·60	0·3225	100·1080
810	0·1352	23·2584	109·50	10714·49	0·3078	100·4305
820	0·1306	24·3936	107·07	10823·99	0·2937	100·7383
830	0·1201	23·5242	104·68	10931·06	0·2803	101·0320

Table VIII—*continued.*

Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
840	0·1218	23·6503	102·33	11035·74	0·2675	101·3123
850	0·1177	23·7721	100·01	11138·07	0·2553	101·5798
860	0·1137	23·8898	97·76	11238·08	0·2438	101·8351
870	0·1098	24·0035	95·53	11335·84	0·2328	102·0789
880	0·1062	24·1133	93·44	11431·37	0·2225	102·3117
890	0·1026	24·2195	91·35	11524·81	0·2127	102·5342
900	0·0993	24·3221	89·33	11616·16	0·2034	102·7469
910	0·0959	24·4214	87·32	11705·49	0·1945	102·9503
920	0·0928	24·5173	85·37	11792·81	0·1860	103·1448
930	0·0898	24·6101	83·48	11878·18	0·1780	103·3308
940	0·0869	24·6999	81·65	11961·66	0·1704	103·5088
950	0·0840	24·7868	79·83	12043·31	0·1631	103·6792
960	0·0813	24·8708	78·01	12123·14	0·1561	103·8423
970	0·0785	24·9521	76·19	12201·15	0·1493	103·9984
980	0·0759	25·0306	74·43	12277·34	0·1429	104·1477
990	0·0734	25·1065	72·67	12351·77	0·1368	104·2906
1000	0·0709	25·1799	70·87	12424·44	0·1307	104·4274
1010	0·0684	25·2508	69·08	12495·31	0·1249	104·5581
1020	0·0660	25·3192	67·31	12564·39	0·1193	104·6830
1030	0·0636	25·3852	65·55	12631·70	0·1140	104·8023
1040	0·0614	25·4488	63·81	12697·25	0·1088	104·9163
1050	0·0591	25·5102	62·08	12761·06	0·1039	105·0251
1060	0·0570	25·5693	60·42	12823·14	0·0992	105·1290
1070	0·0550	25·6263	58·82	12883·56	0·0948	105·2282
1080	0·0531	25·6813	57·31	12942·38	0·0906	105·3230
1090	0·0513	25·7344	55·89	12999·69	0·0868	105·4136
1100	0·0496	25·7857	54·59	13055·58	0·0832	105·5004
1110	0·0481	25·8353	53·36	13110·17	0·0799	105·5836
1120	0·0466	25·8834	52·21	13163·53	0·0768	105·6635
1130	0·0453	25·9300	51·15	13215·74	0·0739	105·7403
1140	0·0440	25·9753	50·16	13266·89	0·0712	105·8142
1150	0·0428	26·0193	49·23	13317·05	0·0687	105·8854
1160	0·0417	26·0621	48·35	13366·28	0·0663	105·9541
1170	0·0406	26·1038	47·53	13414·63	0·0640	106·0204
1180	0·0396	26·1444	46·73	13462·16	0·0619	106·0844
1190	0·0386	26·1840	45·97	13508·89	0·0599	106·1463
1200	0·0377	26·2226	45·27	13554·86	0·0580	106·2062
1210	0·0369	26·2603	44·61	13600·13	0·0562	106·2642
1220	0·0361	26·2972	44·00	13644·74	0·0545	106·3204
1230	0·0353	26·3333	43·43	13688·74	0·0529	106·3749
1240	0·0346	26·3686	42·87	13732·17	0·0514	106·4278
1250	0·0339	26·4032	42·36	13775·04	0·0500	106·4792
1260	0·0332	26·4371	41·85	13817·40	0·0486	106·5292
1270	0·0326	26·4703	41·39	13859·25	0·0473	106·5778
1280	0·0320	26·5029	40·94	13900·64	0·0461	106·6251
1290	0·0314	26·5349	40·49	13941·58	0·0449	106·6712
1300	0·0308	26·5663	40·04	13982·07	0·0437	106·7131
1310	0·0302	26·5971	39·59	14022·11	0·0425	106·7598
1320	0·0297	26·6273	39·18	14061·70	0·0415	106·8023
1330	0·0291	26·6570	38·75	14100·88	0·0404	106·8438
1340	0·0286	26·6861	38·33	14139·63	0·0394	106·8842
1350	0·0281	26·7147	37·92	14177·96	0·0384	106·9236
1360	0·0276	26·7428	37·52	14215·88	0·0374	106·9620
1370	0·0271	26·7704	37·15	14253·40	0·0365	106·9994
1380	0·0267	26·7975	36·80	14290·55	0·0356	107·0359
1390	0·0262	26·8242	36·45	14327·35	0·0348	107·0715
1400	0·0258	26·8504	36·11	14363·80	0·0340	107·1063
1410	0·0254	26·8762	35·77	14399·91	0·0332	107·1403
1420	0·0250	26·9016	35·48	14435·68	0·0324	107·1735
1430	0·0246	26·9266	35·16	14471·16	0·0317	107·2059

Table VIII—*continued.*

Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
1440	0·0242	26·9512	34·85	14506·32	0·0310	107·2376
1450	0·0238	26·9754	34·54	14541·17	0·0303	107·2686
1460	0·0235	26·9992	34·24	14575·71	0·0296	107·2989
1470	0·0231	27·0227	33·98	14609·95	0·0290	107·3285
1480	0·0228	27·0458	33·69	14643·93	0·0284	107·3575
1490	0·0224	27·0686	33·41	14677·62	0·0278	107·3859
1500	0·0221	27·0910	33·14	14711·03	0·0272	107·4137
1510	0·0218	27·1131	32·85	14744·17	0·0266	107·4409
1520	0·0214	27·1349	32·59	14777·02	0·0260	107·4675
1530	0·0211	27·1563	32·34	14809·61	0·0255	107·4935
1540	0·0208	27·1774	32·06	14841·95	0·0249	107·5190
1550	0·0205	27·1982	31·82	14874·01	0·0244	107·5439
1560	0·0202	27·2187	31·58	14905·83	0·0239	107·5683
1570	0·0200	27·2389	31·33	14937·41	0·0234	107·5922
1580	0·0197	27·2589	31·10	14968·74	0·0230	107·6156
1590	0·0194	27·2786	30·86	14999·84	0·0225	107·6386
1600	0·0191	27·2980	30·64	15030·70	0·0221	107·6611
1610	0·0189	27·3171	30·42	15061·34	0·0216	107·6832
1620	0·0186	27·3360	30·19	15091·76	0·0212	107·7048
1630	0·0184	27·3546	29·99	15121·95	0·0208	107·7260
1640	0·0182	27·3730	29·79	15151·94	0·0204	107·7468
1650	0·0179	27·3912	29·60	15181·73	0·0201	107·7672
1660	0·0177	27·4091	29·38	15211·33	0·0197	107·7873
1670	0·0175	27·4268	29·20	15240·71	0·0193	107·8070
1680	0·0173	27·4443	29·02	15269·91	0·0190	107·8263
1690	0·0171	27·4616	28·84	15298·93	0·0186	107·8453
1700	0·0168	27·4787	28·64	15327·77	0·0183	107·8639
1710	0·0167	27·4955	28·47	15356·41	0·0180	107·8822
1720	0·0165	27·5122	28·31	15384·88	0·0176	107·9002
1730	0·0163	27·5287	28·13	15413·19	0·0173	107·9178
1740	0·0161	27·5450	27·97	15441·32	0·0170	107·9351
1750	0·0159	27·5611	27·81	15469·29	0·0168	107·9521
1760	0·0157	27·5770	27·64	15497·10	0·0165	107·9689
1770	0·0155	27·5927	27·49	15524·74	0·0162	107·9854
1780	0·0154	27·6082	27·33	15552·23	0·0159	108·0016
1790	0·0152	27·6236	27·16	15579·56	0·0156	108·0175
1800	0·0150	27·6388	27·03	15606·72	0·0154	108·0331
1810	0·0148	27·6538	26·87	15633·75	0·0151	108·0485
1820	0·0147	27·6686	26·72	15660·62	0·0149	108·0636
1830	0·0145	27·6833	26·54	15687·34	0·0146	108·0785
1840	0·0143	27·6978	26·40	15713·88	0·0144	108·0931
1850	0·0142	27·7121	26·25	15740·28	0·0141	108·1075
1860	0·0140	27·7263	26·09	15766·53	0·0139	108·1216
1870	0·0139	27·7403	25·93	15792·62	0·0137	108·1355
1880	0·0137	27·7542	25·79	15818·55	0·0135	108·1492
1890	0·0136	27·7679	25·64	15844·34	0·0132	108·1627
1900	0·0134	27·7815	25·48	15869·98	0·0130	108·1759

TABLE IX.

BALLISTIC TABLE

for Flat-Headed Projectiles.

The laws upon which the S Table for flat-headed projectiles has been constructed are deduced from experiments carried out at Shoeburyness in 1904–6, and are as follows:—

$$5000 \text{ f/s} > v > 1556 \text{ f/s},$$
$$Cr = [6\ 4683473 - 10]\, v^2.$$

$$1556 \text{ f/s} > v > 1130 \text{ f/s},$$
$$Cr = [3 \cdot 2763377 - 10]\, v^3.$$

$$1130 \text{ f/s} > v > 1000 \text{ f/s},$$
$$Cr = [0 \cdot 2232593 - 10]\, v^4.$$

$$1000 \text{ f/s} > v > 800 \text{ f/s},$$
$$Cr = [3 \cdot 2232593 - 10]\, v^3.$$

$$800 \text{ f/s} > v > 0 \text{ f/s},$$
$$Cr = [6 \cdot 1263493 - 10]\, v^2.$$

In the above equations, $Cr = pg = \mathrm{K}\left(\frac{v}{1000}\right)^3$. The figures in the brackets are the logarithms of the coefficients of v.

Table IX—*continued.*

$s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
50	19 090·1	105·0	119·9	134·8	149·6	164·4	179·2	194·0	208·7	223·4	14·8
51	238·1	252·7	267·3	281·9	296·5	311·0	325·5	340·0	354·5	368·9	14·5
52	383·3	397·7	412·0	426·3	440·6	454·8	469·0	483·2	497·4	511·5	14·3
53	19 525·6	539·7	553·8	567·8	581·8	595·8	609·8	623·7	637·6	651·5	14·0
54	665·4	679·2	693·0	706·8	720·6	734·3	748·0	761·7	775·3	788·9	13·7
55	802·5	816·1	829·7	843·2	856·7	870·2	883·7	897·1	910·5	923·9	13·5
56	19 937·3	950·6	963·9	977·2	990·5	*003·7	*016·9	*030·1	*043·3	*056·5	13·2
57	20 069·6	082·7	095·8	108·9	121·9	134·9	147·9	160·9	173·8	186·7	13·0
58	199·6	212·5	225·4	238·2	251·0	263·8	276·6	289·3	302·0	314·7	12·8
59	20 327·4	340·1	352·7	365·3	377·9	390·5	403·0	415·5	428·0	440·5	12·6
60	453·0	465·5	477·9	490·3	502·7	515·1	527·4	539·7	552·0	564·3	12·4
61	576·6	588·8	601·0	613·2	625·4	637·6	649·8	661·9	674·0	686·1	12·2
62	20 698·2	710·2	722·2	734·2	746·2	758·2	770·2	782·1	794·0	805·9	12·0
63	817·8	829·7	841·5	853·3	865·1	876·9	888·7	900·4	912·1	923·8	11·8
64	935·5	947·2	958·9	970·5	982·1	993·7	*005·3	*016·9	*028·4	*039·9	11·6
65	21 051·4	062·9	074·4	085·9	097·3	108·7	120·1	131·5	142·9	154·2	11·4
66	165·5	176·8	188·1	199·4	210·7	221·9	233·1	244·3	255·5	266·7	11·2
67	277·9	289·1	300·2	311·3	322·4	333·5	344·6	355·7	366·7	377·7	11·1
68	21 388·7	399·7	410·7	421·7	432·6	443·5	454·4	465·3	476·2	487·0	10·9
69	497·8	508·6	519·4	530·2	541·0	551·8	562·6	573·3	584·0	594·7	10·8
70	605·4	616·1	626·8	637·4	648·0	658·6	669·2	679·8	690·4	700·9	10·6
71	21 711·4	721·9	732·4	742·9	753·4	763·9	774·4	784·8	795·2	805·6	10·5
72	816·0	826·4	836·8	847·1	857·4	867·7	878·0	888·3	898·6	908·9	10·3
73	919·1	929·3	939·5	949·7	959·9	970·1	980·3	990·5	*000·6	*010·7	10·2
74	22 020·8	030·9	041·0	051·1	061·2	071·2	081·2	091·2	101·2	111·2	10·0
75	121·2	131·2	141·2	151·1	161·0	170·9	180·8	190·7	200·6	210·4	9·9
76	220·2	230·0	239·8	249·6	259·4	269·2	279·0	288·8	298·5	308·2	9·8
77	22 317·9	327·6	337·3	347·0	356·7	366·3	376·0	385·6	395·2	404·8	9·6
78	414·4	424·0	433·6	443·1	452·6	462·1	471·6	481·1	490·6	500·1	9·5
79	509·6	519·1	528·6	538·0	547·4	556·8	566·2	575·6	585·0	594·4	9·4

80	22 603 ·7	613 ·0	622 ·3	631 ·6	640 ·9	650 ·1	659 ·3	668 ·5	677 ·7	686 ·8	9 ·2
81	695 ·9	705 ·0	714 ·1	723 ·2	732 ·2	741 ·2	750 ·2	759 ·2	768 ·2	777 ·1	9 ·0
82	786 ·0	794 ·9	803 ·8	812 ·6	821 ·4	830 ·2	839 ·0	847 ·8	856 ·5	865 ·2	8 ·8
83	22 873 ·9	882 ·6	891 ·2	899 ·8	908 ·4	917 ·0	925 ·6	934 ·1	942 ·6	951 ·1	8 ·6
84	959 ·6	968 ·1	976 ·6	985 ·0	993 ·4	*001 ·8	*010 ·2	*018 ·5	*026 ·8	*035 ·1	8 ·4
85	23 043 ·4	051 ·7	059 ·9	068 ·1	076 ·3	084 ·5	092 ·7	100 ·9	109 ·0	117 ·1	8 ·2
86	23 125 ·2	133 ·3	141 ·4	149 ·4	157 ·4	165 ·4	173 ·4	181 ·4	189 ·3	197 ·2	8 ·0
87	205 ·1	213 ·0	220 ·9	228 ·8	236 ·6	244 ·4	252 ·2	260 ·0	267 ·8	275 ·6	7 ·8
88	283 ·3	291 ·0	298 ·7	306 ·4	314 ·0	321 ·6	329 ·2	336 ·8	344 ·4	352 ·0	7 ·6
89	23 359 ·6	367 ·2	374 ·7	382 ·2	389 ·7	397 ·2	404 ·7	412 ·1	419 ·5	426 ·9	7 ·5
90	434 ·3	441 ·7	449 ·1	456 ·4	463 ·7	471 ·0	478 ·3	485 ·6	492 ·9	500 ·1	7 ·3
91	507 ·3	514 ·5	521 ·7	528 ·9	536 ·1	543 ·2	550 ·3	557 ·4	564 ·5	571 ·6	7 ·1
92	23 578 ·7	585 ·8	592 ·9	599 ·9	606 ·9	613 ·9	620 ·9	627 ·9	634 ·8	641 ·7	7 ·0
93	648 ·6	655 ·5	662 ·4	669 ·3	676 ·2	683 ·0	689 ·8	696 ·6	703 ·4	710 ·2	6 ·8
94	717 ·0	723 ·8	730 ·6	737 ·3	744 ·0	750 ·7	757 ·4	764 ·1	770 ·8	777 ·4	6 ·7
95	23 784 ·0	790 ·6	797 ·2	803 ·8	810 ·4	817 ·0	823 ·6	830 ·1	836 ·6	843 ·1	6 ·6
96	849 ·6	856 ·1	862 ·6	869 ·1	875 ·5	881 ·9	888 ·3	894 ·7	901 ·1	907 ·5	6 ·4
97	913 ·8	920 ·2	926 ·5	932 ·8	939 ·1	945 ·4	951 ·7	958 ·0	964 ·3	970 ·5	6 ·3
98	23 976 ·7	982 ·9	989 ·1	995 ·3	*001 ·5	*007 ·7	*013 ·9	*020 ·0	*026 ·1	*032 ·2	6 ·2
99	24 038 ·3	044 ·4	050 ·5	056 ·6	062 ·7	068 ·7	074 ·8	080 ·8	086 ·8	092 ·8	6 ·0
100	098 ·8	104 ·8	110 ·8	116 ·7	122 ·6	128 ·5	134 ·4	140 ·3	146 ·1	151 ·9	5 ·9
101	24 157 ·7	163 ·5	169 ·3	175 ·1	180 ·8	186 ·5	192 ·2	197 ·9	203 ·6	209 ·3	5 ·7
102	214 ·9	220 ·5	226 ·1	231 ·7	237 ·3	242 ·9	248 ·4	253 ·9	259 ·4	264 ·9	5 ·6
103	270 ·4	275 ·9	281 ·4	286 ·8	292 ·2	297 ·6	303 ·0	308 ·4	313 ·8	319 ·1	5 ·4
104	24 324 ·4	329 ·7	335 ·0	340 ·3	345 ·6	350 ·8	356 ·0	361 ·2	366 ·4	371 ·6	5 ·2
105	376 ·8	382 ·0	387 ·1	392 ·2	397 ·3	402 ·4	407 ·5	412 ·6	417 ·7	422 ·7	5 ·1
106	427 ·7	432 ·7	437 ·7	442 ·7	447 ·7	452 ·7	457 ·6	462 ·5	467 ·4	472 ·3	5 ·0
107	24 477 ·2	482 ·1	487 ·0	491 ·9	496 ·7	501 ·5	506 ·3	511 ·1	515 ·9	520 ·7	4 ·8
108	525 ·4	530 ·2	534 ·9	539 ·6	544 ·3	549 ·0	553 ·7	558 ·4	563 ·0	567 ·6	4 ·7
109	572 ·2	576 ·8	581 ·4	586 ·0	590 ·6	595 ·1	599 ·7	604 ·3	608 ·8	613 ·3	4 ·6
110	24 617 ·8	622 ·3	626 ·8	631 ·3	635 ·7	640 ·1	644 ·5	648 ·9	653 ·3	657 ·7	4 ·4
111	662 ·1	666 ·5	670 ·9	675 ·2	679 ·5	683 ·8	688 ·1	692 ·4	696 ·7	701 ·0	4 ·3
112	705 ·2	709 ·5	713 ·8	718 ·0	722 ·2	726 ·4	730 ·6	734 ·8	739 ·0	743 ·1	4 ·2
113	24 747 ·2	751 ·4	755 ·5	759 ·6	763 ·7	767 ·8	771 ·9	776 ·0	780 ·1	784 ·2	4 ·1
114	788 ·3	792 ·4	796 ·4	800 ·5	804 ·5	808 ·6	812 ·6	816 ·6	820 ·7	824 ·7	4 ·0
115	828 ·7	832 ·7	836 ·7	840 ·7	844 ·7	848 ·7	852 ·6	856 ·6	860 ·5	864 ·4	4 ·0

Table IX—*continued.*

= C[S(V) − S(*v*)] (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
116	24 868 ·4	872 ·3	876 ·2	880 ·1	884 ·0	887 ·9	891 ·8	895 ·7	899 ·6	903 ·5	3 ·9
117	907 ·4	911 ·3	915 ·1	919 ·0	922 ·8	926 ·6	930 ·5	934 ·3	938 ·1	941 ·9	3 ·8
118	945 ·7	949 ·5	953 ·3	957 ·1	960 ·9	964 ·7	968 ·4	972 ·2	975 ·9	979 ·6	3 ·8
119	24 983 ·4	987 ·1	990 ·8	994 ·5	998 ·2	*001 ·9	*005 ·6	*009 ·3	*013 ·0	*016 ·7	3 ·7
120	25 020 ·4	024 ·1	027 ·8	031 ·5	035 ·1	038 ·8	042 ·4	046 ·0	049 ·7	053 ·3	3 ·7
121	056 ·9	060 ·5	064 ·1	067 ·7	071 ·3	074 ·9	078 ·5	082 ·0	085 ·6	089 ·2	3 ·6
122	25 092 ·7	096 ·3	099 ·8	103 ·4	106 ·9	110 ·4	114 ·0	117 ·5	121 ·0	124 ·5	3 ·5
123	128 ·0	131 ·5	135 ·0	138 ·5	142 ·0	145 ·4	148 ·9	152 ·4	155 ·8	159 ·3	3 ·5
124	162 ·7	166 ·2	169 ·6	173 ·0	176 ·5	179 ·9	183 ·3	186 ·7	190 ·1	193 ·5	3 ·4
125	25 196 ·9	200 ·3	203 ·7	207 ·1	210 ·4	213 ·8	217 ·1	220 ·5	223 ·8	227 ·1	3 ·4
126	230 ·5	233 ·8	237 ·1	240 ·4	243 ·7	247 ·0	250 ·3	253 ·6	256 ·9	260 ·2	3 ·3
127	263 ·5	266 ·7	270 ·0	273 ·3	276 ·5	279 ·8	283 ·1	286 ·3	289 ·6	292 ·9	3 ·3
128	25 296 ·1	299 ·3	302 ·5	305 ·7	308 ·9	312 ·1	315 ·3	318 ·5	321 ·7	324 ·9	3 ·2
129	328 ·1	331 ·3	334 ·4	337 ·6	340 ·7	343 ·9	347 ·1	350 ·2	353 ·4	356. 6	3 ·2
130	359 ·7	362 ·8	365 ·9	369 ·0	372 ·1	375 ·2	378 ·4	381 ·5	384 ·6	387 ·7	3 ·1
131	25 390 ·8	393 ·9	397 ·0	400 ·0	403 ·1	406 ·1	409 ·2	412 ·2	415 ·3	418 ·3	3 ·1
132	421 ·4	424 ·4	427 ·4	430 ·4	433 ·4	436 ·4	439 ·5	442 ·5	445 ·5	448 ·5	3 ·0
133	451 ·5	454 ·5	457 ·5	460 ·5	463 ·5	466 ·5	469 ·4	472 ·4	475 ·3	478 ·3	3 ·0
134	25 481 ·2	484 ·2	487 ·1	490 ·0	492 ·9	495 ·9	498 ·8	501 ·7	504 ·6	507 ·6	2 ·9
135	510 ·5	513 ·4	516 ·3	519 ·2	522 ·1	525 ·0	527 ·9	530 ·8	533 ·6	536 ·5	2 ·9
136	539 ·3	542 ·2	545 ·0	547 ·9	550 ·7	553 ·5	556 ·4	559 ·2	562 ·0	564 ·9	2 ·8
137	25 567 ·7	570 ·5	573 ·3	576 ·1	578 ·9	581 ·7	584 ·5	587 ·3	590 ·1	592 ·9	2 ·8
138	595 ·7	598 ·5	601 ·3	604 ·0	606 ·8	609 ·5	612 ·3	615 ·0	617 ·8	620 ·5	2 ·8
139	623 ·3	626 ·0	628 ·8	631 ·5	634 ·2	636 ·9	639 ·7	642 ·4	645 ·1	647 ·8	2 ·7
140	25 650 ·5	653 ·2	655 ·9	658 ·6	661 ·3	664 ·0	666 ·7	669 ·4	672 ·0	674 ·7	2 ·7
141	677 ·3	680 ·0	682 ·6	685 ·3	687 ·9	690 ·5	693 ·2	695 ·8	698 ·4	701 ·1	2 ·6
142	703 ·7	706 ·3	709 ·0	711 ·6	714 ·2	716 ·8	719 ·4	722 ·0	724 ·6	727 ·2	2 ·6
143	25 729 ·8	732 ·4	735 ·0	737 ·6	740 ·2	742 ·8	745 ·3	747 ·9	750 ·4	753 ·0	2 ·6
144	755 ·5	758 ·1	760 ·6	763 ·2	765 ·7	768 ·2	770 ·8	773 ·3	775 ·8	778 ·4	2 ·5
145	780 ·9	783 ·4	785 ·9	788 ·4	790 ·9	793 ·4	795 ·9	798 ·4	800 ·9	803 ·4	2 ·5

146	25 805·9	808·3	810·8	813·3	815·8	818·2	820·7	823·2	825·6	828·1	2·5
147	830·5	833·0	835·4	837·9	840·3	842·7	845·2	847·6	850·0	852·4	2·4
148	854·9	857·3	859·7	862·1	864·5	866·9	869·3	871·7	874·1	876·5	2·4
149	25 878·9	881·2	883·6	886·0	888·3	890·7	893·1	895·4	897·8	900·1	2·4
150	902·5	904·8	907·2	909·5	911·9	914·2	916·6	918·9	921·2	923·6	2·3
151	925·9	928·2	930·6	932·9	935·2	937·5	939·8	942·1	944·4	946·7	2·3
152	25 949·0	951·3	953·6	955·9	958·1	960·4	962·7	965·0	967·3	969·5	2·3
153	971·8	974·1	976·3	978·6	980·8	983·0	985·3	987·5	989·7	991·9	2·2
154	994·2	996·4	998·6	*000·8	*003·1	*005·3	*007·5	*009·7	*011·9	*014·1	2·2
155	26 016·3	018·5	020·7	022·9	025·1	027·3	029·5	031·6	033·8	036·0	2·2
156	038·2	040·4	042·5	044·7	046·9	049·1	051·3	053·4	055·6	057·8	2·2
157	060·0	062·1	064·3	066·5	068·6	070·8	072·9	075·1	077·2	079·4	2·2
158	26 081·6	083·7	085·9	088·0	090·2	092·3	094·4	096·6	098·7	100·8	2·1
159	103·0	105·1	107·2	109·4	111·5	113·6	115·8	117·9	120·0	122·1	2·1
160	124·3	126·4	128·5	130·6	132·8	134·9	137·0	139·1	141·2	143·4	2·1
161	26 145·5	147·6	149·7	151·8	153·9	156·1	158·2	160·3	162·4	164·5	2·1
162	166·6	168·7	170·8	172·9	175·0	177·1	179·2	181·3	183·4	185·5	2·1
163	187·6	189·6	191·7	193·8	195·9	198·0	200·1	202·1	204·2	206·3	2·1
164	26 208·4	210·4	212·5	214·6	216·7	218·7	220·8	222·9	224·9	227·0	2·1
165	229·1	231·1	233·2	235·2	237·3	239·3	241·4	243·4	245·5	247·5	2·0
166	249·6	251·6	253·7	255·7	257·8	259·8	261·9	263·9	265·9	268·0	2·0
167	26 270·0	272·0	274·1	276·1	278·1	280·2	282·2	284·2	286·2	288·3	2·0
168	290·3	292·3	294·3	296·4	298·4	300·4	302·4	304·4	306·5	308·5	2·0
169	310·5	312·5	314·5	316·5	318·6	320·6	322·6	324·6	326·6	328·6	2·0
170	26 330·6	332·6	334·6	336·6	338·6	340·6	342·6	344·6	346·6	348·6	2·0
171	350·6	352·6	354·5	356·5	358·5	360·5	362·5	364·4	366·4	368·4	2·0
172	370·4	372·3	374·3	376·3	378·3	380·2	382·2	384·2	386·1	388·1	2·0
173	26 390·1	392·0	394·0	396·0	397·9	399·9	401·8	403·8	405·8	407·7	2·0
174	409·7	411·6	413·6	415·5	417·5	419·4	421·4	423·3	425·3	427·2	1·9
175	429·2	431·1	433·1	435·0	437·0	438·9	440·8	442·8	444·7	446·6	1·9
176	26 448·6	450·5	452·4	454·3	456·3	458·2	460·1	462·0	463·9	465·8	1·9
177	467·8	469·7	471·6	473·5	475·4	477·3	479·2	481·2	483·1	485·0	1·9
178	486·9	488·8	490·7	492·6	494·5	496·5	498·4	500·3	502·2	504·1	1·9
179	26 506·0	507·9	509·8	511·7	513·6	515·5	517·4	519·3	521·2	523·1	1·9
180	525·0	526·9	528·8	530·7	532·6	534·5	536·3	538·2	540·1	542·0	1·9
181	543·9	545·7	547·6	549·5	551·4	553·2	555·1	557·0	558·8	560·7	1·9

Table IX—*continued.*

$s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
182	26 562·6	564·4	566·3	568·2	570·0	571·9	573·8	575·6	577·5	579·3	1·9
183	581·2	583·0	584·9	586·7	588·6	590·4	592·3	594·1	596·0	597·8	1·8
184	599·7	601·5	603·4	605·2	607·1	608·9	610·8	612·6	614·4	616·3	1·8
185	26 618·1	619·9	621·8	623·6	625·4	627·3	629·1	630·9	632·8	634·6	1·8
186	636·4	638·3	640·1	641·9	643·8	645·6	647·4	649·3	651·1	652·9	1·8
187	654·7	656·6	658·4	660·2	662·0	663·8	665·7	667·5	669·3	671·1	1·8
188	26 672·9	674·7	676·5	678·4	680·2	682·0	683·8	685·6	687·4	689·2	1·8
189	691·0	692·8	694·6	696·4	698·2	700·0	701·8	703·6	705·3	707·1	1·8
190	708·9	710·7	712·5	714·3	716·1	717·8	719·6	721·4	723·2	725·0	1·8
191	26 726·7	728·5	730·3	732·1	733·8	735·6	737·4	739·2	740·9	742·7	1·8
192	744·5	746·3	748·0	749·8	751·6	753·3	755·1	756·9	758·6	760·4	1·8
193	762·2	763·9	765·7	767·5	769·2	771·0	772·8	774·5	776·3	778·0	1·8
194	26 779·8	781·6	783·3	785·1	786·8	788·6	790·3	792·1	793·8	795·6	1·8
195	797·3	799·1	800·8	802·6	804·3	806·0	807·8	809·5	811·2	813·0	1·7
196	814·7	816·4	818·2	819·9	821·6	823·4	825·1	826·8	828·5	830·3	1·7
197	26 832·0	833·7	835·4	837·2	838·9	840·6	842·3	844·0	845·8	847·5	1·7
198	849·2	850·9	852·6	854·3	856·1	857·8	859·5	861·2	862·9	864·6	1·7
199	866·3	868·1	869·8	871·5	873·2	874·9	876·6	878·3	880·0	881·7	1·7
200	26 883·4	885·1	886·8	888·5	890·2	891·9	893·6	895·3	896·9	898·6	1·7
201	900·3	902·0	903·7	905·4	907·1	908·7	910·4	912·1	913·8	915·5	1·7
202	917·2	918·8	920·5	922·2	923·9	925·6	927·2	928·9	930·6	932·3	1·7
203	26 934·0	935·6	937·3	939·0	940·7	942·3	944·0	945·7	947·3	949·0	1·7
204	950·7	952·3	954·0	955·7	957·3	959·0	960·7	962·3	964·0	965·7	1·7
205	967·3	969·0	970·7	972·3	974·0	975·6	977·3	979·0	980·6	982·3	1·7
206	26 983·9	985·6	987·2	988·9	990·5	992·2	993·8	995·5	997·1	998·8	1·7
207	27 000·4	002·1	003·7	005·4	907·0	008·6	010·3	011·9	013·5	015·2	1·6
208	016·8	018·4	020·1	021·7	023·3	025·0	026·6	028·2	029·9	031·5	1·6
209	27 033·1	034·7	036·4	038·0	039·6	041·2	042·9	044·5	046·1	047·7	1·6
210	049·3	051·0	052·6	054·2	055·8	057·4	059·0	060·7	062·3	063·9	1·6
211	065·5	067·1	068·7	070·3	072·0	073·6	075·2	076·8	078·4	080·0	1·6

212	27 081·6	083·2	084·8	086·4	088·0	089·6	091·2	092·8	094·4	096·0	1·6
213	097·6	099·2	100·8	102·4	104·0	105·6	107·2	108·7	110·3	111·9	1·6
214	113·5	115·1	116·7	118·3	119·8	121·4	123·0	124·6	126·2	127·8	1·6
215	27 129·3	130·9	132·5	134·1	135·7	137·2	138·8	140·4	142·0	143·5	1·6
216	145·1	146·7	148·3	149·8	151·4	153·0	154·5	156·1	157·7	159·2	1·6
217	160·8	162·4	163·9	165·5	167·1	168·6	170·2	171·8	173·3	174·9	1·6
218	27 176·5	178·0	179·6	181·1	182·7	184·2	185·8	187·3	188·9	190·4	1·5
219	192·0	193·5	195·1	196·6	198·2	199·7	201·3	202·8	204·4	205·9	1·5
220	207·5	209·0	210·6	212·1	213·7	215·2	216·8	218·3	219·9	221·4	1·5
221	27 222·9	224·5	226·0	227·6	229·1	230·6	232·2	233·7	235·2	236·8	1·5
222	238·3	239·8	241·4	242·9	244·4	246·0	247·5	249·0	250·6	252·1	1·5
223	253·6	255·1	256·7	258·2	259·7	261·2	262·8	264·3	265·8	267·3	1·5
224	27 268·8	270·4	271·9	273·4	274·9	276·4	277·9	279·5	281·0	282·5	1·5
225	284·0	285·5	287·0	288·5	290·1	291·6	293·1	294·6	296·1	297·6	1·5
226	299·1	300·6	302·1	303·6	305·1	306·6	308·1	309·6	311·1	312·6	1·5
227	27 314·1	315·6	317·1	318·6	320·1	321·6	323·1	324·6	326·0	327·5	1·5
228	329·0	330·5	332·0	333·5	335·0	336·5	337·9	339·4	340·9	342·4	1·5
229	343·9	345·4	346·9	348·3	349·8	351·3	352·8	354·3	355·8	357·2	1·5
230	27 358·7	360·2	361·7	363·2	364·6	366·1	367·6	369·1	370·5	372·0	1·5
231	373·5	375·0	376·4	377·9	379·4	380·8	382·3	383·8	385·2	386·7	1·5
232	388·2	389·6	391·1	392·6	394·0	395·5	396·9	398·4	399·9	401·3	1·5
233	27 402·8	404·2	405·7	407·2	408·6	410·1	411·5	413·0	414·5	415·9	1·5
234	417·4	418·8	420·3	421·7	423·2	424·6	426·1	427·5	429·0	430·4	1·4
235	431·9	433·3	434·8	436·2	437·7	439·1	440·6	442·0	443·5	444·9	1·4
236	27 446·3	447·8	449·2	450·7	452·1	453·5	455·0	456·4	457·9	459·3	1·4
237	460·7	462·2	463·6	465·0	466·5	467·9	469·3	470·7	472·2	473·6	1·4
238	475·0	476·4	477·9	479·3	480·7	482·1	483·6	485·0	486·4	487·8	1·4
239	27 489·3	490·7	492·1	493·5	494·9	496·4	497·8	499·2	500·6	502·0	1·4
240	503·5	504·9	506·3	507·7	509·1	510·5	512·0	513·4	514·8	516·2	1·4
241	517·6	519·0	520·4	521·9	523·3	524·7	526·1	527·5	528·9	530·3	1·4
242	27 531·7	533·1	534·5	535·9	537·3	538·7	540·1	541·5	542·9	544·3	1·4
243	545·7	547·1	548·5	549·9	551·3	552·7	554·1	555·5	556·9	558·3	1·4
244	559·7	561·1	562·5	563·9	565·3	566·7	568·1	569·5	570·9	572·2	1·4
245	27 573·6	575·0	576·4	577·8	579·2	580·6	581·9	583·3	584·7	586·1	1·4
246	587·5	588·9	590·2	591·6	593·0	594·4	595·8	597·1	598·5	599·9	1·4
247	601·3	602·6	604·0	605·4	606·8	608·1	609·5	610·9	612·3	613·6	1·4

Table IX—*continued.*

$s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
248	27 615 ·0	616 ·4	617 ·8	619 ·1	620 ·5	621 ·9	623 ·3	624 ·6	626 ·0	627 ·4	1 ·4
249	628 ·7	630 ·1	631 ·5	632 ·9	634 ·2	635 ·6	637 ·0	638 ·3	639 ·7	641 ·1	1 ·4
250	642 ·4	643 ·8	645 ·2	646 ·5	647 ·9	649 ·2	650 ·6	651 ·9	653 ·3	654 ·6	1 ·4
251	27 656 ·0	657 ·3	658 ·7	660 ·0	661 ·4	662 ·7	664 ·1	665 ·4	666 ·8	668 ·1	1 ·3
252	669 ·5	670 ·8	672 ·2	673 ·5	674 ·9	676 ·2	677 ·6	678 ·9	680 ·3	681 ·6	1 ·3
253	682 ·9	684 ·3	685 ·6	687 ·0	688 ·3	689 ·6	691 ·0	692 ·3	693 ·7	695 ·0	1 ·3
254	27 696 ·3	697 ·7	699 ·0	700 ·3	701 ·7	703 ·0	704 ·3	705 ·7	707 ·0	708 ·3	1 ·3
255	709 ·7	711 ·0	712 ·3	713 ·7	715 ·0	716 ·3	717 ·7	719 ·0	720 ·3	721 ·7	1 ·3
256	723 ·0	724 ·3	725 ·7	727 ·0	728 ·3	729 ·6	731 ·0	732 ·3	733 ·6	735 ·0	1 ·3
257	27 736 ·3	737 ·6	738 ·9	740 ·3	741 ·6	742 ·9	744 ·2	745 ·6	746 ·9	748 ·2	1 ·3
258	749 ·5	750 ·9	752 ·2	753 ·5	754 ·8	756 ·1	757 ·5	758 ·8	760 ·1	761 ·4	1 ·3
259	762 ·7	764 ·0	765 ·3	766 ·7	768 ·0	769 ·3	770 ·6	771 ·9	773 ·2	774 ·5	1 ·3
260	27 775 ·8	777 ·1	778 ·4	779 ·7	781 ·0	782 ·3	783 ·6	784 ·9	786 ·2	787 ·5	1 ·3
261	788 ·8	790 ·1	791 ·4	792 ·7	794 ·0	795 ·3	796 ·6	797 ·9	799 ·2	800 ·5	1 ·3
262	801 ·8	803 ·1	804 ·4	805 ·7	807 ·0	808 ·3	809 ·6	810 ·9	812 ·2	813 ·5	1 ·3
263	27 814 ·8	816 ·1	817 ·4	818 ·7	820 ·0	821 ·3	822 ·6	823 ·9	825 ·2	826 ·5	1 ·3
264	827 ·7	829 ·0	830 ·3	831 ·6	832 ·9	834 ·2	835 ·5	836 ·8	838 ·0	839 ·3	1 ·3
265	840 ·6	841 ·9	843 ·2	844 ·4	845 ·7	847 ·0	848 ·3	849 ·5	850 ·8	852 ·1	1 ·3
266	27 853 ·4	854 ·6	855 ·9	857 ·2	858 ·4	859 ·7	861 ·0	862 ·3	863 ·5	864 ·8	1 ·3
267	866 ·1	867 ·3	868 ·6	869 ·9	871 ·2	872 ·4	873 ·7	875 ·0	876 ·3	877 ·5	1 ·3
268	878 ·8	880 ·1	881 ·3	882 ·6	883 ·9	885 ·1	886 ·4	887 ·7	888 ·9	890 ·2	1 ·3
269	27 891 ·5	892 ·7	894 ·0	895 ·3	896 ·5	897 ·8	899 ·1	900 ·3	901 ·6	902 ·9	1 ·3
270	904 ·1	905 ·4	906 ·7	907 ·9	909 ·2	910 ·5	911 ·7	913 ·0	914 ·2	915 ·5	1 ·3
271	916 ·7	918 ·0	919 ·2	920 ·5	921 ·7	923 ·0	924 ·2	925 ·5	926 ·7	928 ·0	1 ·3
272	27 929 ·2	930 ·5	931 ·7	933 ·0	934 ·2	935 ·5	936 ·7	938 ·0	939 ·2	940 ·5	1 ·3
273	941 ·7	943 ·0	944 ·2	945 ·5	946 ·7	947 ·9	949 ·2	950 ·4	951 ·7	952 ·9	1 ·2
274	954 ·1	955 ·4	956 ·6	957 ·9	959 ·1	960 ·4	961 ·6	962 ·8	964 ·1	965 ·3	1 ·2
275	27 966 ·5	967 ·8	969 ·0	970 ·2	971 ·5	972 ·7	973 ·9	975 ·2	976 ·4	977 ·6	1 ·2
276	978 ·9	980 ·1	981 ·3	982 ·6	983 ·8	985 ·0	986 ·3	987 ·5	988 ·7	990 ·0	1 ·2
277	991 ·2	992 ·4	993 ·6	994 ·9	996 ·1	997 ·3	998 ·5	999 ·8	*001 ·0	*002 ·2	1 ·2

278	28 003 ·4	004 ·7	005 ·9	007 ·1	008 ·3	009 ·5	010 ·8	012 ·0	013 ·2	014 ·4	1 ·2
279	015 ·6	016 ·9	018 ·1	019 ·3	020 ·5	021 ·8	023 ·0	024 ·2	025 ·4	026 ·6	1 ·2
280	027 ·8	029 ·1	030 ·3	031 ·5	032 ·7	033 ·9	035 ·1	036 ·3	037 ·5	038 ·7	1 ·2
281	28 039 ·9	041 ·2	042 ·4	043 ·6	044 ·8	046 ·0	047 ·2	048 ·4	049 ·6	050 ·8	1 ·2
282	052 ·0	053 ·3	054 ·5	055 ·7	056 ·9	058 ·1	059 ·3	060 ·5	061 ·7	062 ·9	1 ·2
283	064 ·1	065 ·3	066 ·5	067 ·7	068 ·9	070 ·1	071 ·3	072 ·5	073 ·7	074 ·9	1 ·2
284	28 076 ·1	077 ·3	078 ·5	079 ·7	080 ·9	082 ·1	083 ·3	084 ·5	085 ·7	086 ·9	1 ·2
285	088 ·0	089 ·2	090 ·4	091 ·6	092 ·8	094 ·0	095 ·2	096 ·4	097 ·6	098 ·8	1 ·2
286	099 ·9	101 ·1	102 ·3	103 ·5	104 ·7	105 ·9	107 ·1	108 ·3	109 ·5	110 ·6	1 ·2
287	28 111 ·8	113 ·0	114 ·2	115 ·4	116 ·6	117 ·7	118 ·9	120 ·1	121 ·3	122 ·5	1 ·2
288	123 ·6	124 ·8	126 ·0	127 ·2	128 ·4	129 ·5	130 ·7	131 ·9	133 ·1	134 ·3	1 ·2
289	135 ·4	136 ·6	137 ·8	139 ·0	140 ·1	141 ·3	142 ·5	143 ·7	144 ·9	146 ·0	1 ·2
290	28 147 ·2	148 ·4	149 ·6	150 ·7	151 ·9	153 ·1	154 ·3	155 ·4	156 ·6	157 ·8	1 ·2
291	158 ·9	160 ·1	161 ·3	162 ·4	163 ·6	164 ·8	166 ·0	167 ·1	168 ·3	169 ·5	1 ·2
292	170 ·6	171 ·8	172 ·9	174 ·1	175 ·2	176 ·4	177 ·6	178 ·7	179 ·9	181 ·0	1 ·2
293	28 182 ·2	183 ·3	184 ·5	185 ·7	186 ·8	188 ·0	189 ·1	190 ·3	191 ·4	192 ·6	1 ·2
294	193 ·8	194 ·9	196 ·1	197 ·2	198 ·4	199 ·5	200 ·7	201 ·8	203 ·0	204 ·1	1 ·1
295	205 ·3	206 ·4	207 ·6	208 ·7	209 ·9	211 ·0	212 ·2	213 ·3	214 ·5	215 ·6	1 ·1
296	28 216 ·8	217 ·9	219 ·1	220 ·2	221 ·4	222 ·5	223 ·7	224 ·8	226 ·0	227 ·1	1 ·1
297	228 ·3	229 ·4	230 ·6	231 ·7	232 ·9	234 ·0	235 ·2	236 ·3	237 ·5	238 ·6	1 ·1
298	239 ·7	240 ·9	242 ·0	243 ·2	244 ·3	245 ·4	246 ·6	247 ·7	248 ·9	250 ·0	1 ·1
299	28 251 ·1	252 ·3	253 ·4	254 ·6	255 ·7	256 ·8	258 ·0	259 ·1	260 ·3	261 ·4	1 ·1
300	262 ·5	263 ·7	264 ·8	265 ·9	267 ·1	268 ·2	269 ·3	270 ·5	271 ·6	272 ·7	1 ·1
301	273 ·8	275 ·0	276 ·1	277 ·2	278 ·4	279 ·5	280 ·6	281 ·7	282 ·9	284 ·0	1 ·1
302	28 285 ·1	286 ·2	287 ·4	288 ·5	289 ·6	290 ·7	291 ·9	293 ·0	294 ·1	295 ·2	1 ·1
303	296 ·4	297 ·5	298 ·6	299 ·7	300 ·9	302 ·0	303 ·1	304 ·2	305 ·4	306 ·5	1 ·1
304	307 ·6	308 ·7	309 ·8	311 ·0	312 ·1	313 ·2	314 ·3	315 ·4	316 ·5	317 ·7	1 ·1
305	28 318 ·8	319 ·9	321 ·0	322 ·1	323 ·2	324 ·3	325 ·5	326 ·6	327 ·7	328 ·8	1 ·1
306	329 ·9	331 ·0	332 ·1	333 ·2	334 ·4	335 ·5	336 ·6	337 ·7	338 ·8	339 ·9	1 ·1
307	341 ·0	342 ·1	343 ·2	344 ·3	345 ·4	346 ·5	347 ·6	348 ·7	349 ·8	350 ·9	1 ·1
308	28 352 ·0	353 ·1	354 ·2	355 ·3	356 ·4	357 ·5	358 ·6	359 ·7	360 ·8	361 ·9	1 ·1
309	363 ·0	364 ·1	365 ·2	366 ·3	367 ·4	368 ·5	369 ·6	370 ·7	371 ·8	372 ·9	1 ·1
310	374 ·0	375 ·1	376 ·2	377 ·3	378 ·4	379 ·5	380 ·6	381 ·7	382 ·8	383 ·9	1 ·1
311	28 385 ·0	386 ·1	387 ·2	388 ·3	389 ·4	390 ·5	391 ·6	392 ·6	393 ·7	394 ·8	1 ·1
312	395 ·9	397 ·0	398 ·1	399 ·2	400 ·3	401 ·4	402 ·4	403 ·5	404 ·6	405 ·7	1 ·1
313	406 ·8	407 ·9	409 ·0	410 ·0	411 ·1	412 ·2	413 ·3	414 ·4	415 ·5	416 ·5	1 ·1

Table IX—*continued.*

$s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
314	28 417 ·6	418 ·7	419 ·8	420 ·9	422 ·0	423 ·0	424 ·1	425 ·2	426 ·3	427 ·4	1 ·1
315	428 ·4	429 ·5	430 ·6	431 ·7	432 ·8	433 ·8	434 ·9	436 ·0	437 ·1	438 ·1	1 ·1
316	439 ·2	440 ·3	441 ·4	442 ·4	443 ·5	444 ·6	445 ·7	446 ·7	447 ·8	448 ·9	1 ·1
317	28 450 ·0	451 ·0	452 ·1	453 ·2	454 ·2	455 ·3	456 ·4	457 ·5	458 ·5	459 ·6	1 ·1
318	460 ·7	461 ·7	462 ·8	463 ·9	464 ·9	466 ·0	467 ·1	468 ·1	469 ·2	470 ·3	1 ·1
319	471 ·3	472 ·4	473 ·4	474 ·5	475 ·6	476 ·6	477 ·7	478 ·8	479 ·8	480 ·9	1 ·1
320	28 482 ·0	483 ·0	484 ·1	485 ·2	486 ·2	487 ·3	488 ·4	489 ·4	490 ·5	491 ·6	1 ·1
321	492 ·6	493 ·7	494 ·8	495 ·8	496 ·9	497 ·9	499 ·0	500 ·1	501 ·1	502 ·2	1 ·1
322	503 ·2	504 ·3	505 ·4	506 ·4	507 ·5	508 ·5	509 ·6	510 ·6	511 ·7	512 ·7	1 ·1
323	28 513 ·8	514 ·8	515 ·9	516 ·9	518 ·0	519 ·0	520 ·1	521 ·1	522 ·2	523 ·2	1 ·0
324	524 ·3	525 ·3	526 ·4	527 ·4	528 ·5	529 ·5	530 ·6	531 ·6	532 ·7	533 ·7	1 ·0
325	534 ·8	535 ·8	536 ·9	537 ·9	539 ·0	540 ·0	541 ·1	542 ·1	543 ·2	544 ·2	1 ·0
326	28 545 ·2	546 ·3	547 ·3	548 ·4	549 ·4	550 ·4	551 ·5	552 ·5	553 ·6	554 ·6	1 ·0
327	555 ·6	556 ·7	557 ·7	558 ·8	559 ·8	560 ·8	561 ·9	562 ·9	563 ·9	565 ·0	1 ·0
328	566 ·0	567 ·1	568 ·1	569 ·1	570 ·2	571 ·2	572 ·2	573 ·3	574 ·3	575 ·3	1 ·0
329	28 576 ·4	577 ·4	578 ·4	579 ·5	580 ·5	581 ·5	582 ·6	583 ·6	584 ·6	585 ·7	1 ·0
330	586 ·7	587 ·7	588 ·8	589 ·8	590 ·8	591 ·9	592 ·9	593 ·9	594 ·9	596 ·0	1 ·0
331	597 ·0	598 ·0	599 ·0	600 ·1	601 ·1	602 ·1	603 ·1	604 ·2	605 ·2	606 ·2	1 ·0
332	28 607 ·2	608 ·3	609 ·3	610 ·3	611 ·4	612 ·4	613 ·4	614 ·4	615 ·5	616 ·5	1 ·0
333	617 ·5	618 ·5	619 ·6	620 ·6	621 ·6	622 ·6	623 ·7	624 ·7	625 ·7	626 ·7	1 ·0
334	627 ·7	628 ·8	629 ·8	630 ·8	631 ·8	632 ·8	633 ·8	634 ·9	635 ·9	636 ·9	1 ·0
335	28 637 ·9	638 ·9	639 ·9	640 ·9	642 ·0	643 ·0	644 ·0	645 ·0	646 ·0	647 ·0	1 ·0
336	648 ·0	649 ·0	650 ·1	651 ·1	652 ·1	653 ·1	654 ·1	655 ·1	656 ·1	657 ·1	1 ·0
337	658 ·1	659 ·2	660 ·2	661 ·2	662 ·2	663 ·2	664 ·2	665 ·2	666 ·2	667 ·2	1 ·0
338	28 668 ·2	669 ·2	670 ·2	671 ·2	672 ·2	673 ·2	674 ·2	675 ·2	676 ·2	677 ·2	1 ·0
339	678 ·2	679 ·2	680 ·2	681 ·2	682 ·2	683 ·2	684 ·2	685 ·2	686 ·2	687 ·2	1 ·0
340	688 ·2	689 ·2	690 ·2	691 ·2	692 ·2	693 ·2	694 ·2	695 ·2	696 ·2	697 ·2	1 ·0
341	28 698 ·2	699 ·2	700 ·2	701 ·2	702 ·2	703 ·2	704 ·2	705 ·2	706 ·2	707 ·2	1 ·0
342	708 ·2	709 ·2	710 ·2	711 ·2	712 ·2	713 ·2	714 ·2	715 ·2	716 ·2	717 ·2	1 ·0
343	718 ·1	719 ·1	720 ·1	721 ·1	722 ·1	723 ·1	724 ·1	725 ·1	726 ·1	727 ·0	1 ·0

344	28 728·0	729·0	730·0	731·0	732·0	733·0	734·0	734·9	735·9	736·9	1·0
345	737·9	738·9	739·9	740·8	741·8	742·8	743·8	744·8	745·8	746·7	1·0
346	747·7	748·7	749·7	750·7	751·7	752·6	753·6	754·6	755·6	756·6	1·0
347	28 757·5	758·5	759·5	760·5	761·5	762·4	763·4	764·4	765·4	766·4	1·0
348	767·3	768·3	769·3	770·3	771·2	772·2	773·2	774·2	775·1	776·1	1·0
349	777·1	778·1	779·0	780·0	781·0	782·0	782·9	783·9	784·9	785·9	1·0
350	28 786·8	787·8	788·8	789·8	790·7	791·7	792·7	793·6	794·6	795·6	1·0
351	796·5	797·5	798·5	799·4	800·4	801·4	802·3	803·3	804·3	805·2	1·0
352	806·2	807·2	808·1	809·1	810·1	811·0	812·0	813·0	813·9	814·9	1·0
353	28 815·9	816·8	817·8	818·8	819·7	820·7	821·7	822·6	823·6	824·5	1·0
354	825·5	826·5	827·4	828·4	829·3	830·3	831·3	832·2	833·2	834·1	1·0
355	835·1	836·1	837·0	838·0	838·9	839·9	840·8	841·8	842·7	843·7	1·0
356	28 844·6	845·6	846·6	847·5	848·5	849·4	850·4	851·3	852·3	853·2	1·0
357	854·2	855·1	856·1	857·0	858·0	858·9	859·9	860·8	861·8	862·7	0·9
358	863·7	864·6	865·6	866·5	867·5	868·4	869·4	870·3	871·3	872·2	0·9
359	28 873·2	874·1	875·1	876·0	877·0	877·9	878·9	879·8	880·8	881·7	0·9
360	882·6	883·6	884·5	885·5	886·4	887·4	888·3	889·3	890·2	891·1	0·9
361	892·1	893·0	894·0	894·9	895·8	896·8	897·7	898·7	899·6	900·5	0·9
362	28 901·5	902·4	903·4	904·3	905·2	906·2	907·1	908·0	909·0	909·9	0·9
363	910·9	911·8	912·7	913·7	914·6	915·5	916·5	917·4	918·3	919·3	0·9
364	920·2	921·2	922·1	923·1	924·0	924·9	925·9	926·8	927·7	928·7	0·9
365	28 929·6	930·5	931·5	932·4	933·3	934·3	935·2	936·1	937·1	938·0	0·9
366	938·9	939·9	940·8	941·7	942·6	943·6	944·5	945·4	946·3	947·3	0·9
367	948·2	949·1	950·0	951·0	951·9	952·8	953·7	954·7	955·6	956·5	0·9
368	28 957·4	958·3	959·3	960·2	961·1	962·0	963·0	963·9	964·8	965·7	0·9
369	966·6	967·6	968·5	969·4	970·3	971·2	972·2	973·1	974·0	974·9	0·9
370	975·8	976·8	977·7	978·6	979·5	980·4	981·4	982·3	983·2	984·1	0·9
371	28 985·0	985·9	986·9	987·8	988·7	989·6	990·5	991·4	992·3	993·3	0·9
372	994·2	995·1	996·0	996·9	997·8	998·7	999·6	*000·5	*001·5	*002·4	0·9
373	29 003·3	004·2	005·1	006·0	006·9	007·8	008·7	009·6	010·6	011·5	0·9
374	29 012·4	013·3	014·2	015·1	016·0	016·9	017·8	018·7	019·6	020·6	0·9
375	021·5	022·4	023·3	024·2	025·1	026·0	026·9	027·8	028·7	029·6	0·9
376	030·5	031·5	032·4	033·3	034·2	035·1	036·0	036·9	037·8	038·7	0·9
377	29 039·6	040·5	041·4	042·3	043·2	044·1	045·0	045·9	046·8	047·7	0·9
378	048·6	049·5	050·4	051·3	052·2	053·1	054·0	054·9	055·8	056·7	0·9
379	057·6	058·5	059·4	060·3	061·2	062·1	063·0	063·9	064·8	065·7	0·9

Table IX—*continued.*

$s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
380	29 066 ·5	067 ·4	068 ·3	069 ·2	070 ·1	071 ·0	071 ·9	072 ·8	073 ·7	074 ·6	0 ·9
381	075 ·5	076 ·4	077 ·3	078 ·2	079 ·1	079 ·9	080 ·8	081 ·7	082 ·6	083 ·5	0 ·9
382	084 ·4	085 ·3	086 ·2	087 ·1	088 ·0	088 ·9	089 ·7	090 ·6	091 ·5	092 ·4	0 ·9
383	29 093 ·3	094 ·2	095 ·1	096 ·0	096 ·9	097 ·7	098 ·6	099 ·5	100 ·4	101 ·3	0 ·9
384	102 ·2	103 ·1	103 ·9	104 ·8	105 ·7	106 ·6	107 ·5	108 ·4	109 ·2	110 ·1	0 ·9
385	111 ·0	111 ·9	112 ·8	113 ·7	114 ·5	115 ·4	116 ·3	117 ·2	118 ·1	119 ·0	0 ·9
386	29 119 ·8	120 ·7	121 ·6	122 ·5	123 ·4	124 ·2	125 ·1	126 ·0	126 ·9	127 ·8	0 ·9
387	128 ·6	129 ·5	130 ·4	131 ·3	132 ·2	133 ·0	133 ·9	134 ·8	135 ·7	136 ·6	0 ·9
388	137 ·4	138 ·3	139 ·2	140 ·1	140 ·9	141 ·8	142 ·7	143 ·6	144 ·4	145 ·3	0 ·9
389	29 146 ·2	147 ·1	147 ·9	148 ·8	149 ·7	150 ·6	151 ·4	152 ·3	153 ·2	154 ·1	0 ·9
390	154 ·9	155 ·8	156 ·7	157 ·6	158 ·4	159 ·3	160 ·2	161 ·1	161 ·9	162 ·8	0 ·9
391	163 ·7	164 ·5	165 ·4	166 ·3	167 ·1	168 ·0	168 ·9	169 ·7	170 ·6	171 ·5	0 ·9
392	29 172 ·3	173 ·2	174 ·0	174 ·9	175 ·8	176 ·6	177 ·5	178 ·4	179 ·2	180 ·1	0 ·9
393	181 ·0	181 ·8	182 ·7	183 ·5	184 ·4	185 ·3	186 ·1	187 ·0	187 ·8	188 ·7	0 ·9
394	189 ·6	190 ·4	191 ·3	192 ·1	193 ·0	193 ·9	194 ·7	195 ·6	196 ·4	197 ·3	0 ·9
395	29 198 ·2	199 ·0	199 ·9	200 ·7	201 ·6	202 ·5	203 ·3	204 ·2	205 ·0	205 ·9	0 ·9
396	206 ·8	207 ·6	208 ·5	209 ·4	210 ·2	211 ·1	211 ·9	212 ·8	213 ·7	214 ·5	0 ·9
397	215 ·4	216 ·2	217 ·1	218 ·0	218 ·8	219 ·7	220 ·6	221 ·4	222 ·3	223 ·2	0 ·9
398	29 224 ·0	224 ·9	225 ·7	226 ·6	227 ·4	228 ·3	229 ·1	230 ·0	230 ·8	231 ·7	0 ·9
399	232 ·5	233 ·4	234 ·2	235 ·1	235 ·9	236 ·8	237 ·6	238 ·5	239 ·3	240 ·2	0 ·9
400	241 ·0										
500	30 000 ·0										

TABLE X.

Four Figure Logarithms.

											Fourth Figure.								
No.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	34
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	7	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	5	6	7

Table X—*continued.*

Four Figure Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	Fourth Figure. 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9975	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	4	4

TABLE XI.

Anti-logarithms or Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1235	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	5	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	5	6

Table XI—*continued.*

Anti-logarithms or Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	13
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE XII.

Natural Sines.

	0′	10′	20′	30′	40′	50′	1 2 3	4 5 6	7 8 9
0°	0000	0029	0058	0087	0116	0145	3 6 9	12 15 17	20 23 26
1	0175	0204	0233	0262	0291	0320	3 6 9	12 15 17	20 23 26
2	0349	0378	0407	0436	0465	0494	3 6 9	12 15 17	20 23 26
3	0523	0552	0581	0610	0640	0669	3 6 9	12 15 17	20 23 26
4	0698	0727	0756	0785	0814	0843	3 6 9	12 15 17	20 23 26
5	0872	0901	0929	0958	0987	1016	3 6 9	12 14 17	20 23 26
6	1045	1074	1103	1132	1161	1190	3 6 9	12 14 17	20 23 26
7	1219	1248	1276	1305	1334	1363	3 6 9	12 14 17	20 23 26
8	1392	1421	1449	1478	1507	1536	3 6 9	12 14 17	20 23 26
9	1564	1593	1622	1650	1679	1708	3 6 9	12 14 17	20 23 26
10	1736	1765	1794	1822	1851	1880	3 6 9	12 14 17	20 23 26
11	1908	1937	1965	1994	2022	2051	3 6 9	11 14 17	20 23 26
12	2079	2108	2136	2164	2193	2221	3 6 9	11 14 17	20 23 26
13	2250	2278	2306	2334	2363	2391	3 6 8	11 14 17	20 23 25
14	2419	2447	2476	2504	2532	2560	3 6 8	11 14 17	20 23 25
15	2588	2616	2644	2672	2700	2728	3 6 8	11 14 17	20 22 25
16	2756	2784	2812	2840	2868	2896	3 6 8	11 14 17	20 22 25
17	2924	2952	2979	3007	3035	3062	3 6 8	11 14 17	19 22 25
18	3090	3118	3145	3173	3201	3228	3 6 8	11 14 17	19 22 25
19	3256	3283	3311	3338	3365	3393	3 5 8	11 14 16	19 22 25
20	3420	3448	3475	3502	3529	3557	3 5 8	11 14 16	19 22 25
21	3584	3611	3638	3665	3692	3719	3 5 8	11 14 16	19 22 24
22	3746	3773	3800	3827	3854	3881	3 5 8	11 14 16	19 21 24
23	3907	3934	3961	3987	4014	4041	3 5 8	11 14 16	19 21 24
24	4067	4094	4120	4147	4173	4200	3 5 8	11 13 16	19 21 24
25	4226	4253	4279	4305	4331	4358	3 5 8	11 13 16	18 21 24
26	4384	4410	4436	4462	4488	4514	3 5 8	10 13 16	18 21 23
27	4540	4566	4592	4617	4643	4669	3 5 8	10 13 15	18 21 23
28	4695	4720	4746	4772	4797	4823	3 5 8	10 13 15	18 20 23
29	4848	4874	4899	4924	4950	4975	3 5 8	10 13 15	18 20 23
30	5000	5025	5050	5075	5100	5125	3 5 8	10 13 15	18 20 23
31	5150	5175	5200	5225	5250	5275	2 5 7	10 12 15	17 20 22
32	5299	5324	5348	5373	5398	5422	2 5 7	10 12 15	17 20 22
33	5446	5471	5495	5519	5544	5568	2 5 7	10 12 15	17 19 22
34	5592	5616	5640	5664	5688	5712	2 5 7	10 12 14	17 19 22
35	5736	5760	5783	5807	5831	5854	2 5 7	10 12 14	17 19 21
36	5878	5901	5925	5948	5972	5995	2 5 7	9 12 14	16 19 21
37	6018	6041	6065	6088	6111	6134	2 5 7	9 12 14	16 18 21
38	6157	6180	6202	6225	6248	6271	2 5 7	9 11 14	16 18 20
39	6293	6316	6338	6361	6383	6406	2 4 7	9 11 13	16 18 20
40	6428	6450	6472	6494	6517	6539	2 4 7	9 11 13	15 18 20
41	6561	6583	6604	6626	6648	6670	2 4 7	9 11 13	15 17 20
42	6691	6713	6734	6756	6777	6799	2 4 6	9 11 13	15 17 19
43	6820	6841	6862	6884	6905	6926	2 4 6	8 11 13	15 17 19
44	6947	6967	6988	7009	7030	7050	2 4 6	8 10 12	15 17 19

Table XII—*continued.*

Natural Sines.

	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
45°	7071	7092	7112	7133	7153	7173	2	4	6	8	10	12	14	16	18
46	7193	7214	7234	7254	7274	7294	2	4	6	8	10	12	14	16	18
47	7314	7333	7353	7373	7392	7412	2	4	6	8	10	12	14	16	18
48	7431	7451	7470	7490	7509	7528	2	4	6	8	10	12	13	15	17
49	7547	7566	7585	7604	7623	7642	2	4	6	8	9	11	13	15	17
50	7660	7679	7698	7716	7735	7753	2	4	6	7	9	11	13	15	17
51	7771	7790	7808	7826	7844	7862	2	4	5	7	9	11	13	14	16
52	7880	7898	7916	7934	7951	7969	2	4	5	7	9	11	12	14	16
53	7986	8004	8021	8039	8056	8073	2	3	5	7	9	10	12	14	16
54	8090	8107	8124	8141	8158	8175	2	3	5	7	8	10	12	14	15
55	8192	8208	8225	8241	8258	8274	2	3	5	7	8	10	12	13	15
56	8290	8307	8323	8339	8355	8371	2	3	5	6	8	10	11	13	14
57	8387	8403	8418	8434	8450	8465	2	3	5	6	8	9	11	12	14
58	8480	8496	8511	8526	8542	8557	2	3	5	6	8	9	11	12	14
59	8572	8587	8601	8616	8631	8646	1	3	4	6	7	9	10	12	13
60	8660	8675	8689	8704	8718	8732	1	3	4	6	7	9	10	11	13
61	8746	8760	8774	8788	8802	8816	1	3	4	6	7	8	10	11	12
62	8829	8843	8857	8870	8884	8897	1	3	4	5	7	8	9	11	12
63	8910	8923	8936	8949	8962	8975	1	3	4	5	6	8	9	10	12
64	8988	9001	9013	9026	9038	9051	1	3	4	5	6	8	9	10	11
65	9063	9075	9088	9100	9112	9124	1	2	4	5	6	7	8	10	11
66	9135	9147	9159	9171	9182	9194	1	2	3	5	6	7	8	9	10
67	9205	9216	9228	9239	9250	9261	1	2	3	4	6	7	8	9	10
68	9272	9283	9293	9304	9315	9325	1	2	3	4	5	6	7	9	10
69	9336	9346	9356	9367	9377	9387	1	2	3	4	5	6	7	8	9
70	9397	9407	9417	9426	9436	9446	1	2	3	4	5	6	7	8	9
71	9455	9465	9474	9483	9492	9502	1	2	3	4	5	6	6	7	8
72	9511	9520	9528	9537	9546	9555	1	2	3	4	4	5	6	7	8
73	9563	9572	9580	9588	9596	9605	1	2	2	3	4	5	6	7	7
74	9613	9621	9628	9636	9644	9652	1	2	2	3	4	5	5	6	7
75	9659	9667	9674	9681	9689	9696	1	1	2	3	4	4	5	6	7
76	9703	9710	9717	9724	9730	9737	1	1	2	3	3	4	5	5	6
77	9744	9750	9757	9763	9769	9775	1	1	2	3	3	4	4	5	6
78	9781	9787	9793	9799	9805	9811	1	1	2	2	3	3	4	5	5
79	9816	9822	9827	9833	9838	9843	1	1	2	2	3	3	4	4	5
80	9848	9853	9858	9863	9868	9872	0	1	1	2	2	3	3	4	4
81	9877	9881	9886	9890	9894	9899	0	1	1	2	2	3	3	3	4
82	9903	9907	9911	9914	9918	9922	0	1	1	2	2	2	3	3	3
83	9925	9929	9932	9936	9939	9942	0	1	1	1	2	2	3	3	3
84	9945	9948	9951	9954	9957	9959	0	1	1	1	1	2	2	2	2
85	9962	9964	9967	9969	9971	9974	0	0	1	1	1	1	2	2	2
86	9976	9978	9980	9981	9983	9985	0	0	1	1	1	1	1	1	2
87	9986	9988	9989	9990	9992	9993	0	0	0	1	1	1	1	1	1
88	9994	9995	9996	9997	9997	9998	0	0	0	0	0	0	1	1	1
89	9998	9999	9999	1·0000	1·0000	1·0000	0	0	0	0	0	0	0	0	0

TABLE XIII.

Natural Cosines.

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
0	1·0000	1·0000	1·0000	1·0000	9999	9999	0	0	0	0	0	0	0	0	0
1	9998	9998	9997	9997	9996	9995	0	0	0	0	0	0	0	0	0
2	9994	9993	9992	9990	9989	9988	0	0	0	0	0	0	1	1	1
3	9986	9985	9983	9981	9980	9978	0	0	1	1	1	1	1	1	1
4	9976	9974	9971	9969	9967	9964	0	0	1	1	1	1	1	1	2
5	9962	9959	9957	9954	9951	9948	0	1	1	1	1	1	2	2	2
6	9945	9942	9939	9936	9932	9929	0	1	1	1	2	2	2	2	2
7	9925	9922	9918	9914	9911	9907	0	1	1	2	2	2	3	3	3
8	9903	9899	9894	9890	9886	9881	0	1	1	2	2	2	3	3	3
9	9877	9872	9868	9863	9858	9853	0	1	1	2	2	3	3	3	4
10	9848	9843	9838	9833	9827	9822	1	1	2	2	2	3	3	4	4
11	9816	9811	9805	9799	9793	9787	1	1	2	2	3	3	4	4	5
12	9781	9775	9769	9763	9757	9750	1	1	2	2	3	3	4	5	5
13	9744	9737	9730	9724	9717	9710	1	1	2	3	3	4	4	5	6
14	9703	9696	9689	9681	9674	9667	1	1	2	3	4	4	5	5	6
15	9659	9652	9644	9636	9628	9621	1	2	2	3	4	4	5	6	7
16	9613	9605	9596	9588	9580	9572	1	2	2	3	4	5	5	6	7
17	9563	9555	9546	9537	9528	9520	1	2	3	3	4	5	6	7	7
18	9511	9502	9492	9483	9474	9465	1	2	3	4	4	5	6	7	8
19	9455	9446	9436	9426	9417	9407	1	2	3	4	5	6	6	7	8
20	9397	9387	9377	9367	9356	9346	1	2	3	4	5	6	7	8	9
21	9336	9325	9315	9304	9293	9283	1	2	3	4	5	6	7	8	9
22	9272	9261	9250	9239	9228	9216	1	2	3	4	6	6	7	9	10
23	9205	9194	9182	9171	9159	9147	1	2	3	5	6	7	8	9	10
24	9135	9124	9112	9100	9088	9075	1	2	4	5	6	7	8	9	10
25	9063	9051	9038	9026	9013	9001	1	3	4	5	6	7	8	10	11
26	8988	8975	8962	8949	8936	8923	1	3	4	5	6	8	9	10	11
27	8910	8897	8884	8870	8857	8843	1	3	4	5	7	8	9	11	12
28	8829	8816	8802	8788	8774	8760	1	3	4	6	7	8	9	11	12
29	8746	8732	8718	8704	8689	8675	1	3	4	6	7	8	10	11	12
30	8660	8646	8631	8616	8601	8587	1	3	4	6	7	9	10	11	13
31	8572	8557	8542	8526	8511	8496	2	3	5	6	8	9	10	12	13
32	8480	8465	8450	8434	8418	8403	2	3	5	6	8	9	11	12	14
33	8387	8371	8355	8339	8323	8307	2	3	5	6	8	9	11	12	14
34	8290	8274	8258	8241	8225	8208	2	3	5	7	8	10	11	13	14
35	8192	8175	8158	8141	8124	8107	2	3	5	7	8	10	12	13	15
36	8090	8073	8056	8039	8021	8004	2	3	5	7	9	10	12	14	15
37	7986	7969	7951	7934	7916	7898	2	4	5	7	9	10	12	14	16
38	7880	7862	7844	7826	7808	7790	2	4	5	7	9	11	12	14	16
39	7771	7753	7735	7716	7698	7679	2	4	6	7	9	11	13	14	16
40	7660	7642	7623	7604	7585	7566	2	4	6	8	9	11	13	15	17
41	7547	7528	7509	7490	7470	7451	2	4	6	8	10	11	13	15	17
42	7431	7412	7392	7373	7353	7333	2	4	6	8	10	12	13	15	17
43	7314	7294	7274	7254	7234	7214	2	4	6	8	10	12	14	16	18
44	7193	7173	7153	7132	7112	7092	2	4	6	8	10	12	14	16	18

Table XIII.—*continued.*

Natural Cosines.

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
45	7071	7050	7030	7009	6988	6967	2	4	6	8	10	12	15	17	19
46	6947	6926	6905	6884	6862	6841	2	4	6	8	11	13	15	17	19
47	6820	6799	6777	6756	6734	6713	2	4	6	9	11	13	15	17	19
48	6691	6670	6648	6626	6604	6583	2	4	7	9	11	13	15	17	19
49	6561	6539	6517	6494	6472	6450	2	4	7	9	11	13	15	17	20
50	6428	6406	6383	6361	6338	6316	2	4	7	9	11	13	15	18	20
51	6293	6271	6248	6225	6202	6180	2	5	7	9	11	13	16	18	20
52	6157	6134	6111	6088	6065	6041	2	5	7	9	12	14	16	18	20
53	6018	5995	5972	5948	5925	5901	2	5	7	9	12	14	16	18	21
54	5878	5854	5831	5807	5783	5760	2	5	7	9	12	14	16	19	21
55	5736	5712	5688	5664	5640	5616	2	5	7	10	12	14	17	19	21
56	5592	5568	5544	5519	5495	5471	2	5	7	10	12	14	17	19	22
57	5446	5422	5398	5373	5348	5324	2	5	7	10	12	15	17	19	22
58	5299	5275	5250	5225	5200	5175	2	5	7	10	12	15	17	20	22
59	5150	5125	5100	5075	5050	5025	3	5	8	10	13	15	17	20	22
60	5000	4975	4950	4924	4899	4874	3	5	8	10	13	15	18	20	23
61	4848	4823	4797	4772	4746	4720	3	5	8	10	13	15	18	20	23
62	4695	4669	4643	4617	4592	4566	3	5	8	10	13	15	18	20	23
63	4540	4514	4488	4462	4436	4410	3	5	8	10	13	15	18	21	23
64	4384	4358	4331	4305	4279	4253	3	5	8	11	13	16	18	21	23
65	4226	4200	4173	4147	4120	4094	3	5	8	11	13	16	18	21	24
66	4067	4041	4014	3987	3961	3934	3	5	8	11	14	16	19	21	24
67	3907	3881	3854	3827	3800	3773	3	5	8	11	14	16	19	21	24
68	3746	3719	3692	3665	3638	3611	3	5	8	11	14	16	19	21	24
69	3584	3557	3529	3502	3475	3448	3	5	8	11	14	16	19	22	24
70	3420	3393	3365	3338	3311	3283	3	5	8	11	14	16	19	22	25
71	3256	3228	3201	3173	3145	3118	3	6	8	11	14	16	19	22	25
72	3090	3062	3035	3007	2979	2952	3	6	8	11	14	17	19	22	25
73	2924	2896	2868	2840	2812	2784	3	6	8	11	14	17	19	22	25
74	2756	2728	2700	2672	2644	2616	3	6	8	11	14	17	20	22	25
75	2588	2560	2532	2504	2476	2447	3	6	8	11	14	17	20	22	25
76	2419	2391	2363	2334	2306	2278	3	6	8	11	14	17	20	23	25
77	2250	2221	2193	2164	2136	2108	3	6	9	11	14	17	20	23	25
78	2079	2051	2022	1994	1965	1937	3	6	9	11	14	17	20	23	26
79	1908	1880	1851	1822	1794	1765	3	6	9	12	14	17	20	23	26
80	1736	1708	1679	1650	1622	1593	3	6	9	12	14	17	20	23	26
81	1564	1536	1507	1478	1449	1421	3	6	9	12	14	17	20	23	26
82	1392	1363	1334	1305	1276	1248	3	6	9	12	14	17	20	23	26
83	1219	1190	1161	1132	1103	1074	3	6	9	12	14	17	20	23	26
84	1045	1016	0987	0958	0929	0901	3	6	9	12	14	17	20	23	26
85	0872	0843	0814	0785	0756	0727	3	6	9	12	15	17	20	23	26
86	0698	0669	0640	0610	0581	0552	3	6	9	12	15	17	20	23	26
87	0523	0494	0465	0436	0407	0378	3	6	9	12	15	17	20	23	26
88	0349	0320	0291	0262	0233	0204	3	6	9	12	15	17	20	23	26
89	0175	0145	0116	0087	0058	0029	3	6	9	12	15	17	20	23	26

TABLE XIV.

Natural Tangents.

	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	1 2	3 4
0°	·0000	0015	0029	0044	0058	0073	0087	0102	0116	0131	0145	0160	3 6	9 12
1	·0175	0189	0204	0218	0233	0247	0262	0276	0291	0306	0320	0335	3 6	9 12
2	·0349	0364	0378	0393	0407	0422	0437	0451	0466	0480	0495	0509	3 6	9 12
3	·0524	0539	0553	0568	0582	0597	0612	0626	0641	0655	0670	0685	3 6	9 12
4	·0699	0714	0729	0743	0758	0772	0787	0802	0816	0831	0846	0860	3 6	9 12
5	·0875	0890	0904	0919	0934	0948	0963	0978	0992	1007	1022	1036	3 6	9 12
6	·1051	1066	1080	1095	1110	1125	1139	1154	1169	1184	1198	1213	3 6	9 12
7	·1228	1243	1257	1272	1287	1302	1317	1331	1346	1361	1376	1391	3 6	9 12
8	·1405	1420	1435	1450	1465	1480	1495	1509	1524	1539	1554	1569	3 6	9 12
9	·1584	1599	1614	1629	1644	1658	1673	1688	1703	1718	1733	1748	3 6	9 12
10	·1763	1778	1793	1808	1823	1838	1853	1868	1883	1899	1914	1929	3 6	9 12
11	·1944	1959	1974	1989	2004	2019	2035	2050	2065	2080	2095	2110	3 6	9 12
12	·2126	2141	2156	2171	2186	2202	2217	2232	2247	2263	2278	2293	3 6	9 12
13	·2309	2324	2339	2355	2370	2385	2401	2416	2432	2447	2462	2478	3 6	9 12
14	·2493	2509	2524	2540	2555	2571	2586	2602	2617	2633	2648	2664	3 6	9 12
15	·2679	2695	2711	2726	2742	2758	2773	2789	2805	2820	2836	2852	3 6	9 13
16	·2867	2883	2899	2915	2931	2946	2962	2978	2994	3010	3026	3041	3 6	9 13
17	·3057	3073	3089	3106	3121	3137	3153	3169	3185	3201	3217	3233	3 6	10 13
18	·3249	3265	3281	3298	3314	3330	3346	3362	3378	3395	3411	3427	3 6	10 13
19	·3443	3460	3476	3492	3508	3525	3541	3558	3574	3590	3607	3623	3 6	10 13
20	·3640	3656	3673	3689	3706	3722	3739	3755	3772	3789	3805	3822	3 7	10 13
21	·3839	3855	3872	3889	3906	3922	3939	3956	3973	3990	4006	4023	3 7	10 13
22	·4040	4057	4074	4091	4108	4125	4142	4159	4176	4193	4210	4228	3 7	10 14
23	·4245	4262	4279	4296	4314	4331	4348	4365	4383	4400	4417	4435	3 7	10 14
24	·4452	4470	4487	4505	4522	4540	4557	4575	4592	4610	4628	4645	4 7	10 14
25	·4663	4681	4699	4716	4734	4752	4770	4788	4806	4823	4841	4859	4 7	11 14
26	·4877	4895	4913	4931	4950	4968	4986	5004	5022	5040	5059	5077	4 7	11 15
27	·5095	5114	5132	5150	5169	5187	5206	5224	5243	5261	5280	5298	4 7	11 15
28	·5317	5336	5354	5373	5392	5411	5430	5448	5467	5486	5505	5524	4 8	11 15
29	·5543	5562	5581	5600	5619	5639	5658	5677	5696	5715	5735	5754	4 8	12 15
30	·5774	5793	5812	5832	5851	5871	5890	5910	5930	5949	5969	5989	4 8	12 16
31	·6009	6028	6048	6068	6088	6108	6128	6148	6168	6188	6208	6228	4 8	12 16
32	·6249	6269	6289	6310	6330	6350	6371	6391	6412	6432	6453	6473	4 8	12 16
33	·6494	6515	6536	6556	6577	6598	6619	6640	6661	6682	6703	6724	4 8	13 17
34	·6745	6766	6787	6809	6830	6851	6873	6894	6916	6937	6959	6980	4 9	13 17
35	·7002	7024	7046	7067	7089	7111	7133	7155	7177	7199	7221	7243	4 9	13 18
36	·7265	7288	7310	7332	7355	7377	7400	7422	7445	7467	7490	7513	5 9	15 18
37	·7536	7558	7581	7604	7627	7650	7673	7696	7720	7743	7766	7789	5 9	14 18
38	7813	7836	7860	7883	7907	7931	7954	7978	8002	8026	8050	8074	5 10	14 19
39	·8098	8122	8146	8170	8195	8219	8243	8268	8292	8317	8342	8366	5 10	15 20
40	·8391	8416	8441	8466	8491	8516	8541	8566	8591	8617	8642	8667	5 10	15 20
41	·8693	8718	8744	8770	8796	8821	8847	8873	8899	8925	8952	8978	5 10	16 21
42	·9004	9030	9057	9083	9110	9137	9163	9190	9217	9244	9271	9298	5 11	16 21
43	·9325	9352	9380	9407	9435	9462	9490	9517	9545	9573	9601	9629	6 11	17 22
44	·9657	9685	9713	9742	9770	9798	9827	9856	9884	9913	9942	9971	6 11	17 23

Table XIV—*continued.*

Natural Tangents.

	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	1	2	3	4
45°	1·000	0029	0058	0088	0117	0147	0176	0206	0235	0265	0295	0325	6	12	18	24
46	1·035	0385	0416	0446	0477	0507	0538	0569	0599	0630	0661	0692	6	12	18	25
47	1·072	0755	0786	0818	0850	0881	0913	0945	0977	1009	1041	1074	6	13	19	25
48	1·111	1139	1171	1204	1237	1270	1303	1336	1369	1403	1436	1470	7	13	20	26
49	1·150	1538	1571	1606	1640	1674	1708	1743	1778	1812	1847	1882	7	14	21	28
50	1·192	1953	1988	2024	2059	2095	2131	2167	2203	2239	2276	2312	7	14	22	29
51	1·235	2386	2423	2460	2497	2534	2572	2609	2647	2685	2723	2761	8	15	23	30
52	1·280	2838	2876	2915	2954	2993	3032	3072	3111	3151	3190	3230	8	16	23	31
53	1·327	3311	3351	3392	3432	3473	3514	3555	3597	3638	3680	3722	8	16	25	33
54	1·376	3806	3848	3891	3934	3976	4019	4063	4106	4150	4193	4237	9	17	26	34
55	1·428	4326	4370	4415	4460	4505	4550	4596	4641	4687	4733	4779	9	18	27	36
56	1·483	4872	4919	4966	5013	5061	5108	5156	5204	5253	5301	5350	10	19	29	38
57	1·540	5448	5497	5547	5597	5647	5697	5747	5798	5849	5900	5952	10	20	30	40
58	1·600	6055	6107	6160	6212	6265	6319	6372	6426	6479	6534	6588	11	21	32	43
59	1·664	6698	6753	6808	6864	6920	6977	7033	7090	7147	7205	7262	11	23	34	45
60	1·732	7379	7437	7496	7556	7615	7675	7735	7796	7856	7917	7979	12	24	36	48
61	1·804	8103	8165	8228	8291	8354	8418	8482	8546	8611	8676	8741	13	26	38	51
62	1·881	8873	8940	9007	9074	9142	9210	9278	9347	9416	9486	9556	14	27	41	55
63	1·963	9697	9768	9840	9912	9984	$\overline{0}057$	$\overline{0}130$	$\overline{0}204$	$\overline{0}278$	$\overline{0}353$	$\overline{0}428$	15	29	44	58
64	2·050	0579	0655	0732	0809	0887	0965	1044	1123	1203	1283	1364	16	31	47	63
65	2·144	1527	1609	1692	1775	1859	1943	2028	2113	2199	2286	2373	17	34	51	68
66	2·246	2549	2637	2727	2817	2907	2998	3090	3183	3276	3369	3464	18	37	55	74
67	2·356	3654	3750	3847	3945	4043	4142	4242	4342	4443	4545	4648	20	40	60	79
68	2·475	4855	4960	5065	5172	5279	5386	5495	5605	5715	5826	5938	22	43	65	87
69	2·605	6165	6279	6395	6511	6628	6746	6865	6985	7106	7228	7351	24	47	71	95
70	2·747	2·760	2·773	2·785	2·798	2·811	2·824	2·837	2·850	2·864	2·877	2·891	3	5	8	10
71	2·904	2·918	2·932	2·946	2·960	2·974	2·989	3·003	3·018	3·033	3·047	3·063	3	6	9	11
72	3·078	3·093	3·108	3·124	3·140	3·156	3·172	3·188	3·204	3·221	3·237	3·254	3	6	10	13
73	3·271	3·288	3·305	3·323	3·340	3·358	3·376	3·394	3·412	3·431	3·450	3·468	4	7	11	14
74	3·487	3·507	3·526	3·546	3·566	3·586	3·606	3·626	3·647	3·668	3·689	3·710	4	8	12	16
75	3·732	3·754	3·776	3·798	3·821	3·844	3·867	3·890	3·914	3·938	3·962	3·986	5	9	14	19
76	4·011	4·036	4·061	4·087	4·113	4·139	4·165	4·192	4·219	4·247	4·275	4·303	5	11	16	21
77	4·331	4·360	4·390	4·419	4·449	4·480	4·511	4·542	4·574	4·606	4·638	4·671	6	12	19	25
78	4·705	4·739	4·773	4·808	4·843	4·879	4·915	4·952	4·989	5·027	5·066	5·105	7	15	22	29
79	5·145	5·185	5·226	5·267	5·309	5·352	5·396	5·410	5·485	5·530	5·576	5·623	9	17	26	35
80	5·671	5·720	5·769	5·820	5·871	5·923	5·976	6·030	6·084	6·140	6·197	6·255				
81	6·314	6·374	6·435	6·497	6·561	6·625	6·691	6·758	6·827	6·897	6·968	7·041				
82	7·115	7·191	7·269	7·348	7·429	7·511	7·596	7·682	7·770	7·861	7·953	8·048				
83	8·144	8·243	8·345	8·449	8·556	8·665	8·777	8·892	9·010	9·131	9·255	9·383				
84	9·514	9·649	9·788	9·931	10·08	10·23	10·39	10·55	10·71	10·88	11·06	11·24				
85	11·43	11·62	11·83	12·03	12·25	12·47	12·71	12·95	13·20	13·46	13·73	14·01				
86	14·30	14·61	14·92	15·26	15·60	15·97	16·35	16·75	17·17	17·61	18·07	18·56				
87	19·08	19·63	20·21	20·82	21·47	22·16	22·90	23·69	24·54	25·45	26·43	27·49				
88	28·64	29·88	31·24	32·73	34·37	36·18	38·19	40·44	42·96	45·83	49·10	52·88				
89	57·29	62·50	68·75	76·39	85·94	98·22	114·6	137·5	171·9	229·2	343·8	687·5				

Difference columns cease to be useful.

TABLE XV.

Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff.	Cosec.	Tan.	Diff.	Cotan.	Secant.	Diff.	Cosine.	
0° 0′	Infin. neg.	Infin.	Infinite.	Infin neg.	Infin.	Infinite.	10·0000	0	10·0000	90° 0′
0 1	6·4637	3010	13·5363	6·4637	3010	13·5363	10·0000	0	10·0000	89 59
0 2	6·7648	1761	13·2352	6·7648	1761	13·2352	10·0000	0	10·0000	89 58
0 3	6·9408	1249	13·0592	6·9408	1249	13·0592	10·0000	0	10·0000	89 57
0 4	7·0658	969	12·9342	7·0658	969	12·9342	10·0000	0	10·0000	89 56
0 5	7·1627	792	12·8373	7·1627	792	12·8373	10·0000	0	10·0000	89 55
0 6	7·2419	669	12·7581	7·2419	669	12·7581	10·0000	0	10·0000	99 54
0 8	7·3668	512	12·6332	7·3668	512	12·6332	10·0000	0	10·0000	89 52
0 10	7·4637	414	12·5363	7·4637	414	12·5363	10·0000	0	10·0000	89 50
0 12	7·5429	348	12·4571	7·5429	348	12·4571	10·0000	0	10·0000	89 48
0 14	7·6099	300	12·3901	7·6099	300	12·3901	10·0000	0	10·0000	89 46
0 16	7·6678	263	12·3322	7·6678	263	12·3322	10·0000	0	10·0000	89 44
0 18	7·7190	235	12·2810	7·7190	235	12·2810	10·0000	0	10·0000	89 42
0 20	7·7648	212	12·2352	7·7648	212	12·2352	10·0000	0	10·0000	89 40
0 22	7·8061	193	12·1939	7·8062	193	12·1938	10·0000	0	10·0000	89 38
0 24	7·8439	177	12·1561	7·8439	177	12·1561	10·0000	0	10·0000	89 36
0 26	7·8787	164	12·1213	7·8787	164	12·1213	10·0000	0	10·0000	89 34
0 28	7·9109	152	12·0891	7·9109	152	12·0891	10·0000	0	10·0000	89 32
0 80	7·9408	137	12·0592	7·9409	137	12·0591	10·0000	0	10·0000	89 30
0 35	8·0078	118	11·9922	8·0078	118	11·9922	10·0000	0	10·0000	89 25
0 40	8·0658	104	11·9342	8·0658	104	11·9342	10·0000	0	10·0000	89 20
0 45	8·1169	93	11·8831	8·1170	93	11·8830	10·0000	0	10·0000	89 15
0 50	8·1627	84	11·8373	8·1627	84	11·8373	10·0000	0	10·0000	89 10
0 55	8·2041	77	11·7959	8·2041	77	11·7959	10·0001	0	9·9999	89 5
1 0	8·2419	70	11·7581	8·2419	70	11·7581	10·0001	0	9·9999	89 0
1 5	8·2766	65	11·7234	8·2767	65	11·7233	10·0001	0	9·9999	88 55
1 10	8·3088	60	11·6912	8·3089	60	11·6911	10·0001	0	9·9999	88 50
1 15	8·3388	56	11·6612	8·3389	56	11·6611	10·0001	0	9·9999	88 45
1 20	8·3668	53	11·6332	8·3669	53	11·6331	10·0001	0	9·9999	88 40
1 25	8·3931	50	11·6069	8·3932	50	11·6068	10·0001	0	9·9999	88 35
1 30	8·4179	46	11·5821	8·4181	46	11·5819	10·0001	0	9·9999	88 30
1 40	8·4637	42	11·5363	8·4638	42	11·5362	10·0002	0	9·9998	88 20
1 50	8·5050	38	11·4950	8·5053	38	11·4947	10·0002	0	9·9998	88 10
2 0	8·5428	35	11·4572	8·5431	35	11·4569	10·0003	0	9·9997	88 0
2 10	8·5776	32	11·4224	8·5779	32	11·4221	10·0003	0	9·9997	87 50
2 20	8·6097	30	11·3903	8·6101	30	11·3899	10·0004	0	9·9996	87 40
2 30	8·6397	28	11·3603	8·6401	28	11·3599	10·0004	0	9·9996	87 30
2 40	8·6677	26	11·3323	8·6682	26	11·3318	10·0005	0	9·9995	87 20
2 50	8·6940	25	11·3060	8·6945	25	11·3055	10·0005	0	9·9995	87 10
3 0	8·7188	24	11·2812	8·7194	24	11 2806	10·0006	0	9·9994	87 0
3 10	8·7423	22	11·2577	8·7429	22	11·2571	10·0007	0	9·9993	86 50
3 20	8·7645	21	11·2355	8·7652	21	11·2348	10·0007	0	9·9993	86 40
3 30	8·7857	20	11·2143	8·7865	20	11·2135	10·0008	0	9·9992	86 30
3 40	8·8059	19	11·1941	8·8067	19	11·1933	10·0009	0	9·9991	86 20
3 50	8·8251	18	11·1749	8·8261	18	11·1739	10·0010	0	9·9990	86 10
4 0	8·8436	18	11·1564	8·8446	18	11·1554	10·0011	0	9·9989	86 0
4 10	8·8613	17	11·1387	8·8624	17	11·1376	10·0012	0	9·9989	85 50
4 20	8·8783	16	11·1217	8·8795	16	11·1205	10·0011	0	9·9988	85 40
4 30	8·8946	16	11·1054	8·8960	16	11·1040	10·0013	0	9·9987	85 30
4 40	8·9104	15	11·0896	8·9118	15	11·0882	10·0014	0	9·9986	85 20
4 50	8·9256	15	11·0744	8·9272	15	11·0728	10·0015	0	9·9985	85 10
5 0	8·9403	14	11·0597	8·9420	14	11·0580	10·0017	0	9·9983	85 0
5 10	8·9545	14	11·0455	8·9563	14	11·0437	10·0018	0	9·9982	84 50
5 20	8·9682	13	11·0318	8·9701	13	11·0299	10·0019	0	9·9981	84 40
5 30	8·9816	13	11·0184	8·9836	13	11·0164	10·0020	0	9·9980	84 30
5 40	8·9945	13	11·0055	8·9966	13	11·0034	10·0021	0	9·9979	84 20
5 50	9·0070	12	10·9930	9·0093	12	10·9907	10·0023	0	9·9977	84 10
	Cosine.	Diff. for 1′.	Secant.	Cotan.	Diff. for 1′.	Tan.	Cosec.	Diff. for 1′.	Sine.	Angle.

Table XV.—*continued.*

Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff.	Cosec.	Tan.	Diff.	Cotan.	Secant.	Diff.	Cosine.	...
6° 0′	9·0192	12	10·9808	9·0216	12	10·9784	10·0024	0	9·9976	84° 0′
6 10	9·0311	12	10·9689	9·0336	12	10·9664	10·0025	0	9·9975	83 50
6 20	9·0426	11	10·9574	9·0453	11	10·9547	10·0027	0	9·9973	83 40
6 30	9·0539	11	10·9461	9·0567	11	10·9433	10·0028	0	9·9972	83 30
6 40	9·0648	11	10·9352	9·0678	11	10·9322	10·0029	0	9·9971	83 20
6 50	9·0755	10	10·9245	9·0786	11	10·9214	10·0031	0	9·9969	83 10
7 0	9·0859	10	10·9141	9·0891	10	10·9109	10·0032	0	9·9968	83 0
7 10	9·0961	10	10·9039	9·0995	10	10·9005	10·0034	0	9·9966	82 50
7 20	9·1060	10	10·8940	9·1096	10	10·8904	10·0036	0	9·9964	82 40
7 30	9·1157	10	10·8843	9·1194	10	10·8806	10·0037	0	9·9963	82 30
7 40	9·1252	9	10·8748	9·1291	9	10·8709	10·0039	0	9·9961	82 20
7 50	9·1345	9	10·8655	9·1385	9	10·8615	10·0041	0	9·9959	82 10
8 0	9·1436	9	10·8564	9·1478	9	10·8522	10·0042	0	9·9958	82 6
8 10	9·1525	9	10·8475	9·1569	9	10·8431	10·0044	0	9·9956	81 50
8 20	9·1612	9	10·8388	9·1658	9	10·8342	10·0046	0	9·9954	81 40
8 30	9·1697	83	10·8303	9·1745	85	10·8255	10·0048	2	9·9952	81 30
9 0	9·1943	78	10·8057	9·1997	80	10·8003	10·0054	2	9·9946	81 0
9 30	9·2176	74	10·7824	9·2236	76	10·7764	10·0060	2	9·9940	80 30
10 0	9·2397	70	10·7603	9·2463	72	10·7537	10·0066	2	9·9934	80 0
10 30	9·2606	67	10·7394	9·2680	69	10·7320	10·0073	2	9·9927	79 30
11 0	9·2806	64	10·7194	9·2887	66	10·7113	10·0081	2	9·9919	79 0
11 30	9·2997	61	10·7003	9·3085	63	10·6915	10·0088	3	9·9912	78 30
12 0	9·3179	58	10·6821	9·3275	61	10·6725	10·0096	3	9·9904	78 0
12 30	9·3353	56	10·6647	9·3458	59	10·6542	10·0104	3	9·9896	77 30
13 0	9·3521	54	10·6479	9·3634	57	10. 6366	10·0113	3	9·9887	77 0
13 30	9·3682	52	10·6318	9·3804	55	10·6196	10·0122	3	9·9878	76 30
14 0	9·3837	50	10·6163	9·3968	53	10·6032	10·0131	3	9·9869	76 0
14 30	9·3986	48	10·6014	9·4127	51	10·5873	10·0141	3	9·9859	75 30
15 0	9·4130	46	10·5870	9·4281	50	10·5719	10·0151	3	9·9849	75 0
15 30	9·4269	45	10·5731	9·4430	48	10·5570	10·0161	3	9·9839	74 30
16° 0′	9·4403	43	10·5597	9·4575	47	10·5425	10·0172	4	9·9828	74 0
17 0	9·4659	40	10·5341	0·4853	44	10·5147	10·0194	4	9·9806	73 0
18 0	9·4900	38	10·5100	9·5118	42	10·4882	10·0218	4	9·9782	72 0
19 0	9·5126	36	10·4874	9·5370	40	10·4630	10·0243	4	9·9757	71 0
20 0	9·5341	34	10·4659	9·5611	39	10·4389	10·0270	5	9·9730	70 0
21 0	9·5543	32	10·4457	9·5842	37	10·4158	10·0298	5	9·9702	69 0
22 0	9·5736	31	10·4264	9·6064	36	10·3936	10·0328	5	9·9672	68 0
23 0	9·5919	29	10·4081	9·6279	35	10·3721	10·0360	5	9·9640	67 0
24 0	9·6093	28	10·3907	9·6486	34	10·3514	10·0393	6	9·9607	66 0
25 0	9·6259	27	10·3741	9·6687	33	10·3313	10·0427	6	9·9573	65 0
26 0	9·6418	25	10·3582	9·6882	32	10·3118	10·0463	6	9·9537	64 0
27 0	9·6570	24	10·3430	9·7072	31	10·2928	10·0501	7	9·9499	63 0
28 0	9·6716	23	10·3284	9·7257	30	10·2743	10·0541	7	9·9459	62 0
29 0	9·6856	22	10·3144	9·7438	29	10·2562	10·0582	7	9·9418	61 0
30 0	9·6990	21	10·3010	9·7614	29	10·2386	10·0625	7	9·9375	60 0
31 0	9·7118	21	10·2882	9·7788	28	10·2212	10·0669	8	9·9331	59 0
32 0	9·7242	20	10·2758	9·7958	28	10·2042	10·0716	8	9·9284	58 0
33 0	9·7361	19	10·2639	9·8125	28	10·1875	10·0764	8	9·9236	57 0
34 0	9·7476	19	10·2524	9·8290	27	10·1710	10·0814	9	9·9186	56 0
35 0	9·7586	18	10·2414	9·8452	27	10·1548	10·0866	9	9·9134	55 0
36 0	9·7692	17	10·2308	9·8613	26	10·1387	10·0920	9	9·9080	54 0
37 0	9·7795	16	10·2205	9·8771	26	10·1229	10·0977	10	9·9023	53 0
38 0	9·7893	16	10·2107	9·8928	26	10·1072	10·1035	10	9·8965	52 0
39 0	9·7989	15	10·2011	9·9084	26	10·0916	10·1095	10	9·8905	51 0
40 0	9·8081	15	10·1919	9·9238	26	10·0762	10·1157	11	9·8843	50 0
41 0	9·8169	14	10·1831	9·9392	25	10·0608	10·1222	11	9·8778	49 0
42 0	9·8255	14	10·1745	9·9544	25	10·0456	10·1289	12	9·8711	48 0
43 0	9·8338	13	10·1662	9·9697	25	10·0303	10·1359	12	9·8641	47 0
44 0	9·8418	13	10·1582	9·9848	25	10·0152	10·1431	12	9·8569	46 0
45 0	9·8495	12	10·1505	10·0000	25	10·0000	10·1505	13	9·8495	45 0
...	Cosine.	Diff. for 10′.	Secant.	Cotan.	Diff. for 10′.	Tan.	Cosec.	Diff. for 10′.	Sine.	Angle.

TABLE XVI.

Conversion of Measures.

(Chiefly based on data contained in Col. Noble's Useful Tables.)

Length.

Metric to British.				*British to Metric.*					
Mètres.	Yards.	Feet.	Inches.	Yds.	Mètres.	Ft.	Mètres.	Ins.	Centi-mètres.
1	1·0936	3·2809	39·37	1	0·91438	1	0·30479	1	2·5400
2	2·1873	6·5618	78·74	2	1·82877	2	0·60959	2	5·0799
3	3·2809	9·8427	118·11	3	2·74315	3	0·91438	3	7·6199
4	4·3745	13·1236	157·48	4	3·65753	4	1·21918	4	10·1598
5	5·4682	16·4045	196·85	5	4·57192	5	1·52397	5	12·6998
6	6·5618	19·6854	236·22	6	5·48630	6	1·82877	6	15·2397
7	7·6554	22·9663	275·60	7	6·40068	7	2·13356	7	17·7797
8	8·7491	26·2472	314·97	8	7·31507	8	2·43836	8	20·3196
9	9·8427	29·5281	354·34	9	8·22945	9	2·74315	9	22·8596

Metric Table of Length.

Milli-mètres.
10 = 1 centimètre.
100 = 1 décimètre.
1000 = 1 mètre.
Mètres.
10 = 1 décamètre.
100 = 1 hectomètre.
1000 = 1 kilomètre.

Explanation.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of yards in 2354 mètres (see cols. 1 and 2)	of feet in 12·4 mètres (see cols. 1 and 3).	of inches in 30·5 centimètres (see cols. 1 and 4). Note, 1 m.=100 cm.	of mètres in 1026 yards (see cols. 5 and 6).	of mètres in 1742 feet (see cols. 7 and 8).	of centimètres in 17·72 ins. (see cols. 9 and 10).
mètres. yards.	mètres. feet.	cms. inches.	yards. mètres.	feet. mètres.	inches. cms.
2000=2187·3	10 =32·809	30·0=11·811	1000=914·38	1000=304·79	10 =25·400
300= 328·09	2 = 6·562	·5= 0·197	20= 18·29	700=213·36	7 =17·780
50= 54·68	0·4= 1·312		6= 5·49	40= 12·19	0·7 = 1·778
4= 4·37				2= 0·61	0·02= 0·051
∴ 2354=2574·44	∴ 12·4=40·683	∴ 30·5=12·008	∴ 1026=938·16	∴ 1742=530·95	∴ 17·72=45·009

Note.—If a table of conversion is not at hand, an approximation to the equivalent in inches of a distance measured in centimètres may be obtained by multiplying by 0·4: thus, 30·5 cm. multiply by 0·4, and we have 12·2 inches; the real equivalent as shown above is 12·008 inches.

Weight.

Metric to British.				*British to Metric.*							
Kilo-grammes.	Tons.	Pounds Avoir-dupois.	Grains Troy.	Tons.	Metric tons or milliers	Pounds Avoir-dupois.	Kilo-grammes	Grains Troy.	Grammes.		
1	·000984	2·2046	15432·3	1	1·016	1	0·4536	1	·0648		
2	·001968	4·4092	30864·7	2	2·032	2	0·9072	2	·1296		
3	·002953	6·6139	46297·0	3	3·048	3	1·3608	3	·1944		
4	·003937	8·8185	61729·4	4	4·064	4	1·8144	4	·2592		
5	·004921	11·0231	77161·7	5	5·080	5	2·2680	5	·3240		
6	·005905	13·2277	92594·1	6	6·096	6	2·7216	6	·3888		
7	·006889	15·4323	108026·4	7	7·112	7	3·1751	7	·4536		
8	·007874	17·6370	123458·8	8	8·128	8	3·6287	8	·5184		
9	008858	19·8416	138891·1	9	9·144	9	4·0823	9	·5832		

Metric Table of Weight.

Milli-grammes.
10 = 1 centigramme.
100 = 1 décigramme.
1000 = 1 gramme.
Grammes.
10 = 1 décagramme.
100 = 1 hectogramme.
1000 = 1 kilogramme.
Kilo-grammes.
10 = 1 myriagramme.
100 = 1 quintal.
1000 = 1 millier, or tonne, or metric ton.

Explanation.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of tons in 35 tonnes. (see cols. 1 and 2).	of pounds in 56·3 kgms. (see cols. 1 and 3).	of grains in 120 grammes (see cols. 1 and 4).	of tonnes in 38 tons. (see cols. 5 and 6).	of kilogrammes in 68 pounds. (see cols. 7 and 8).	of grammes in 85 grains. (see cols. 9 and 10)
tonnes. tons.	kgms. lbs.	grammes. grains.	tons. tonnes.	lbs. kgs.	grains. grammes
30 = 29·53	50 =110·231	100 = 1543·23	30 = 30·48	60 = 27·216	80 = 5·184
5 = 4·92	6 = 13·228	20 = 308·65	8 = 8·13	8 = 3·629	5 = 0·324
	0·3= 0·661				
∴ 35 = 34·45	∴ 56·3=124·120	∴ 120 = 1851·88	∴ 38 = 38·61	∴ 68 = 30·845	∴ 85 = 5·508

Note.—7000 grains troy = 1 pound avoirdupois.

Table XVI—*continued.*

Pressure.

Metric and Atmospheric to British.

Kilogrammes per sq. cm.	Pounds per square inch.	Tons per square inch.	Atmospheres.	Pounds per square inch.	Tons per square inch.
1	14·223	·00635	1	14·7	·00656
2	28·446	·01270	2	29·4	·01313
3	42·668	·01905	3	44·1	·01969
4	56·891	·02540	4	58·8	·02625
5	71·114	·03175	5	73·5	·03281
6	85·337	·03810	6	88·2	·03938
7	99·560	·04445	7	102·9	·04594
8	113·783	·05080	8	117·6	·05250
9	128·005	·05715	9	132·3	·05906

British to Metric and Atmospheric.

Pounds per square inch.	Kilogrammes per sq. cm.	Atmospheres.	Tons per square inch.	Kilogrammes per sq. cm.	Atmospheres.
1	·07031	·068	1	157·49	152·38
2	·14062	·136	2	314·99	304·76
3	·21093	·204	3	472·48	457·14
4	·28124	·272	4	629·97	609·52
5	·35155	·340	5	787·47	761·91
6	·42186	·408	6	944·96	914·29
7	·49217	·476	7	1102·45	1066·67
8	·56248	·544	8	1259·95	1219·05
9	·63279	·612	9	1417·44	1371·43

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of pounds per square inch in 32·1 kilogrammes per square centimètre (see cols. 1 and 2).

kgs. per sq. cm.		lbs. per sq. in.
30	=	426·68
2	=	28·45
0·1	=	1·42
∴ 32·1	=	456·55

of tons per square inch in 3210 kilogrammes per square centimètre (see cols. 1 and 3).

kgs. per sq. cm.		tons per sq. in.
3000	=	19·05
200	=	1·27
10	=	0·06
∴ 3210	=	20·38

of tons per square inch in 3254 atmospheres (see cols. 4 and 6).

atmospheres.		tons per sq. in.
3000	=	19·69
200	=	1·31
50	=	0·33
4	=	0·03
∴ 3254	=	21·36

of kilogrammes per square centimètre in 15 lbs. on the square inch (see cols. 7 and 8).

lbs. per sq. in.		kgs. per sq. cm.
10	=	0·7031
5	=	0·3516
∴ 15	=	1·0547

of kilogrammes per square centimètre in 18·3 tons per square inch (see cols. 10 and 11).

tons per sq. in.		kgs. per sq. cm.
10	=	1574·9
8	=	1259·95
0·3	=	47·25
∴ 18·3	=	2882·1

of atmospheres in 14·6 tons per square inch (see cols. 10 and 12).

tons per sq. in.		atmospheres.
10	=	1523·8
4	=	609·5
0·6	=	91·4
∴ 14·6	=	2224·7

Energy.

Metric to British.

Mètre-tons.	Foot-tons.
1	3·2291
2	6·4581
3	9·6872
4	12·9162
5	16·1453
6	19·3743
7	22·6034
8	25·8324
9	29·0615

British to Metric.

Foot-tons.	Mètre tons.
1	0·3097
2	0·6194
3	0·9291
4	1·2388
5	1·5484
6	1·8581
7	2·1678
8	2·4775
9	2·7872

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of foot-tons in 4367 mètre-tonnes (see cols. 1 and 2).

mètre-tonnes.		foot-tons.
4000	=	12916·2
300	=	968·72
60	=	193·74
7	=	22·60
∴ 4367	=	14101·26

of mètre-tonnes in 3592 foot-tons (see cols. 3 and 4).

foot-tons.		mètre-tonnes.
3000	=	929·1
500	=	154·84
90	=	27·87
2	=	0·62
∴ 3592	=	1112·43

NOTE.—1000 mètre-tons is called a dinamode in Italy.

TABLE XVII.

Double Entry for v, $\frac{R}{C}$, and a.

$\frac{R}{C}$. Velocity f/s.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.
500	03909	07920	12032	16250	20579	25022	29583	34266	39072	44010	49086	54300	59661	65175	70845	76689	82683	88864	95228																	
510	03757	07611	11555	15594	19760	24020	28399	32886	37526	42288	47118	52134	57275	62561	67995	73584	79332	85250	91345	97620																
520	03613	07318	11106	14990	18994	23085	27290	31595	36064	40645	45265	50086	55019	60090	65301	70659	76167	81837	87676	93689																
530	03477	07042	10684	14429	18276	22211	26252	30389	34682	39080	43522	48150	52886	57754	62755	67895	73177	78614	84213	89976																
540	03349	06782	10296	13903	17602	21392	25279	29261	33375	37594	41881	46322	50872	55548	60350	65285	70355	75572	80944	86472																
550	03229	06538	09929	13406	16968	20619	24363	28202	32141	36184	40334	44594	48969	53463	58078	62820	67690	72700	77858	83164																
560	03116	06309	09580	12933	16367	19886	23495	27202	30974	34847	38871	42961	47171	51493	55931	60491	65174	69989	74945	80042																
570	03008	06089	09246	12481	15793	19187	22668	26251	29871	33585	37482	41418	45471	49632	53904	58292	62798	67430	72193	77094																
580	02905	05879	08927	12049	15245	18520	21879	25339	28827	32401	36164	39959	43864	47872	51986	56213	60552	65011	69586	74309																
590	02807	05679	08623	11637	14723	17885	21130	24460	27838	31299	34913	38577	42343	46206	50172	54245	58428	62723	67139	71676																
600	02714	05489	08333	11245	14227	17281	20409	23616	26901	30270	33724	37267	40901	44628	48153	52382	56416	60557	64812	69183																
610	02626	05309	08057	10873	13757	16707	19720	22811	26007	29251	32594	36022	39531	43130	46822	50614	54505	58500	62604	66818																
620	02542	05137	07795	10520	13310	16163	19073	22060	25154	28284	31516	34835	38225	41702	45272	48929	52684	56540	60499	64565																
630	02461	04974	07547	10185	12885	15647	18464	21356	24342	27369	30492	33703	36980	40340	43794	47323	50919	54672	58495	62419																
640	02384	04819	07312	09868	12482	15157	17889	20691	23570	26506	29523	32625	35794	39043	42382	45792	49293	52893	56585	60375																
650	02311	04672	07090	09566	12100	14691	17343	20058	22838	25687	28606	31597	34663	37806	41028	44333	47721	51198	54766	58129	62190	66052	70021	74088	[illegible]	82594										
660	02242	04532	06881	09278	11739	14246	16821	19451	22143	24905	27736	30617	33585	36627	39750	42943	46221	49583	53033	56575	60222	63956	67791	71730	[illegible]	79154										
670	02175	04397	06680	09001	11390	13820	16319	18866	21475	24154	26903	29682	32557	35503	38529	41617	44700	48043	51381	54809	58341	61952	65690	69467	73392	77419										
680	02111	04268	06486	08736	11053	13411	15835	18304	20834	23433	26099	28791	31577	34431	37361	40352	43427	46576	49807	53125	56543	60067	63623	67304	[illegible]	74389										
690	02050	04145	06299	08482	10728	13018	15367	17764	20220	22740	25321	27989	30640	33407	36244	39146	42125	45175	48305	51518	54824	58207	61676	65239	68901	72066										
700	01992	04028	06119	08239	10415	12642	14917	17247	19632	22072	24570	27126	29746	32429	35176	37993	40882	43838	46871	49985	53181	56457	59816	63265	66893	70146										
710	01936	03917	05946	08006	10113	12275	14487	16752	19069	21430	23851	26348	28890	31494	34154	36891	39693	42560	45501	48519	51610	54784	58038	61879	64816	68331										
720	01883	03809	05779	07783	09829	11929	14079	16280	18530	20820	23168	25603	28071	30598	33177	35836	38555	41337	44189	47117	50108	53185	56338	59577	62905	66506										
730	01832	03705	05619	07569	09560	11601	13691	15830	18015	20241	22521	24888	27285	29739	32242	34824	37464	40164	42932	45772	48670	51655	54713	57853	61075	64870										
740	01783	03605	05466	07364	09304	11289	13321	15400	17523	19691	21910	24201	26530	28913	31346	33852	36414	39038	41725	44480	47295	50191	53157	56204	59625	62524										
750	01736	03509	05319	07168	09059	10991	12967	14988	17055	19167	21328	23539	25802	28118	30488	32917	35404	37954	40564	43237	45977	48788	51668	54625	57656	60767										
760	01691	03416	05178	06980	08823	10705	12626	14592	16611	18665	20768	22908	25110	27384	29665	32030	34447	36925	39462	42060	44714	47444	50241	53112	56068	59097										
770	01647	03327	05043	06798	08594	10428	12296	14210	16178	18177	20224	22301	24443	26637	28874	31174	33524	35932	38398	40924	43501	46153	48871	51660	54534	57477										
780	01604	03242	04913	06623	08373	10150	11978	13841	15756	17702	19694	21715	23800	25935	28114	30348	32634	34975	37373	39828	42336	44913	47555	50265	53053	55998										
790	01563	03160	04789	06455	08159	10898	11671	13486	15346	17241	19178	21151	23181	25259	27381	29553	31777	34054	36386	38773	41215	43720	46289	48923	51625	54395										
800	01524	03081	04670	06293	07952	09645	11374	13144	14950	16794	18678	20608	22585	24608	26675	28788	30952	33168	35436	37758	40134	42570	45069	47629	50251	52988	55095	58524	61425	64401	67458	70593	73815	77121	80517	84000
810	01486	03005	04556	06137	07752	09399	11088	12814	14570	16361	18198	20086	22012	23982	25994	28052	30159	32316	34523	36782	39094	41463	43894	46384	48930	51549	54218	56966	59783	62676	65648	68694	71818	75023	78310	81708
820	01449	02932	04445	05986	07560	09166	10814	12497	14207	15950	17741	19585	21462	23380	25341	27345	29397	31498	33646	35845	38095	40401	42766	45188	47663	50201	52803	55474	58212	61025	63915	66875	69907	73022	76223	79513
830	01414	02862	04338	05841	07377	08945	10551	12193	13860	15561	17307	19104	20934	22803	24714	26666	28666	30713	32805	34946	37137	39382	41684	44042	46460	48920	51450	54048	56711	59448	62258	65136	68082	71112	74223	77419
840	01381	02794	04235	05702	07202	08734	10300	11901	13529	15193	16896	18644	20427	22249	24112	26016	27966	29961	31998	34084	36219	38406	40647	42944	45291	47697	50150	52688	55281	57944	60677	63477	66343	69291	72316	75423
850	01349	02729	04136	05569	07035	08532	10030	11622	13213	14842	16504	18202	19942	21719	23536	25393	27295	29241	31227	33260	35341	37472	39656	41894	44186	46532	48931	51394	53921	56513	59172	61897	64690	67555	70498	73520
860	01318	02666	04041	05442	06875	08338	09830	11355	12910	14505	16128	17780	19478	21212	22985	24797	26654	28553	30490	32472	34502	36580	38709	40891	43135	45426	47765	50165	52631	55155	57743	60396	63122	65900	68765	71707
870	01288	02605	03949	05320	06721	08150	09609	11099	12619	14179	15765	17376	19034	20727	22458	24228	26041	27895	29786	31721	33702	35730	37807	39935	42130	44365	46646	48987	51393	53852	56374	58959	61618	64322	67114	69979
880	01259	02547	03861	05202	06572	07969	09396	10853	12340	13864	15416	16990	18611	20265	21956	23686	25457	27268	29115	31005	32940	34920	36947	39025	41170	43349	45575	47859	50205	52605	55064	57585	60178	62818	65540	68382
890	01231	02491	03776	05088	06428	07795	09191	10616	12072	13561	15080	16623	18207	19824	21477	23169	24930	26672	28477	30324	32215	34150	36130	38161	40251	42377	44551	46781	49070	51413	53813	56274	58802	61384	64030	66762
900	01204	02437	03694	04978	06289	07627	08994	10388	11813	13269	14756	16273	17822	19405	21022	22678	24371	26104	27871	29678	31527	33420	35356	37341	39372	41449	43574	45752	47987	50276	52621	55025	57490	60016	62608	65266
910	01178	02384	03615	04872	06154	07465	08804	10168	11562	12988	14444	15932	17452	19002	20584	22205	23862	25557	27288	29057	30866	32718	34612	36554	38530	40561	42643	44772	46956	49194	51487	53838	56242	58711	61242	63839
920	01153	02333	03538	04769	06024	07308	08620	09956	11321	12717	14144	15605	17094	18612	20162	21749	23373	25030	26726	28459	30230	32043	33895	35797	37726	39716	41750	43833	45968	48157	50401	52700	55048	57464	59939	62476
930	01129	02283	03464	04669	05899	07157	08442	09751	11089	12457	13856	15289	16748	18236	19755	21310	22901	24524	26185	27883	29619	31393	33208	35070	36957	38905	40896	42935	45024	47165	49361	51610	53907	56273	58692	61174
940	01105	02235	03392	04572	05779	07012	08270	09554	10866	12208	13579	14984	16414	17873	19362	20887	22446	24036	25664	27328	29029	30768	32547	34370	36223	38130	40080	42077	44123	46219	48368	50569	52819	55134	57500	59929
950	01082	02189	03322	04478	05663	06872	08105	09365	10652	11969	13313	14687	16091	17522	18983	20478	22007	23566	25162	26794	28462	30166	31910	33697	35523	37301	39302	41250	43264	45318	47422	49577	51783	54042	56358	58737

Table XVII—*continued.*

$\frac{R}{C}$. Velocity f/s.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.
960	01060	02144	03255	04387	05551	06737	07946	09183	10447	11740	13058	14404	15778	17183	18617	20084	21583	23112	24678	26279	27915
970	01038	02101	03190	04299	05441	06604	07790	09005	10247	11515	12809	14127	15476	16855	18263	19703	21174	22675	24211	25783	27388
980	01017	02059	03126	04214	05333	06474	07638	08831	10050	11295	12565	13858	15183	16538	17921	19335	20779	22253	23761	25303	26880
990	00997	02018	03063	04132	05228	06347	07490	08661	09857	11079	12326	13597	14900	16231	17590	18979	20397	21845	23326	24841	26389
1000	00977	01978	03002	04052	05125	06223	07345	08494	09667	10867	12092	13344	14625	15933	17269	18634	20027	21451	22906	24394	25915
1010	00958	01939	02943	03972	05024	06101	07203	08330	09480	10658	11863	13101	14359	15644	16958	18300	19669	21069	22499	23962	25456
1020	00940	01901	02886	03895	04927	05984	07067	08173	09302	10459	11643	12863	14099	15364	16656	17975	19321	20699	22104	23543	25013
1030	00922	01865	02831	03822	04835	05872	06936	08022	09132	10269	11433	12632	13846	15092	16363	17660	18984	20339	21722	23137	24582
1040	00905	01830	02779	03752	04748	05766	06811	07878	08970	10088	11232	12407	13601	14827	16077	17353	18656	19989	21351	22743	24164
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1060	00872	01765	02681	03620	04585	05571	06578	07607	08669	09750	10857	11990	13149	14332	15541	16778	18044	19335	20657	22006	23384
1070	00856	01733	02634	03558	04508	05479	06468	07481	08527	09592	10683	11797	12941	14107	15297	16517	17765	19038	20341	21671	23030
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1110	00797	01618	02464	03333	04224	05136	06074	07036	08019	09030	10061	11119	12201	13307	14437	15595	16780	17989	19226	20492	21785
1120	00783	01591	02424	03281	04158	05058	05985	06937	07906	08904	09922	10969	12037	13130	14247	15392	16562	17759	18981	20232	21511
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1140	00757	01540	02348	03182	04036	04914	05819	06747	07693	08667	09662	10685	11729	12799	13893	15012	16157	17329	18526	19751	21004
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1390	00512	01054	01622	02220	02847	03504	04190	04902	05644	06413	07206	08023	08864	09727	10618	11530	12462	13420	14409	15421	16458
1400	00505	01039	01600	02190	02809	03458	04136	04841	05575	06337	07123	07933	08767	09625	10508	11414	12340	13293	14274	15279	16307
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1430	00484	00998	01536	02104	02701	03325	03982	04665	05376	06116	06880	07670	08485	09325	10187	11073	11985	12921	13882	14863	15867
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1450	00472	00971	01496	02050	02632	03242	03883	04552	05249	05975	06727	07504	08305	09133	09982	10855	11755	12679	13627	14595	15588
1460	00465	00957	01476	02023	02598	03201	03835	04496	05187	05907	06653	07423	08218	09039	09882	10749	11642	12559	13501	14464	15452

$\frac{R}{C}$. Velocity f/s.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.
960	29587	31300	33062	34855	36688	38562	40481	42447	44461	46522	48634	50799	52989	55258	57588							
970	29028	30710	32444	34204	36003	37840	39723	41651	43626	45646	47717	49841	51978	54203	56186							
980	28490	30140	31842	33569	35334	37137	38986	40877	42514	44794	46825	48909	51007	53189	55427							
990	27971	29590	31256	32950	34682	36453	38268	40124	42024	43967	45959	48003	50074	52215	54410							
1000	27469	29059	30685	32347	34047	35788	37569	39392	41256	43164	45118	47123	49175	51277	53431							
1010	26984	28532	30128	31760	33428	35141	36889	38681	40509	42384	44302	46269	48309	50374	52488							
1020	26515	28028	29597	31200	32839	34522	36240	38001	39796	41639	43523	45455	47474	49503	51579							
1030	26060	27547	29091	30666	32278	33932	35621	37353	39117	40929	42781	44679	46666	48061	50701							
1040	25618	27088	28607	30158	31745	33371	35032	36735	38472	40254	42076	43942	45885	47846	49852							
1050	25189	26651	28146	29676	31239	32838	34478	36147	37861	39614	41408	43244	45127	47056	49028	51050	53123	55246	57421	59651	61939	64288
1060	24794	26235	27708	29220	30760	32333	33943	35591	37283	39008	40776	42584	44433	46333	48274	50263	52303	54394	56535	58730	60981	63293
1070	24420	25840	27293	28785	30303	31853	33439	35062	36733	38432	40175	41956	43777	45649	47561	49521	51530	53589	55699	57861	60078	62354
1080	24065	25466	26900	28371	29868	31396	32960	34561	36209	37885	39604	41360	43158	45003	46888	48820	50801	52831	54911	57042	59227	61470
1090	23728	25111	26527	27977	29455	30963	32506	34087	35712	37367	39063	40796	42572	44393	46253	48159	50113	52116	54167	56269	58424	60637
1100	23409	24775	26173	27603	29062	30553	32077	33640	35242	36878	38552	40264	42019	43816	45653	47535	49464	51441	53466	55541	57668	59851
1110	23106	24456	25837	27249	28691	30166	31673	33219	34799	36418	38070	39763	41496	43271	45086	46946	48852	50804	52805	54854	56955	59110
1120	22817	24152	25519	26912	28339	29798	31289	32818	34379	35980	37613	39287	41001	42755	44550	46390	48274	50203	52180	54206	56282	58412
1130	22543	23864	25216	26593	28005	29449	30925	32437	33981	35565	37180	38836	40533	42268	44043	45864	47727	49636	51590	53593	55646	57753
1140	22282	23590	24928	26291	27689	29119	30580	32076	33605	35172	36771	38410	40089	41805	43563	45365	47210	49098	51032	53014	55045	57129
1150	22033	23328	24653	26006	27391	28807	30254	31735	33250	34801	36386	38009	39668	41367	43108	44893	46720	48589	50503	52465	54476	56539
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1170	21566	22838	24139	25476	26836	28228	29649	31104	32592	34114	35674	37268	38885	40553	42263	44016	45810	47644	49522	51448	53422	55446
1180	21347	22608	23898	25222	26571	27951	29361	30804	32278	33787	35334	36915	38520	40174	41868	43606	45386	47204	49065	50974	52931	54987
1190	21135	22387	23666	24976	26314	27683	29082	30513	31974	33471	35005	36573	38170	39810	41490	43213	44978	46782	48627	50520	52460	54450
1200	20930	22172	23441	24738	26065	27423	28812	30231	31681	33166	34687	36242	37832	39460	41126	42834	44585	46375	48205	50082	52007	53981
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1220	20530	21752	23002	24279	25585	26923	28291	29688	31117	32580	34076	35606	37170	38778	40418	42099	43820	45584	47386	49237	51129	53071
1230	20333	21546	22786	24054	25350	26678	28036	29422	30841	32293	33778	35296	36849	38445	40073	41740	43448	45198	46988	48825	50702	52628
1240	20139	21343	22573	23832	25119	26437	27784	29160	30569	32011	33485	34992	36534	38117	39733	41387	43082	44819	46596	48419	50283	52194
1250	19948	21142	22363	23613	24891	26199	27536	28902	30301	31733	33197	34693	36225	37794	39390	41040	42722	44446	46211	48020	49871	51768
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1270	19572	20748	21951	23183	24443	25733	27050	28398	29778	31180	32635	34110	35619	37162	38748	40364	42022	43719	45462	47242	49068	50937
1280	19388	20555	21750	22973	24224	25505	26813	28152	29522	30923	32360	33824	35323	36855	38430	40035	41681	43366	45097	46864	48677	50533
1290	19206	20365	21552	22766	24009	25281	26580	27909	29270	30662	32088	33542	35031	36553	38117	39712	41347	43020	44738	46494	48294	50137
1300	19026	20178	21357	22563	23797	25060	26350	27670	29022	30405	31820	33264	34743	36257	37808	39395	41018	42680	44385	46131	47918	49748
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1330	18499	19630	20788	21971	23180	24415	25679	26975	28299	29656	31041	32457	33908	35396	36912	38472	40062	41693	43362	45075	46828	48621
1340	18329	19453	20603	21778	22980	24207	25463	26750	28066	29414	30790	32197	33640	35117	36625	38173	39754	41375	43033	44734	46475	48257
1350	18161	19278	20421	21588	22782	24002	25251	26529	27836	29175	30543	31942	33376	34843	36343	37879	39451	41061	42709	44398	46127	47899
1360	17996	19106	20242	21400	22586	23800	25042	26311	27610	28940	30300	31691	33116	34574	36066	37589	39152	40752	42390	44067	45784	47547
1370	17834	18936	20065	21215	22393	23600	24835	26096	27388	28709	30061	31444	32860	34308	35792	37304	38857	40448	42077	43743	45448	47201
1380	17674	18768	19890	21033	22203	23403	24631	25884	27168	28481	29825	31200	32607	34046	35521	37024	38568	40149	41769	43425	45119	46861
1390	17516	18603	19717	20853	22016	23209	24429	25675	26951	28257	29593	30959	32357	33788	35254	36740	38283	39855	41465	43112	44796	46527
1400	17360	18440	19547	20676	21832	23017	24229	25468	26737	28036	29364	30721	32110	33534	34990	36479	38003	39565	41166	42804	44480	46199
1410	17205	18279	19379	20501	21650	22827	24031	25264	26526	27818	29138	30486	31866	33284	34730	36213	37727	39279	40871	42501	44169	45877
1420	17052	18120	19213	20329	21471	22641	23837	25064	26318	27603	28915	30256	31628	33038	34475	35950	37455	38999	40581	42202	43862	45559
1430	16902	17963	19049	20160	21295	22458	23647	24867	26114	27391	28696	30030	31395	32796	34225	35691	37188	38723	40296	41908	43559	45246
1440	16755	17809	18888	19993	21122	22278	23460	24673	25913	27183	28480	29807	31165	32558	33979	35436	36925	38452	40016	41619	43260	44938
1450	16610	17658	18730	19828	20952	22101	23277	24482	25715	26978	28268	29587	30938	32323	33737	35185	36667	38185	39741	41334	42965	44634
1460	16468	17509	18575	19666	20784	21927	23097	24294	25520	26776	28060	29371	30716	32090	33498	34938	36413	37922	39470	41053	42674	44335

(9263)

R/C. Velocity f/s.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.
1470	00458	00944	01457	01997	02564	03161	03788	04441	05126	05839	06579	07343	0812	08945	09783	10644	11530	12440
1480	00452	00932	01438	01971	02531	03121	03741	04387	05065	05772	06505	07263	0805	08852	09685	10540	11419	12323
1490	00446	00920	01419	01945	02499	03082	03695	04334	05005	05705	06432	07183	0795	08760	09587	10436	11309	12207
1500	00440	00908	01401	01920	02468	03044	03649	04282	04946	05639	06359	07104	0787	08669	09490	10333	11200	12092
1510	00434	00896	01382	01896	02436	03005	03604	04231	04888	05573	06286	07024	0778	08578	09392	10229	11091	11978
1520	00428	00884	01364	01872	02405	02967	03559	04180	04830	05508	06214	06945	0770	08488	09295	10126	10983	11865
1530	00422	00872	01346	01848	02374	02930	03515	04129	04773	05444	06142	06867	0762	08398	09199	10024	10876	11752
1540	00416	00861	01329	01824	02344	02894	03472	04079	04716	05380	06071	06790	0753	08309	09104	09923	10769	11640
1550	00410	00850	01312	01800	02315	02858	03429	04030	04660	05317	06001	06714	0745	08221	09010	09823	10663	11528
1560	00405	00839	01295	01777	02286	02823	03387	03981	04604	05255	05933	06639	0737	08134	08917	09724	10558	11417
1570	00400	00828	01278	01755	02258	02788	03346	03933	04549	05194	05865	06565	0729	08048	08825	09625	10454	11306
1580	00395	00818	01262	01733	02230	02754	03305	03885	04495	05133	05798	06492	0721	07962	08734	09528	10351	11196
1590	00391	00808	01246	01711	02203	02720	03265	03838	04441	05073	05731	06419	0713	07877	08644	09432	10249	11087
1600	00387	00798	01231	01690	02176	02687	03225	03792	04388	05014	05665	06347	0705	07793	08554	09337	10147	10980
1610	00382	00788	01215	01669	02149	02654	03186	03746	04336	04956	05600	06276	0698	07710	08464	09243	10046	10874
1620	00377	00778	01200	01648	02122	02621	03147	03701	04285	04898	05536	06206	0690	07627	08375	09149	09946	10768
1630	00373	00768	01185	01628	02095	02589	03109	03657	04234	04841	05473	06137	0682	07545	08287	09056	09847	10663
1640	00369	00759	01171	01608	02069	02557	03071	03613	04184	04784	05411	06068	0675	07464	08201	08963	09748	10559
1650	00365	00750	01157	01588	02044	02526	03034	03570	04134	04728	05349	06000	0667	07384	08115	08871	09651	10456
1660	00360	00741	01143	01569	02019	02495	02997	03527	04085	04673	05288	05933	0660	07305	08030	08779	09555	10353
1670	00355	00732	01129	01550	01995	02465	02961	03485	04037	04619	05228	05867	0653	07227	07945	08688	09460	10252
1680	00351	00723	01115	01531	01971	02435	02926	03443	03990	04566	05169	05801	0646	07149	07861	08598	09365	10152
1690	00347	00715	01102	01513	01948	02406	02891	03402	03944	04514	05111	05736	0639	07072	07778	08510	09271	10053
1700	00343	00706	01089	01495	01925	02378	02857	03362	03898	04462	05053	05672	0632	06996	07697	08423	09178	09955
1710	00339	00698	01076	01478	01902	02350	02824	03324	03854	04411	04995	05609	0625	06921	07617	08338	09086	09858
1720	00335	00690	01064	01461	01879	02323	02791	03287	03810	04361	04938	05546	0618	06847	07538	08254	08995	09761
1730	00331	00682	01052	01444	01857	02296	02759	03250	03767	04311	04882	05484	0611	06774	07460	08170	08905	09665
1740	00327	00674	01040	01427	01835	02269	02727	03213	03725	04262	04827	05423	0604	06702	07382	08087	08815	09570
1750	00324	00666	01028	01410	01814	02243	02696	03177	03683	04214	04773	05363	0598	06631	07305	08004	08727	09477
1760	00320	00658	01016	01393	01793	02217	02665	03141	03641	04167	04721	05305	0591	06560	07228	07922	08640	09385
1770	00316	00650	01004	01377	01772	02191	02635	03105	03599	04120	04669	05247	0585	06490	07152	07841	08554	09294
1780	00312	00643	00993	01361	01752	02166	02605	03069	03558	04074	04618	05190	0579	06420	07077	07760	08468	09204
1790	00309	00636	00982	01346	01732	02141	02575	03034	03518	04028	04567	05134	0572	06351	07002	07680	08383	09114
1800	00306	00620	00971	01331	01713	02117	02546	02999	03479	03983	04517	05078	0566	06283	06924	07600	08299	09025
1810	00302	00623	00960	01316	01694	02094	02517	02965	03439	03939	04467	05023	0560	06215	06855	07521	08216	08936
1820	00299	00616	00950	01302	01678	02071	02489	02931	03400	03895	04418	04968	0554	06149	06784	07444	08134	08848
1830	00296	00609	00939	01288	01660	02048	02461	02898	03362	03852	04369	04914	0548	06084	06713	07368	08052	08761
1840	00293	00602	00928	01274	01642	02026	02434	02866	03324	03809	04321	04861	0542	06020	06643	07293	07971	08675
1850	00290	00596	00918	01261	01624	02005	02407	02834	03287	03767	04274	04808	0536	05957	06574	07219	07891	08590
1860	00286	00589	00908	01247	01606	01983	02381	02804	03252	03727	04228	04756	0531	05895	06506	07146	07812	08506
1870	00283	00582	00898	01233	01588	01961	02356	02774	03217	03687	04182	04705	0525	05834	06439	07074	07734	08423
1880	00280	00576	00888	01220	01570	01940	02331	02745	03183	03647	04137	04655	0519	05773	06373	07002	07657	08341
1890	00277	00570	00879	01207	01553	01918	02306	02716	03149	03608	04093	04605	0514	05713	06308	06931	07581	08259
1900	00274	00564	00870	01194	01536	01898	02281	02687	03116	03569	04049	04556	0509	05654	06243	06861	07506	08178
1910	00271	00558	00861	01181	01520	01878	02256	02657	03082	03531	04005	04507	0503	05595	06179	06792	07432	08097
1920	00268	00552	00852	01168	01504	01858	02232	02628	03049	03493	03962	04459	0498	05537	06116	06723	07359	08018
1930	00266	00546	00843	01156	01488	01838	02208	02599	03016	03456	03920	04412	0493	05480	06054	06655	07286	07940
1940	00263	00540	00834	01144	01472	01818	02185	02571	02984	03420	03879	04366	0488	05423	05992	06588	07214	07863
1950	00261	00534	00825	01133	01457	01799	02162	02544	02952	03384	03838	04320	0483	05367	05931	06522	07142	07787
1960	00258	00529	00817	01121	01442	01780	02140	02518	02922	03349	03799	04276	0478	05312	05870	06457	07071	07712
1970	00255	00524	00809	01100	01427	01762	02118	02492	02892	03314	03760	04232	0473	05257	05810	06392	07001	07637

R/C. Velocity f/s.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.
1470	13376	14334	15317	16327	17361	18421	19505	20617	21754	22918	24107	25326	26575	27853	29157	30495	31860	33262
1480	13252	14205	15182	16186	17214	18267	19345	20450	21581	22739	23921	25134	26376	27647	28944	30275	31633	33027
1490	13130	14077	15048	16045	17067	18114	19186	20284	21409	22561	23737	24943	26178	27442	28732	30056	31408	32794
1500	13009	13950	14915	15905	16921	17962	19027	20119	21238	22384	23554	24753	25981	27238	28522	29838	31184	32562
1510	12889	13823	14781	15765	16776	17810	18869	19955	21068	22208	23372	24564	25786	27036	28314	29621	30961	32331
1520	12770	13697	14648	15626	16631	17660	18713	19793	20900	22033	23191	24377	25592	26835	28107	29406	30739	32102
1530	12651	13572	14516	15489	16488	17511	18558	19632	20733	21859	23011	24191	25400	26636	27901	29193	30519	31875
1540	12533	13447	14386	15353	16346	17363	18404	19472	20567	21686	22832	24006	25209	26438	27696	28982	30301	31650
1550	12415	13324	14258	15218	16205	17216	18251	19313	20402	21515	22655	23822	25019	26241	27492	28773	30085	31427
1560	12298	13202	14131	15085	16066	17070	18100	19156	20239	21345	22480	23640	24830	26046	27290	28566	29871	31206
1570	12182	13081	14005	14953	15928	16925	17950	19000	20077	21177	22306	23459	24642	25852	27090	28360	29658	30986
1580	12067	12961	13879	14822	15790	16781	17801	18845	19916	21010	22133	23279	24456	25659	26891	28155	29447	30767
1590	11952	12841	13754	14691	15653	16639	17653	18691	19756	20844	21961	23101	24271	25468	26693	27951	29237	30549
1600	11838	12722	13630	14561	15517	16498	17506	18538	19596	20679	21789	22924	24087	25278	26497	27748	29028	30333
1610	11725	12604	13507	14432	15382	16358	17360	18385	19437	20514	21618	22748	23905	25090	26303	27546	28820	30118
1620	11613	12487	13385	14304	15248	16219	17215	18233	19280	20351	21448	22573	23724	24903	26110	27346	28613	29905
1630	11502	12371	13264	14177	15114	16081	17071	18083	19124	20189	21280	22399	23544	24717	25918	27147	28408	29694
1640	11392	12255	13143	14051	14982	15943	16927	17934	18969	20028	21113	22226	23366	24533	25727	26950	28204	29485
1650	11284	12141	13023	13926	14852	15806	16784	17786	18815	19868	20948	22055	23189	24350	25538	26755	28002	29277
1660	11177	12028	12904	13802	14723	15670	16642	17639	18663	19710	20785	21886	23013	24168	25350	26561	27801	29071
1670	11071	11916	12786	13678	14594	15535	16502	17493	18512	19553	20623	21718	22839	23988	25164	26369	27602	28866
1680	10966	11804	12668	13555	14466	15401	16363	17348	18362	19398	20462	21551	22666	23809	24979	26178	27405	28663
1690	10861	11694	12552	13433	14339	15269	16225	17205	18213	19244	20302	21385	22494	23631	24795	25989	27210	28461
1700	10757	11585	12437	13313	14213	15138	16088	17063	18065	19091	20143	21220	22324	23455	24613	25801	27016	28260
1710	10654	11477	12323	13194	14088	15009	15953	16922	17918	18939	19985	21057	22156	23281	24433	25615	26824	28060
1720	10552	11370	12210	13076	13964	14881	15819	16783	17773	18788	19828	20895	21989	23108	24254	25430	26633	27862
1730	10451	11264	12099	12959	13842	14754	15686	16645	17629	18638	19672	20734	21823	22936	24076	25246	26443	27666
1740	10351	11158	11989	12843	13721	14627	15554	16508	17486	18489	19518	20574	21658	22765	23900	25064	26255	27472
1750	10253	11054	11879	12728	13601	14501	15424	16372	17344	18342	19365	20416	21495	22596	23725	24883	26068	27280
1760	10156	10951	11770	12614	13482	14375	15294	16237	17204	18197	19214	20260	21333	22428	23551	24704	25883	27090
1770	10060	10848	11662	12501	13364	14250	15165	16102	17065	18053	19064	20105	21172	22261	23379	24526	25699	26901
1780	09964	10746	11555	12388	13247	14126	15036	15968	16926	17909	18915	19950	21012	22095	23208	24349	25517	26713
1790	09869	10645	11449	12276	13130	14004	14908	15835	16788	17766	18767	19796	20852	21931	23038	24173	25336	26526
1800	09774	10545	11343	12166	13014	13884	14781	15703	16651	17624	18620	19643	20693	21768	22869	23998	25156	26340
1810	09680	10445	11238	12057	12899	13765	14655	15572	16515	17483	18473	19491	20535	21606	22702	23825	24978	26155
1820	09587	10346	11135	11949	12785	13646	14531	15443	16381	17343	18328	19340	20379	21445	22536	23653	24801	25972
1830	09494	10249	11033	11842	12673	13528	14408	15315	16248	17204	18184	19191	20225	21285	22371	23483	24625	25790
1840	09403	10153	10932	11736	12562	13411	14286	15188	16116	17067	18042	19044	20072	21127	22207	23314	24450	25610
1850	09313	10058	10832	11631	12452	13295	14165	15062	15985	16932	17902	18898	19921	20970	22044	23146	24276	25431
1860	09224	09964	10733	11527	12343	13180	14046	14938	15855	16798	17763	18754	19772	20815	21883	22980	24103	25254
1870	09135	09871	10635	11424	12235	13066	13928	14815	15726	16664	17624	18611	19624	20661	21723	22815	23932	25078
1880	09047	09779	10538	11322	12127	12954	13811	14693	15598	16531	17486	18468	19476	20508	21565	22651	23762	24903
1890	08960	09688	10442	11220	12020	12843	13695	14572	15472	16399	17349	18326	19329	20356	21408	22488	23594	24729
1900	08874	09597	10347	11119	11914	12733	13579	14452	15347	16268	17213	18185	19183	20205	21252	22326	23427	24557
1910	08788	09507	10251	11018	11808	12624	13464	14332	15223	16138	17078	18045	19038	20055	21097	22165	23262	24386
1920	08703	09418	10156	10918	11703	12516	13350	14213	15100	16009	16944	17906	18894	19906	20943	22006	23099	24217
1930	08620	09329	10062	10819	11600	12408	13237	14095	14977	15881	16811	17768	18751	19758	20790	21849	22937	24049
1940	08538	09242	09970	10722	11498	12301	13126	13978	14855	15754	16680	17632	18609	19611	20639	21693	22776	23882
1950	08457	09156	09879	10626	11398	12195	13016	13862	14734	15629	16550	17497	18469	19466	20489	21539	22616	23717
1960	08377	09071	09789	10532	11299	12090	12907	13747	14614	15506	16422	17363	18330	19322	20340	21386	22458	23554
1970	08298	08987	09700	10439	11200	11986	12799	13633	14496	15384	16295	17230	18192	19179	20192	21234	22300	23392

R/C. Velocity f/s.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.	5200.	5300.
1470	34693	36161	37662	39202	[illegible]	42386	44039										
1480	34451	35911	37404	38935	[illegible]	42102	43746										
1490	34211	35662	37148	38670	[illegible]	41821	43456										
1500	33972	35415	36894	38407	[illegible]	41542	43169										
1510	33734	35169	36641	38145	39687	41266	42884										
1520	33498	34925	36389	37886	39421	40992	42601										
1530	33263	34683	36139	37629	39157	40720	42319										
1540	33030	34443	35891	37374	38894	40449	42039										
1550	32799	34205	35645	37121	38633	40180	41761										
1560	32569	33968	35401	36870	38374	39912	41485										
1570	32341	33733	35159	36620	38117	39646	41212										
1580	32115	33500	34918	36372	37861	39383	40941										
1590	31891	33268	34679	36126	37607	39122	40672										
1600	31669	33038	34442	35882	37355	38863	40405										
1610	31448	32810	34207	35639	37105	38606	40140										
1620	31228	32584	33974	35398	36857	38351	39877										
1630	31010	32360	33743	35159	36611	38097	39616										
1640	30794	32137	33513	34922	36367	37845	39357										
1650	30580	31916	33285	34687	36124	37595	39099	40639	42217	43835	45496	47198	48939	50724	52556	54434	56358
1660	30367	31697	33059	34454	35882	37346	38843	40377	41946	43558	45209	46900	48633	50411	52233	54103	56018
1670	30156	31480	32835	34223	35643	37099	38589	40117	41678	43282	44925	46606	48330	50100	51913	53774	55680
1680	29947	31264	32613	33993	35406	36854	38338	39858	41412	43008	44643	46315	48030	49792	51596	53448	55345
1690	29739	31050	32392	33765	35171	36612	38089	39601	41148	42736	44363	46027	47734	49487	51282	53124	55012
1700	29533	30838	32173	33539	34938	36372	37842	39346	40886	42466	44085	45742	47441	49184	50970	52803	54681
1710	29328	30627	31956	33315	34707	36135	37597	39093	40626	42198	43809	45460	47150	48883	50661	52484	54352
1720	29125	30417	31740	33092	34478	35899	37354	38842	40368	41933	43536	45179	46861	48585	50354	52168	54027
1730	28923	30209	31525	32871	34251	35665	37113	38593	40112	41670	43265	44900	46574	48290	50050	51855	53705
1740	28722	30002	31312	32651	34025	35433	36873	38346	39858	41408	42996	44623	46289	47997	49748	51545	53386
1750	28523	29796	31100	32433	33801	35203	36635	38102	39606	41148	42729	44348	46006	47706	49449	51237	53070
1760	28326	29592	30890	32217	33579	34975	36399	37860	39356	40890	42463	44075	45725	47417	49152	50932	52758
1770	28130	29390	30681	32002	33359	34748	36165	37620	39108	40635	42199	43804	45446	47131	48857	50629	52447
1780	27936	29189	30474	31789	33140	34522	35932	37381	38862	40382	41938	43535	45170	46847	48564	50328	52138
1790	27743	28990	30269	31578	32922	34297	35701	37143	38618	40131	41679	43268	44896	46565	48274	50029	51830
1800	27551	28792	30065	31369	32705	34073	35472	36906	38376	39882	41422	43003	44624	46285	47986	49732	51523
1810	27360	28596	29863	31161	32489	33850	35244	36671	38135	39635	41167	42740	44354	46007	47700	49437	51218
1820	27171	28402	29663	30954	32275	33630	35018	36438	37896	39389	40914	42480	44086	45731	47416	49144	50917
1830	26984	28209	29464	30748	32063	33412	34794	36207	37659	39144	40663	42222	43820	45457	47135	48854	50619
1840	26798	28017	29266	30544	31853	33196	34572	35978	37423	38901	40414	41966	43556	45186	46856	48567	50324
1850	26614	27827	29070	30343	31646	32982	34351	35752	37189	38660	40167	41712	43294	44917	46579	48283	50032
1860	26432	27639	28876	30144	31441	32770	34132	35528	36956	38420	39922	41460	43034	44650	46304	48002	49743
1870	26251	27452	28683	29946	31237	32560	33915	35305	36725	38183	39679	41210	42776	44385	46032	47722	49456
1880	26071	27267	28492	29749	31034	32351	33700	35084	36497	37948	39438	40961	42520	44122	45762	47444	49170
1890	25892	27083	28302	29553	30832	32144	33486	34864	36271	37715	39198	40714	42266	43861	45494	47168	48886
1900	25715	26900	28113	29358	30632	31938	33274	34645	36047	37484	38959	40468	42014	43602	45228	46894	48604
1910	25538	26719	27926	29165	30433	31734	33064	34428	35825	37255	38721	40224	41763	43344	44964	46622	48324
1920	25363	26539	27740	28973	30236	31531	32855	34212	35604	37028	38485	39982	41514	43088	44702	46352	48047
1930	25189	26360	27556	28783	30040	31329	32647	33998	35384	36802	38252	39742	41268	42835	44442	46085	47772
1940	25017	26182	27373	28595	29846	31128	32441	33786	35166	36578	38022	39505	41024	42585	44184	45820	47499
1950	24847	26005	27191	28408	29653	30929	32236	33576	34950	36356	37794	39270	40783	42337	43928	45557	47228
1960	24679	25830	27011	28223	29462	30732	32083	33368	34735	36135	37568	39038	40544	42091	43674	45296	46959
1970	24511	25657	26832	28039	29272	30536	31832	33161	34522	35916	37343	38807	40307	41846	43421	45036	46692

R / C	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.	520
Velocity f/s.																																																				
1980	00252	00519	00801	01098	01412	01744	02096	02467	02862	03280	03721	04189	04683	05203	05751	06328	06932	07563	08220	08903	09612	10346	11102	11883	12691	13521	14379	15262	16169	17098	18056	19038	20045	21083	22143	23230	24344	25485	26654	27856	29084	30342	31632	32956	34310	35698	37120	38578	40071	41602	43170	4477
1990	00250	00514	00793	01087	01398	01726	02074	02442	02833	03246	03683	04146	04635	05150	05693	06265	06864	07490	08143	08820	09524	10253	11005	11781	12584	13410	14263	15141	16043	16968	17921	18898	19900	20932	21987	23069	24178	25314	26478	27674	28897	30149	31434	32752	34100	35482	36898	38350	39837	41360	42921	4451
2000	00248	00509	00785	01076	01384	01708	02053	02418	02804	03213	03645	04104	04588	05098	05636	06203	06797	07418	08066	08738	09437	10161	10909	11680	12478	13300	14148	15021	15918	16839	17787	18759	19756	20782	21832	22909	24013	25144	26303	27493	28711	29957	31237	32549	33892	35267	36677	38123	39604	41119	42674	4420
2010	00246	00504	00777	01065	01370	01691	02032	02394	02775	03180	03607	04062	04541	05047	05580	06142	06731	07347	07990	08657	09351	10070	10814	11580	12373	13191	14034	14902	15794	16711	17654	18621	19613	20633	21678	22750	23849	24975	26129	27314	28527	29767	31042	32348	33685	35053	36457	37897	39371	40879	42428	4401
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2060	14320	15188	16081	16998	17943	18912	19904	20925	21973	23049	24149	25279	26436	27622	28837	30083	31361	32670	34008	35379	36791	38231
2070	14206	15069	15957	16870	17810	18774	19762	20778	21821	22802	23987	25112	26264	27444	28654	29895	31167	32471	33803	35168	36574	38008
2080	14093	14951	15834	16743	17678	18637	19621	20632	21670	22736	23826	24946	26093	27267	28472	29708	30974	32272	33599	34959	36358	37786
2090	13981	14834	15712	16617	17546	18501	19481	20487	21520	22581	23666	24781	25923	27092	28292	29522	30782	32074	33396	34751	36143	37566
2100	13869	14718	15591	16491	17415	18366	19342	20343	21371	22427	23507	24617	25754	26918	28113	29337	30591	31877	33194	34544	35929	37347
2110	13758	14603	15471	16366	17285	18232	19204	20200	21223	22274	23350	24454	25586	26745	27934	29153	30402	31681	32993	34338	35717	37129
2120	13648	14488	15352	16242	17156	18099	19066	20058	21076	22122	23193	24202	25419	26573	27757	28970	30214	31487	32793	34133	35506	36912
2130	13539	14374	15233	16119	17028	17967	18929	19916	20930	21971	23037	24131	25253	26402	27581	28788	30027	31294	32595	33929	35296	36696
2140	13431	14261	15115	15997	16901	17836	18793	19775	20785	21824	22882	23971	25088	26232	27406	28607	29841	31102	32398	33726	35087	36481
2150	13323	14149	14998	15876	16775	17706	18658	19635	20641	21672	22728	23812	24924	26063	27232	28428	29656	30911	32202	33524	34879	36267
2160	13216	14038	14882	15755	16650	17576	18524	19496	20497	21524	22575	23654	24761	25895	27059	28250	29472	30721	32007	33323	34672	36055
2170	13110	13927	14767	15635	16526	17447	18391	19358	20354	21377	22423	23497	24599	25728	26887	28073	29289	30533	31814	33124	34467	35844
2180	13005	13817	14653	15516	16403	17319	18259	19221	20212	21230	22272	23341	24438	25562	26716	27897	29107	30347	31622	32927	34264	35635
2190	12900	13708	14540	15398	16281	17192	18127	19085	20071	21084	22122	23186	24278	25397	26546	27722	28927	30162	31431	32731	34062	35427
2200	12796	13600	14428	15281	16160	17066	17996	18950	19931	20939	21973	23032	24118	25234	26377	27548	28748	29978	31241	32536	33861	35220
2210	12693	13493	14317	15165	16039	16941	17866	18816	19792	20795	21825	22879	23959	25072	26210	27376	28570	29795	31052	32342	33661	35014
2220	12591	13387	14207	15050	15919	16817	17737	18683	19654	20652	21678	22727	23802	24910	26044	27205	28393	29613	30865	32149	33462	34809
2230	12489	13282	14097	14936	15800	16694	17609	18551	19517	20510	21532	22576	23646	24749	25879	27035	28217	29433	30679	31957	33264	34606
2240	12388	13177	13988	14822	15682	16571	17482	18419	19381	20369	21387	22426	23491	24589	25714	26865	28042	29253	30494	31766	33068	34404
2250	12288	13073	13880	14709	15565	16449	17356	18288	19246	20229	21242	22277	23337	24430	25550	26696	27868	29074	30310	31576	32873	34203
2260	12180	12969	13772	14597	15448	16328	17230	18158	19112	20090	21098	22129	23184	24272	25387	26528	27695	28896	30127	31388	32680	34004
2270	12091	12866	13665	14486	15332	16208	17105	18029	18978	19952	20955	21982	23032	24115	25225	26361	27523	28719	29945	31201	32488	33807
2280	11993	12764	13558	14375	15217	16088	16981	17901	18845	19815	20813	21835	22881	23959	24064	26195	27352	28543	29764	31015	32297	33611
2290	11896	12662	13452	14265	15103	15969	16858	17773	18713	19679	20672	21689	22731	23804	24904	26030	27182	28368	29584	30830	32107	33416
2300	11800	12561	13346	14156	14990	15851	16736	17646	18582	19543	20532	21544	22582	23650	24745	25866	27014	28194	29405	30646	31919	33222
2310	11704	12461	13241	14048	14877	15734	16615	17520	18451	19408	20393	21400	22434	23496	24586	25703	26846	28021	29227	30463	31732	33029
2320	11609	12361	13137	13941	14765	15617	16494	17395	18321	19274	20254	21257	22287	23343	24428	25541	26679	27850	29050	30281	31546	32837
2330	11515	12262	13034	13834	14654	15501	16374	17271	18192	19141	20116	21115	22141	23191	24271	25380	26514	27680	28874	30100	31361	32646
2340	11422	12164	12932	13728	14544	15386	16255	17148	18064	19009	19979	20974	21995	23040	24116	25220	26350	27511	28700	29920	31177	32456
2350	11329	12067	12831	13623	14435	15272	16137	17026	17937	18878	19843	20834	21850	22890	23962	25061	26187	27343	28527	29741	30993	32267
2360	11237	11971	12731	13519	14327	15159	16020	16904	17811	18748	19708	20695	21706	22742	23809	24903	26025	27176	28355	29564	30810	32079
2370	11146	11876	12632	13416	14219	15047	15904	17783	17686	18619	19574	20557	21563	22595	23657	24746	25864	27010	28184	29388	30628	31892
2380	11056	11782	12534	13313	14112	14936	15788	16663	17562	18490	19441	20420	21421	22449	23506	24590	25704	26845	28014	29213	30447	31706
2390	10967	11680	12437	13211	14006	14826	15673	16544	17439	18362	19309	20283	21280	22304	23356	24436	25545	26681	27845	29039	30267	31522
2400	10879	11597	12341	13110	13901	14717	15559	16426	17317	18235	19178	20147	21140	22160	23207	24283	25387	26518	27677	28867	30088	31339
2410	10792	11505	12245	13010	13797	14608	15446	16309	17196	18108	19047	20012	21001	22017	23059	24131	25230	26356	27510	28696	29910	31157
2420	10705	11414	12150	12910	13693	14500	15334	16193	17075	17982	18917	19877	20863	21875	22912	23980	25073	26195	27344	28525	29733	30976
2430	10619	11323	12055	12811	13590	14393	15223	16078	16955	17857	18788	19743	20725	21734	22766	23829	24917	26035	27179	28355	29557	30796
2440	10533	11233	11961	12713	13488	14287	15112	15963	16836	17733	18660	19610	20588	21593	22621	23679	24762	25875	27015	28186	29382	30617
2450	10448	11144	11868	12616	13386	14181	15002	15840	16718	17610	18532	19478	20452	21453	22477	23530	24608	25716	26852	28018	29209	30439
2460	10364	11056	11775	12510	13285	14076	14893	15736	16601	17488	18405	19347	20317	21314	22334	23382	24455	25558	26689	27851	29037	30262
2470	10280	10968	11683	12423	13185	13972	14785	15624	16485	17367	18279	19217	20183	21176	22191	23235	24303	25401	26527	27685	28867	30086
2480	10197	10881	11592	12328	13086	13868	14677	15512	16369	17247	18154	19088	20050	21038	22049	23088	24152	25245	26367	27520	28698	29911

Velocity f/s. \ R/C.	5000.	5100.	5200.	5300.	5400.	5500.	5600.	5700.	5800.	5900.	6000.	6100.	6200.	6300.	6400.	6500.	6600.	6700.	6800.	6900.	7000.
1980	41602	43170	44778	46427																	
1990	41360	42921	44522	46164																	
2000	41119	42674	44268	45904	47584	49305	51066	52874	54729	56635	58593	60600	62661	64779	66952	69184	71480	73841	76265	78758	81323
2010	40879	42428	44016	45646	47318	49031	50784	52585	54432	56330	58279	60278	62328	64437	66600	68823	71109	73458	75873	78355	80907
2020	40641	42184	43766	45389	47053	48758	50504	52297	54137	56026	57966	59958	61997	64097	66250	68464	70749	73077	75483	77954	80493
2030	40405	41942	43518	45133	46790	48487	50225	52011	53844	55724	57655	59639	61669	63759	65902	68107	70373	72699	75095	77555	80082
2040	40171	41701	43271	44879	46528	48218	49948	51726	53552	55424	57346	59322	61343	63423	65557	67752	70008	72324	74709	77159	79674
2050	39939	41462	43025	44626	46268	47950	49673	51443	53262	55126	57039	59007	61019	63089	65214	67399	69645	71951	74325	76765	79268
2060	39708	41225	42780	44374	46009	47684	49400	51162	52973	54829	56734	58693	60697	62757	64873	67049	69285	71581	73944	76373	78865
2070	39478	40989	42537	44124	45752	47420	49129	50883	52686	54534	56431	58381	60377	62428	64535	66701	68927	71213	73566	75984	78464
2080	39250	40754	42295	43875	45496	47158	48860	50606	52401	54241	56130	58071	60059	62101	64199	66355	68572	70848	73190	75597	78066
2090	39024	40520	42054	43628	45242	46898	48593	50331	52118	53950	55831	57763	59743	61776	63865	66011	68219	70485	72816	75212	77670
2100	38799	40287	41814	43382	44990	46639	48327	50058	51837	53661	55534	57457	59429	61454	63533	65670	67868	70124	72444	74829	77277
2110	38575	40055	41575	43137	44739	46381	48063	49787	51558	53374	55239	57153	59117	61134	63204	65331	67519	69766	72075	74449	76887
2120	38352	39825	41338	42894	44490	46125	47801	49517	51281	53088	54946	56851	58806	60816	62876	64994	67172	69410	71708	74071	76500
2130	38130	39596	41103	42653	44242	45870	47540	49248	51005	52803	54654	56551	58497	60500	62550	64659	66827	69056	71344	73696	76115
2140	37910	39369	40870	42413	43996	45617	47280	48981	50731	52520	54364	56252	58190	60185	62226	64326	66484	68704	70982	73323	75732
2150	37691	39144	40639	42175	43752	45365	47022	48715	50458	52239	54076	55955	57885	59872	61904	63995	66148	68354	70622	72953	75351
2160	37473	38920	40409	41939	43509	45114	46765	48451	50186	51960	53789	55660	57582	59560	61584	63666	65804	68006	70264	72585	74972
2170	37256	38698	40181	41704	43268	44865	46509	48189	49916	51683	53504	55367	57281	59250	61266	63339	65468	67660	69908	72220	74596
2180	37041	38478	39955	41471	43028	44618	46254	47928	49647	51408	53220	55076	56982	58942	60950	63014	65134	67316	69555	71857	74222
2190	36827	38260	39730	41239	42789	44373	46001	47669	49380	51135	52938	54787	56685	58636	60636	62691	64802	66974	69204	71496	73850
2200	36615	38043	39507	41009	42551	44130	45749	47411	49115	50864	52658	54500	56390	58332	60324	62370	64472	66634	68855	71137	73480
2210	36404	37828	39285	40780	42314	43888	45499	47155	48852	50595	52380	54214	56097	58030	60014	62051	64144	66296	68507	70779	73113
2220	36195	37614	39064	40552	42079	43647	45251	46901	48590	50327	52104	53929	55805	57730	59706	61734	63818	65960	68162	70423	72748
2230	35987	37400	38844	40325	41845	43407	45004	46648	48330	50061	51829	53646	55515	57432	59400	61419	63494	65626	67819	70070	72385
2240	35780	37187	38625	40099	41613	43168	44759	46396	48072	49796	51556	53365	55226	57135	59096	61106	63172	65295	67478	69719	72025
2250	35574	36975	38407	39875	41383	42930	44515	46146	47815	49532	51284	53085	54939	56840	58793	60795	62852	64066	67139	69371	71667
2260	35369	36764	38190	39652	41154	42694	44273	45897	47559	49269	51014	52807	54653	56547	58492	60485	62534	64639	66802	69025	71311
2270	35165	36554	37974	39430	40926	42460	44033	45650	47305	49008	50745	52531	54369	56256	58192	60177	62218	64314	66468	68681	70958
2280	34962	36345	37759	39210	40700	42227	43794	45404	47053	48748	50478	52257	54087	55966	57894	59871	61904	63991	66136	68340	70607
2290	34760	36137	37546	38991	40475	41996	43557	45160	46803	48489	50213	51985	53807	55678	57598	59567	61592	63670	65806	68001	70258
2300	34559	35930	37334	38774	40252	41766	43321	44917	46554	48231	49950	51715	53529	55392	57303	59265	61281	63351	65478	67664	69911
2310	34359	35724	37123	38558	40030	41538	43087	44675	46306	47974	49689	51446	53253	55108	57010	58964	60972	63034	65153	67330	69567
2320	34161	35520	36913	38343	39809	41311	42854	44435	46059	47719	49429	51179	52979	54825	56719	58665	60665	62719	64830	66998	69225
2330	33964	35317	36704	38129	39590	41085	42622	44196	45813	47466	49170	50914	52707	54544	56430	58368	60360	62406	64509	66668	68885
2340	33768	35116	36497	37917	39372	40861	42392	43959	45569	47215	48913	50650	52436	54265	56143	58073	60056	62094	64189	66339	68547
2350	33573	34916	36291	37706	39155	40638	42163	43723	45326	46966	48657	50388	52166	53987	55858	57779	59754	61784	63871	66012	68210
2360	33379	34717	36087	37496	38939	40416	41935	43488	45084	46718	48402	50127	51897	53711	55574	57487	59454	61476	63554	65687	67875
2370	33187	34520	35885	37287	38724	40195	41708	43255	44844	46472	48149	49867	51630	53437	55292	57197	59156	61170	63239	65363	67542
2380	32997	34324	35684	37080	38511	39976	41482	43024	44606	46228	47898	49609	51365	53164	55012	56909	58860	60866	62926	65041	67211
2390	32808	34130	35484	36874	38299	39759	41258	42794	44369	45986	47648	49352	51101	52893	54734	56623	58566	60564	62615	64721	66882
2400	32621	33937	35286	36669	38088	39543	41035	42565	44134	45745	47399	49097	50839	52624	54457	56339	58274	60263	62306	64403	66555
2410	32435	33745	35089	36465	37878	39328	40814	42337	43901	45506	47152	48843	50578	52356	54182	56057	57984	59964	61998	64086	66230
2420	32249	33554	34892	36262	37670	39114	40594	42111	43669	45268	46906	48590	50318	52090	53908	55777	57695	59667	61692	63771	65907
2430	32064	33363	34696	36060	37463	38901	40376	41886	43438	45031	46662	48339	50060	51825	53636	55498	57408	59372	61388	63458	65586
2440	31880	33173	34501	35860	37257	38689	40159	41662	43209	44795	46419	48089	49803	51562	53366	55220	57122	59078	61086	63147	65267
2450	31697	32985	34307	35661	37052	38478	39943	41440	42981	44560	46178	47841	49548	51300	53097	54944	56838	58786	60785	62838	64950
2460	31515	32798	34114	35463	36848	38268	39728	41219	42754	44326	45938	47594	49294	51039	52829	54669	56555	58495	60486	62531	64635
2470	31334	32612	33923	35267	36645	38060	39514	40999	42528	44094	45700	47349	49042	50780	52563	54396	56274	58206	60189	62226	64322
2480	31154	32427	33733	35072	36444	37854	39301	40781	42303	43863	45463	47106	48792	50523	52299	54125	55995	57919	59894	61923	64011

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R/δ	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.
Velocity f/s.																									
2490	00160	00326	00501	00687	00881	01086	01301	01527	01768	02023	02293	02579	02878	03199	03538	03897	04280	04687	05114	05565	06045	06550	07081	07637	08220
2500	00159	00323	00497	00682	00874	01077	01290	01515	01753	02006	02273	02556	02853	03171	03507	03863	04242	04645	05068	05516	05992	06493	07019	07571	08150
2510	00157	00321	00493	00676	00866	01068	01279	01502	01738	01989	02253	02534	02829	03144	03477	03830	04204	04603	05023	05468	05940	06436	06958	07506	08081
2520	00156	00318	00489	00671	00859	01059	01269	01490	01724	01972	02234	02512	02804	03117	03447	03797	04167	04561	04978	05420	05888	06380	06898	07441	08012
2530	00155	00316	00485	00665	00852	01050	01258	01477	01710	01956	02215	02490	02780	03090	03418	03764	04130	04520	04934	05373	05837	06325	06838	07377	07944
2540	00153	00313	00481	00660	00845	01042	01248	01465	01695	01939	02197	02468	02756	03063	03389	03732	04094	04480	04890	05326	05786	06270	06779	07314	07877
2550	00152	00310	00477	00655	00838	01033	01238	01453	01681	01923	02179	02447	02733	03037	03360	03700	04058	04440	04847	05279	05735	06216	06721	07252	07810
2560	00150	00308	00473	00649	00832	01025	01228	01442	01668	01907	02160	02426	02709	03011	03331	03668	04022	04401	04805	05233	05685	06162	06663	07190	07744
2570	00149	00306	00470	00644	00825	01016	01218	01430	01654	01891	02142	02405	02686	02985	03303	03636	03987	04363	04763	05187	05635	06109	06606	07129	07679
2580	00148	00303	00466	00639	00818	01008	01208	01419	01641	01876	02124	02385	02663	02960	03275	03605	03953	04325	04722	05142	05586	06056	06549	07069	07614
2590	00147	00301	00463	00634	00812	01000	01198	01407	01628	01860	02106	02365	02641	02935	03247	03574	03919	04288	04681	05097	05538	06004	06493	07009	07550
2600	00146	00299	00459	00629	00806	00992	01189	01396	01614	01845	02089	02346	02619	02910	03219	03543	03886	04252	04641	05053	05490	05952	06438	06950	07487
2610	00144	00296	00456	00624	00799	00984	01179	01385	01601	01830	02072	02326	02597	02885	03191	03512	03853	04216	04602	05009	05443	05901	06383	06892	07425
2620	00143	00294	00452	00619	00793	00976	01169	01373	01588	01815	02055	02307	02576	02861	03163	03482	03821	04181	04563	04966	05397	05851	06329	06834	07363
2630	00142	00292	00449	00614	00787	00968	01160	01362	01576	01800	02038	02288	02555	02837	03135	03452	03789	04146	04525	04924	05351	05802	06275	06777	07302
2640	00141	00289	00445	00609	00781	00960	01151	01351	01564	01786	02021	02269	02534	02813	03108	03423	03757	04111	04487	04882	05306	05753	06222	06720	07241
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R/δ	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	510
Velocity f/s.																										
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R/C. Velocity f/s.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.
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Table XVII—*continued.*

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